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Beam-Beam Issues in KEKB
— Crossing Angle —

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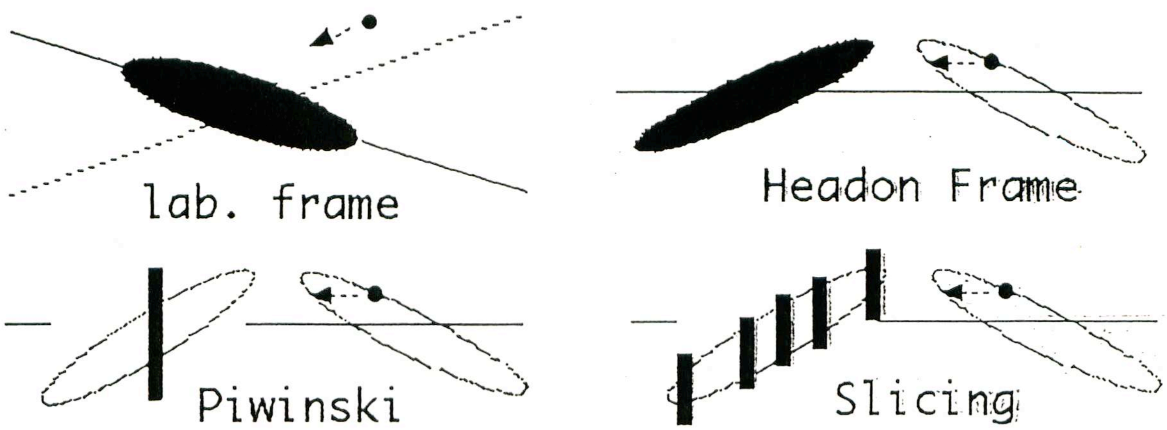
KEKB Accelerator Review (7-10 June 1995)

Contents

1 The Basis of the Simulation

The most detailed beam-beam kick

- Use Syncro-Beam mapping[1], which is symplectic in 6D phase space and x, y and z are treated on equal footing).
- Lorentz boost to make collision headon[2].
- Longitudinal slicing[3].



♡ at leat not worse than anything

♡ can be inserted to any 6D tracking code: can be put in without affecting otherthings in the Ring.

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[1] K. Hirata, H. Moshammer and F. Ruggiero, Part. Accel. **40**, 205 (1993).

[2] K. Hirata, PRL **74** 2228 (1995).

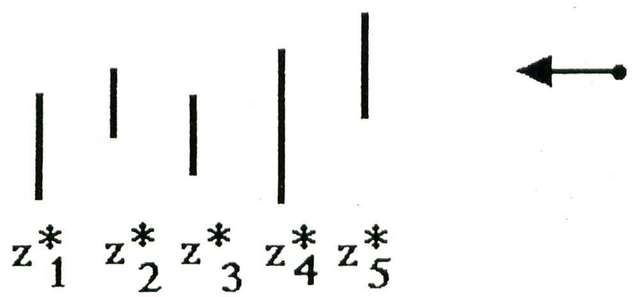
[3] S. Krishnagopal and R. Siemann, Phys. Rev. **D41**, 2312 (1990).

1.1 Synchro-Beam Mapping

The successive interaction of a particle with slices:

$$T_{z^*} \exp \int dz^* : F(\boldsymbol{x}, z^*) :\simeq \prod_i \exp : F(\boldsymbol{x}, z_i^*) :\simeq \exp : H_{bb}$$

$F(\boldsymbol{x}, z^*)$ describes the interaction with the particle and the slice having z^*.



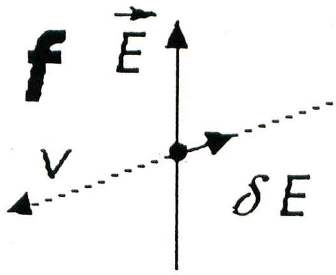
$$\boldsymbol{x}(0) \xrightarrow{Drift\ Back} \boldsymbol{x}(s^*) \xrightarrow{e^{:F:}} \boldsymbol{x}'(s^*) \xrightarrow{Drift} \boldsymbol{x}'(0),$$

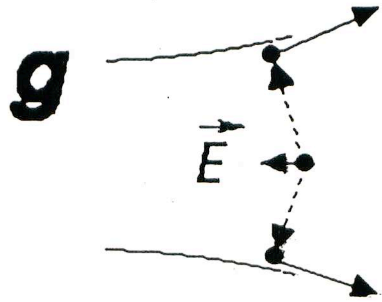
$$s^* \equiv S(z, z^*) = (z - z^*)/2.$$

$$\exp : F(\boldsymbol{x}, z^*) : \boldsymbol{x} \text{ is equivalent to}$$

$$\begin{aligned}
x^{new} &= x + S(z, z_*) f_X, \\
p_x^{new} &= p_x - f_X, \\
y^{new} &= y + S(z, z_*) f_Y, \\
p_y^{new} &= p_y - f_Y, \\
z^{new} &= z, \\
\epsilon^{new} &= \epsilon - \frac{1}{2} f_X [p_x - \frac{1}{2} f_X] - \frac{1}{2} f_Y [p_y - \frac{1}{2} f_Y] - g.
\end{aligned}$$

f: energy loss due to traverse of transverse E field with an angle
g: energy loss due to longitudinal field (important when $\beta \lesssim \sigma_z$)
$S = S(z, z^*) = (z - z^*)/2$: s of the collision between the particle having z and the slice having z^*. The hourglass effect is taken into account through $\sigma(S(z, z^*))$.





$$f_X \equiv \frac{n_* r_e}{\gamma_0} f_x(X, Y; \sigma_x(S), \sigma_y(S)),$$

$$f_Y \equiv \frac{n_* r_e}{\gamma_0} f_y(X, Y; \sigma_x(S), \sigma_y(S)),$$

$$g = g(X, Y; \sigma_x(S), \sigma_y(S))$$

1.2 Lorentz Boost $\mathcal{L}$

accelerator variables $\downarrow$ Cartesian cooridinates $\xrightarrow{\text{Lorentz Boost}}$ Cartesian cooridinates $\uparrow$ accelerator variables

$$
\begin{aligned}
x^* &= \tan\phi z + [1 + h_x^* \sin\phi] x \ , \\
y^* &= y + \sin\phi h_y^* x \ , \\
z^* &= z/\cos\phi + h_z^* \sin\phi x \ , \\
p_x^* &= (p_x - \tan\phi h)/\cos\phi \ , \\
p_y^* &= p_y/\cos\phi \ , \\
p_z^* &= p_z - \tan\phi p_x + \tan^2\phi h.
\end{aligned}
$$

Here $h_i^* = \partial h^*/\partial p_i^*$ and $h^* = h(p_x^*, p_y^*, p_z^*)$.

$$h(p_x, p_y, p_z) = p_z + 1 - \sqrt{(p_z+1)^2 - p_x^2 - p_y^2}.$$

This is nonlinear, because the drift is nonlinear

1.3 Arc $\mathcal{A}$

Linear Lattice with Synchrotron Radiation abstracted from SAD

$$\boldsymbol{x} \longrightarrow M\boldsymbol{x} + \boldsymbol{d},$$

such that $D \equiv < d_i d_j >$ is the one-turn diffusion matrix:

$$\Sigma \longrightarrow M\Sigma M^t + D.$$

Nonlinear Lattice with Synchrotron Radiation Put the beam-beam part into a tracking code (SAD).

2 Machine Parameters

1. Find the "almost optimum" set of parameters ($\nu_{x,y}$, $\varepsilon_{x,y}$, $\beta^*_{x,y}$, σ_z etc) using the weak-strong simulation with nominal linear lattice

   (a) tune survey

      i. Luminosity

      ii. maximum amplitudes (tail?)

      iii. self-crabbing

   (b) iteration between optics people

2. Check the "optimum" parameters by the weak-strong simulation with nominal linear lattice

   (a) nonlinear lattice with errors and corrections

   (b) quasi-strong-strong

   (c) tail with linear lattice

   (d) transient effects (injection)

   (e) strong-strong with linear lattice

   (f) strong-strong with nonlinear lattice

   (g) tail with nonlinear lattice

2.1 Find Machine Parameters and Operation Point

- with Linear Lattice (Weak-Strong).
- It should be consistent with lattice design:
  - dynamic apperture
  - emittance controle

Boundary conditions:

- $N \leq 1.4 \times 10^{10}$ for LER
- emittance coupling ration should be ~ 2 % (or larger)
- geometrical luminosity is $1.16425 \times 10^{34} \mathrm{cm}^{-2}\mathrm{s}^{-1}$ (a little more than $1 \times 10^{34} \mathrm{cm}^{-2}\mathrm{s}^{-1}$).
- geometrical ξ_y should be $\simeq 0.05$
- should be consistent with lattice and apperture issues

After some iterations with Optics people

$\beta_{x,y}$	(0.33. 0.008)m
$\epsilon_{x,y}$	$1.74 \times 10^{-8}, 3.44 \times 10^{-10}$m
σ_z	4 mm
ϕ	11 mrad
tunes	(0.52, 0.08) 0.017)
geometrical luminosity	$1.16425 \times 10^{34} \mathrm{cm}^{-2}\mathrm{s}^{-1}$
geometrical ξ	(0.04,0.052)

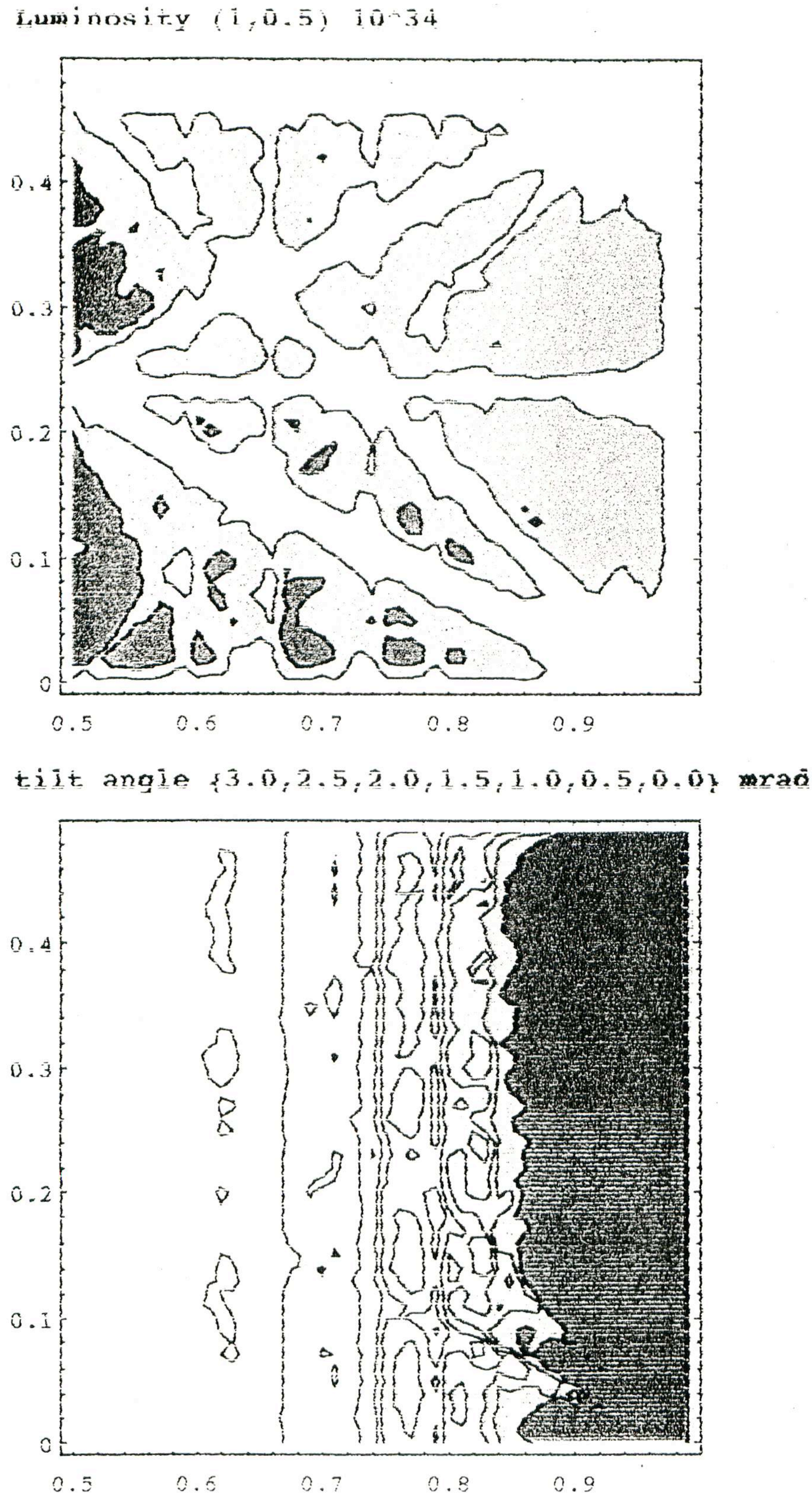


Figure 1: Luminosity and longitudinal tilt as functions of (ν_x, ν_y).

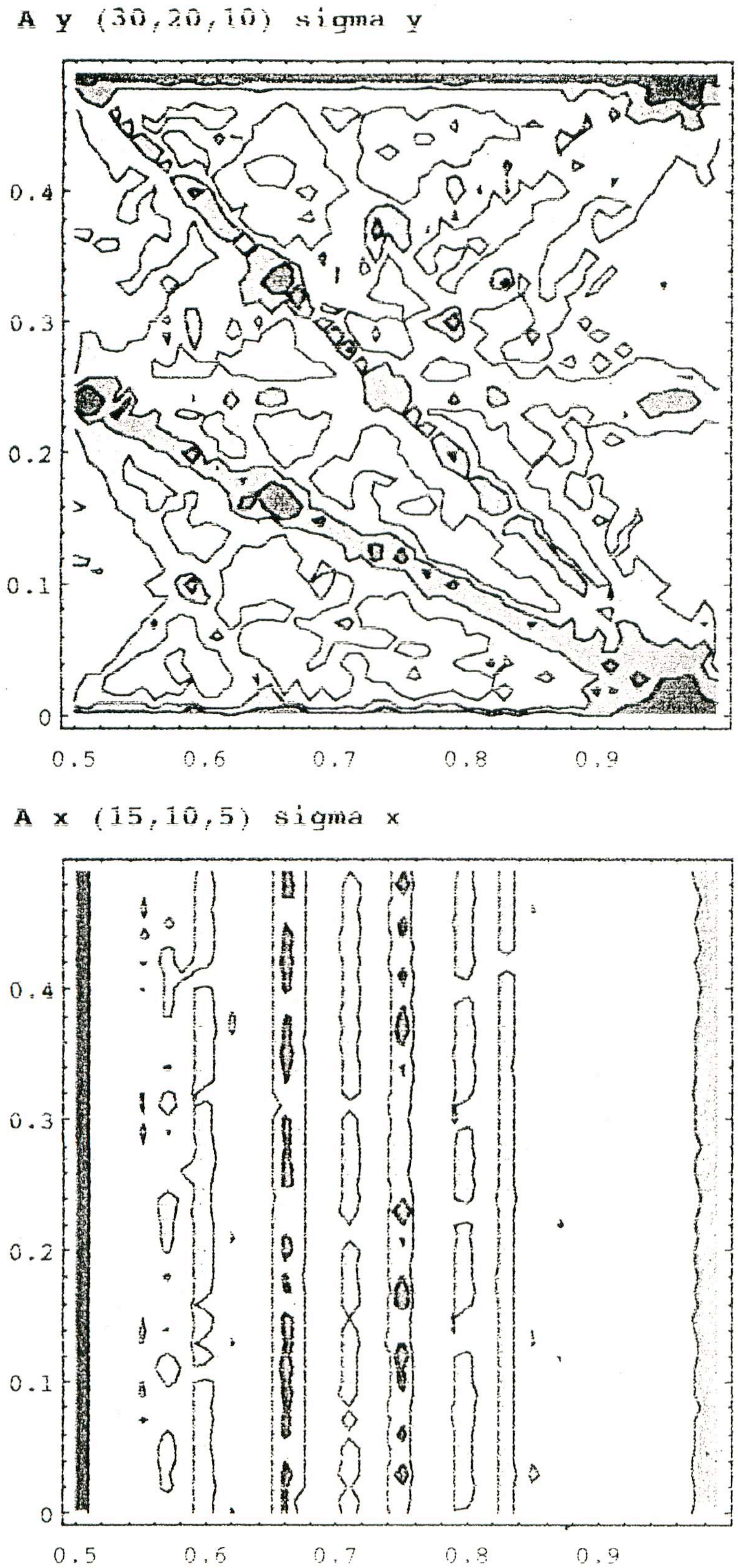


Figure 2: A_y and A_x as functions of (ν_x, ν_y).

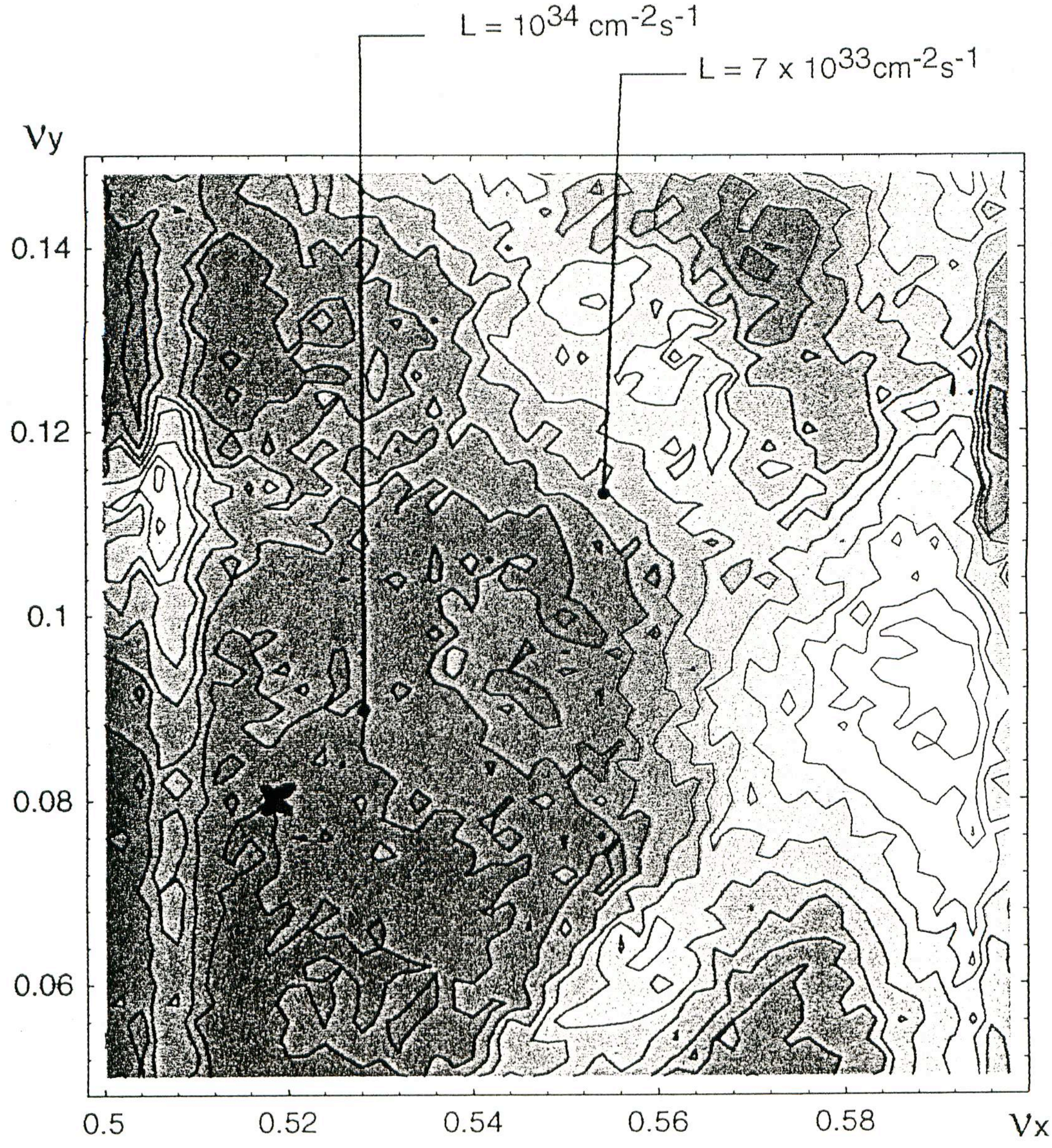


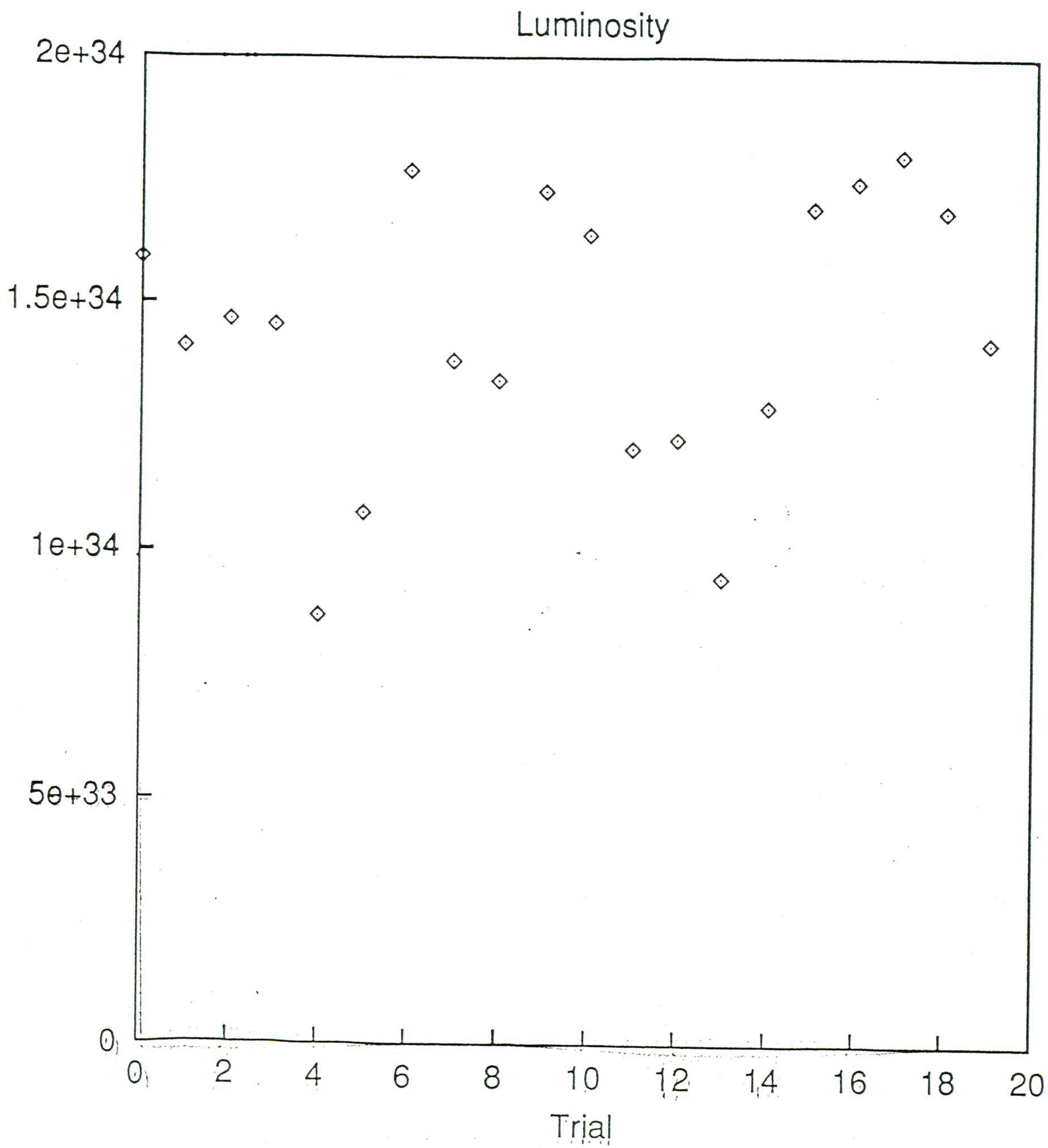
Figure 3: Luminosity evaluated as a function of (ν_x, ν_y).

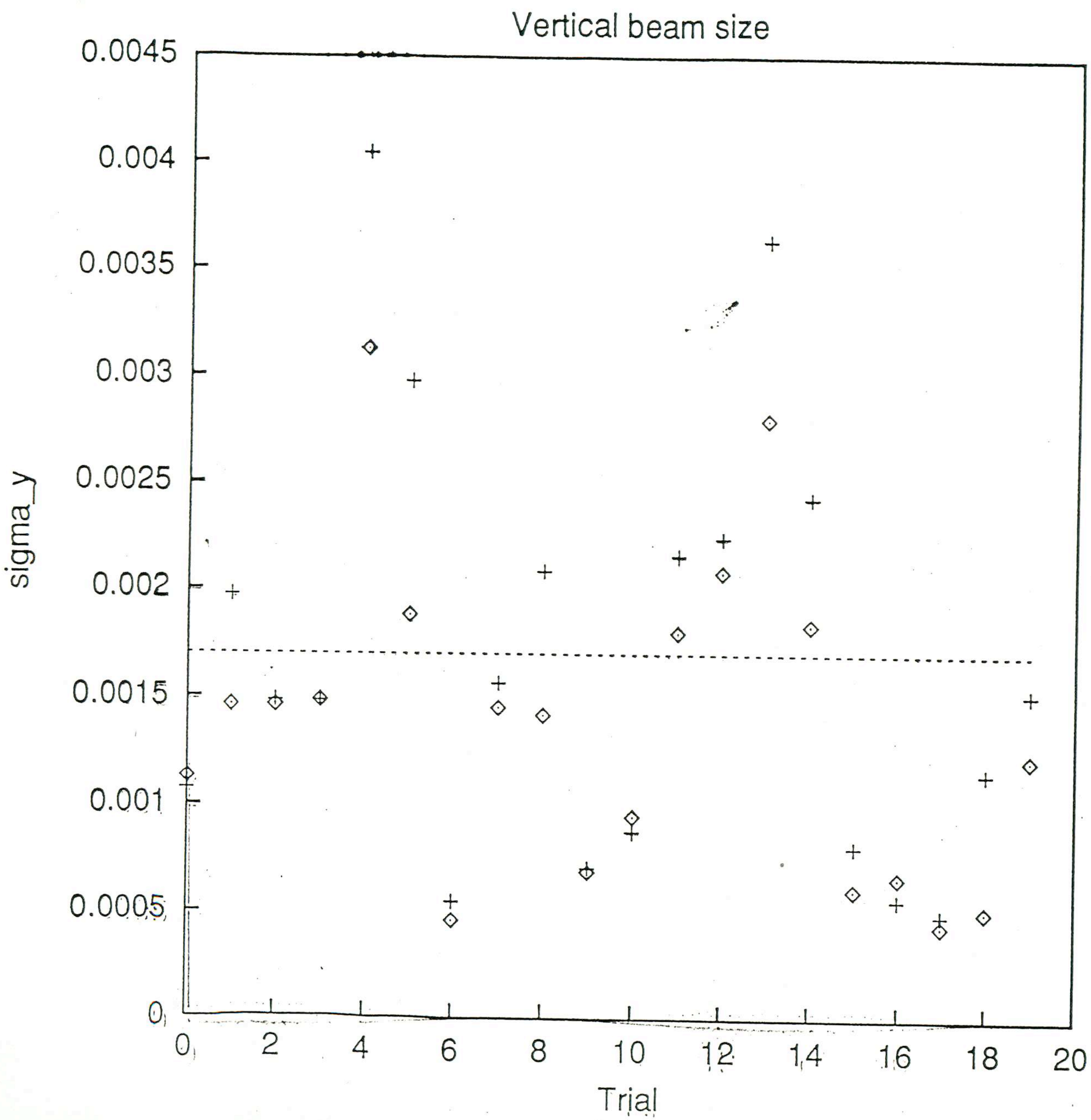
2.2 Check the parameter set by SAD

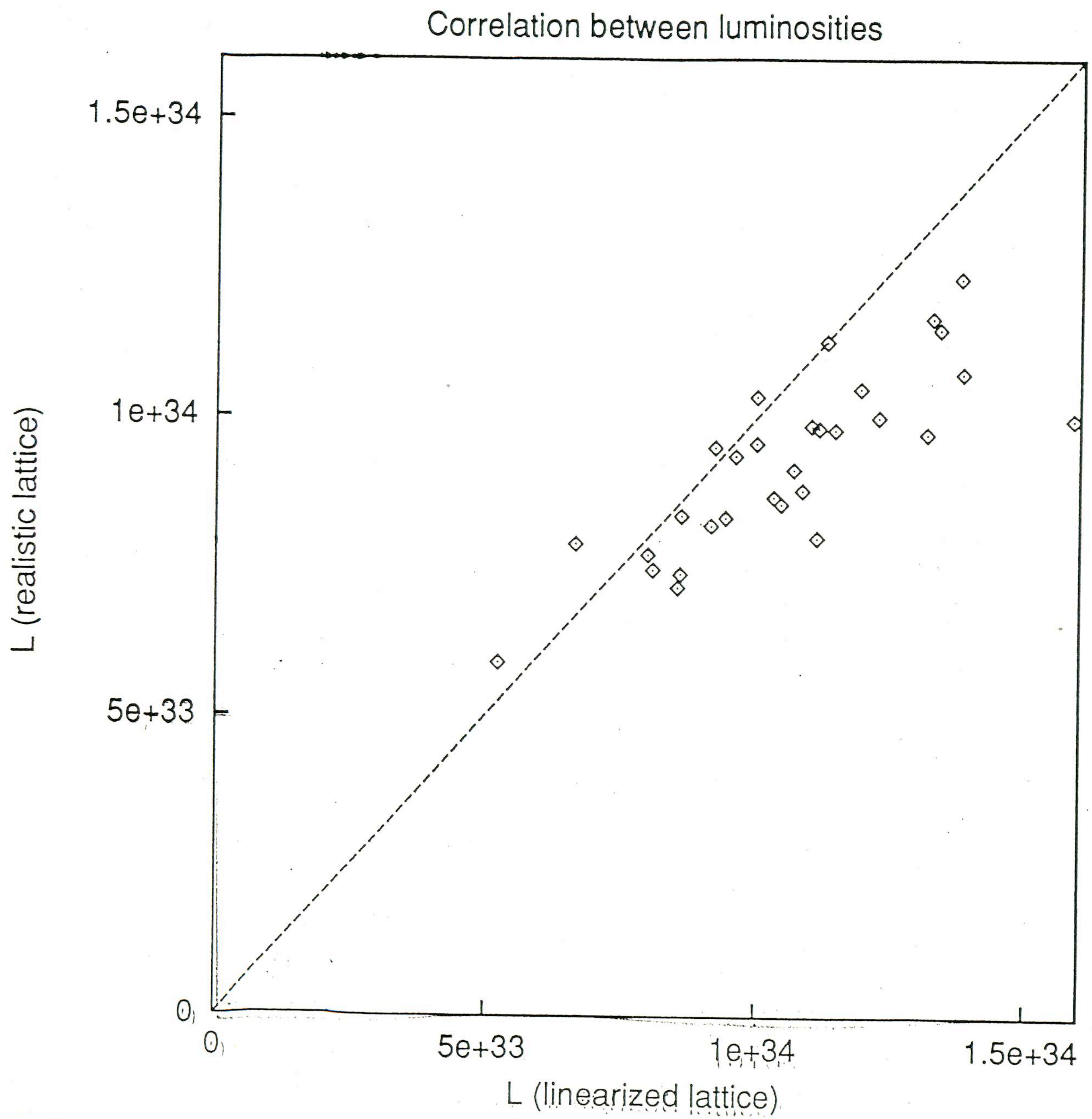
1. nonlinear lattice with errors and corrections
2. quasi-strong-strong
3. tail with linear lattice
4. transient effects (injection)

Nonlinear Lattice with Errors and Corrections When the nominal σ_y is large, the L is reduced.

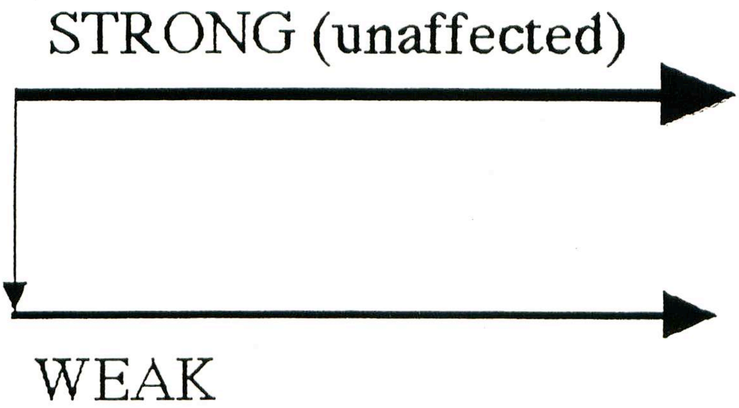
Figs.

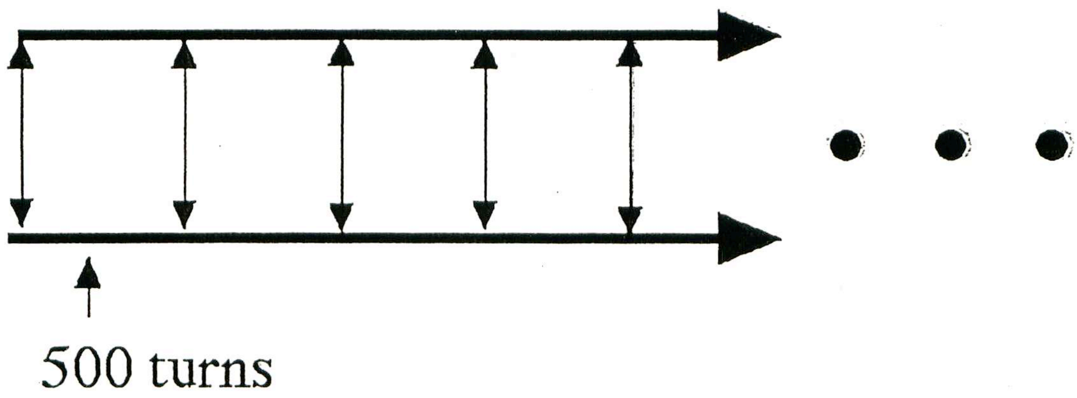

Luminosity
2e+34
1.5e+34
1e+34
5e+33
0
0
2
4
6
8
10
12
14
16
18
20
Trial


Vertical beam size
sigma_y
0.0045
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
Trial
0
2
4
6
8
10
12
14
16
18
20


Correlation between luminosities
L (realistic lattice)
1.5e+34
1e+34
5e+33
0
0
5e+33
1e+34
1.5e+34
L (linearized lattice)

Quasi Strong-Strong simulation Issue is whether T_d should be equalized etc. So-called strong-strong simulation is still difficult.





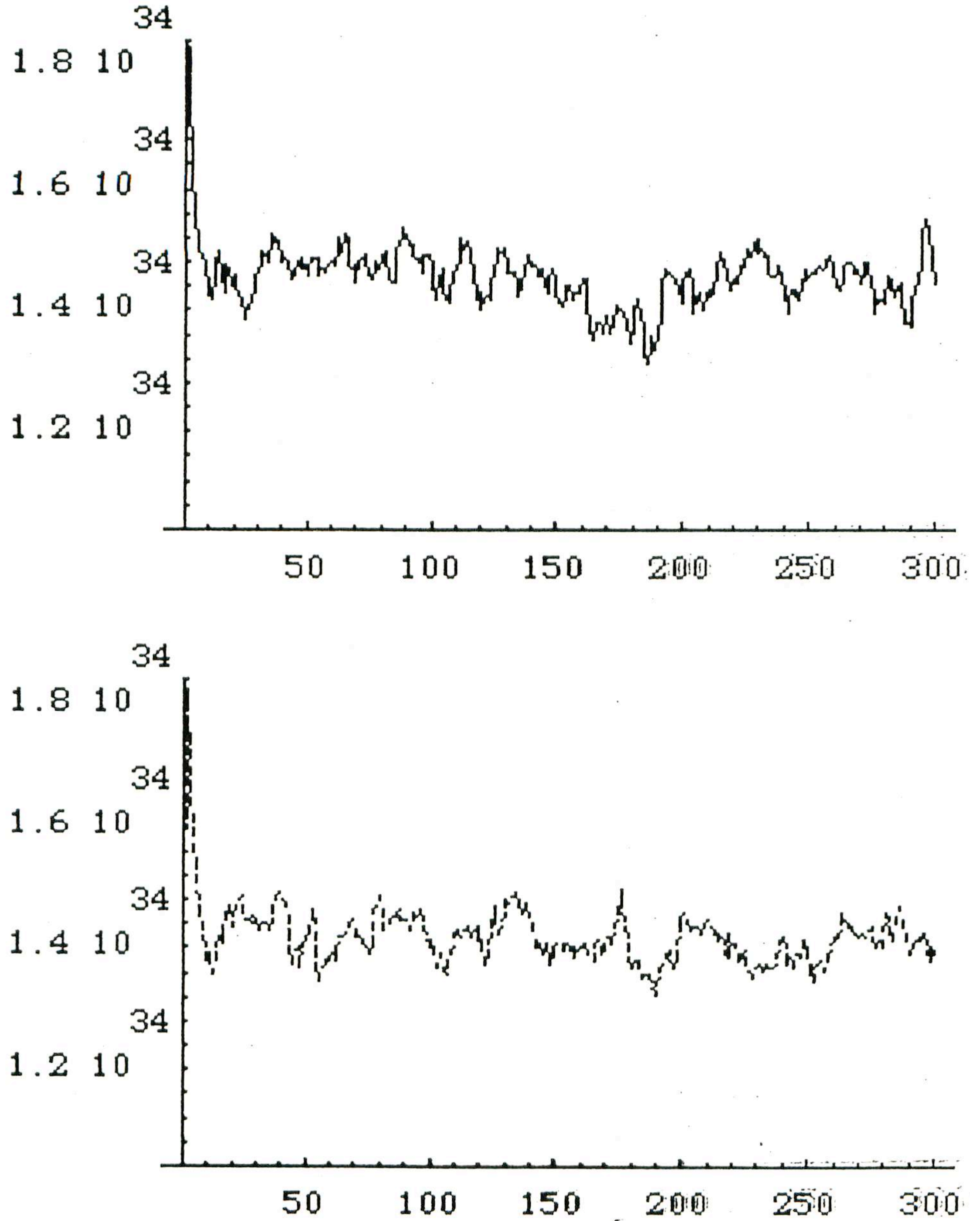


Figure 4: Luminosities for the case with eqaual damping time (ABOVE) and unequal damping time (BELOW).

The non-Gaussian tail will appear in the weak beam (the beam the damping time of which is longer).

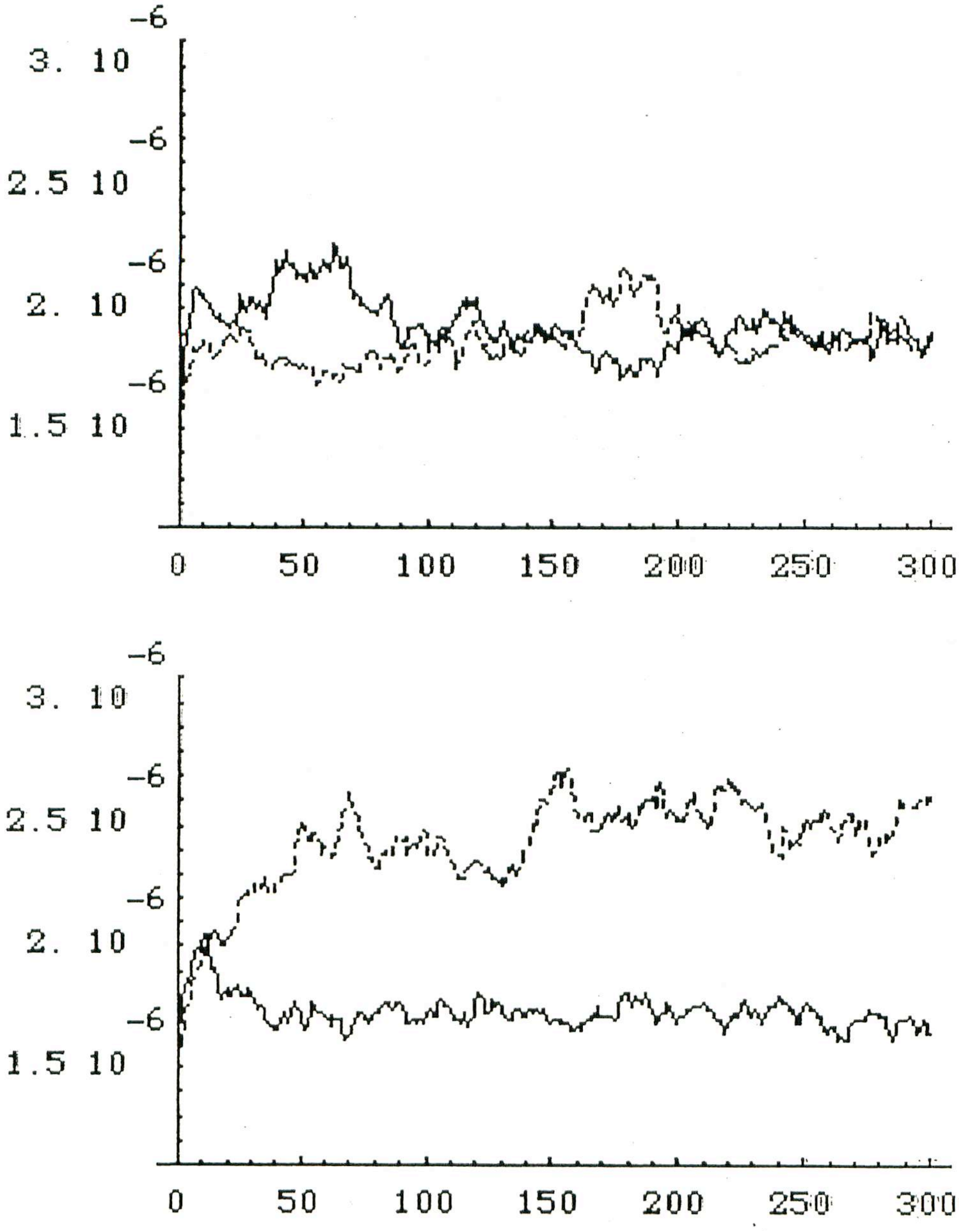


Figure 5: σ_z for the case with eqaual damping time (ABOVE) and unequal damping time (BELOW).

2.3 Beam-Beam Tail

Maximum Amplitude

- The maximum amplitude might indicate the amount of the tail formation
- but no proof for it
- no simple and reliable way

Long term tracking

tracking 50 super particles $\times$ 10^8 turns of revolution.

$$\simeq \begin{cases} \text{1000 seconds for 50 particles} \\ \text{14 hours for a single particle} \end{cases}$$

$$I_x = \frac{1}{2}\left[x^2/(\sigma_x)^2 + p_x^2/(\sigma_{p_x})^2\right]$$

$$I_y = \frac{1}{2}\left[y^2/(\sigma_y)^2 + p_y^2/(\sigma_{p_y})^2\right].$$

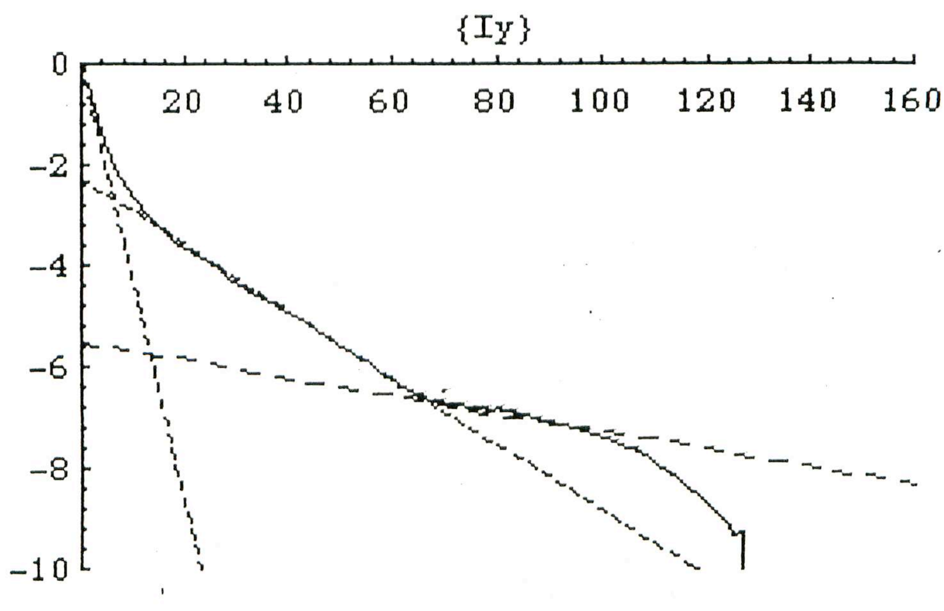


Figure 6: $\log_{10}(\rho(I_y))$ for the ideal linear lattice. $\int I_y \rho(I_y) = 1$.

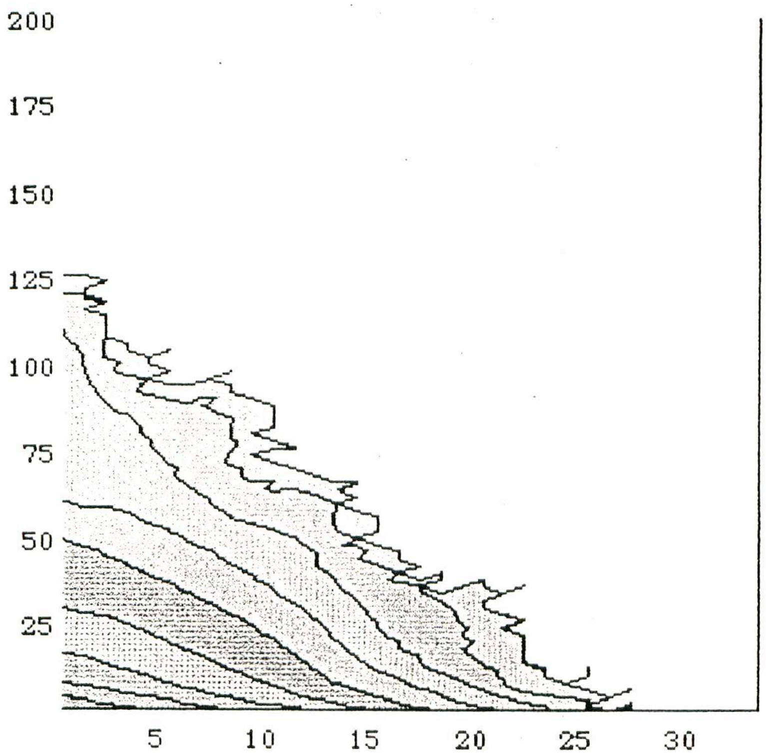
Non-gaussian tails in the particle distribution is seen. In this case the function $\rho(I_y)$ can be fitted with a sum of

$$\rho(I_y) \lesssim e^{-I_y} + 5.2 \times 10^{-3} e^{-0.15 I_y} + 3 \times 10^{-6} e^{-0.04 I_y}. \qquad (1)$$

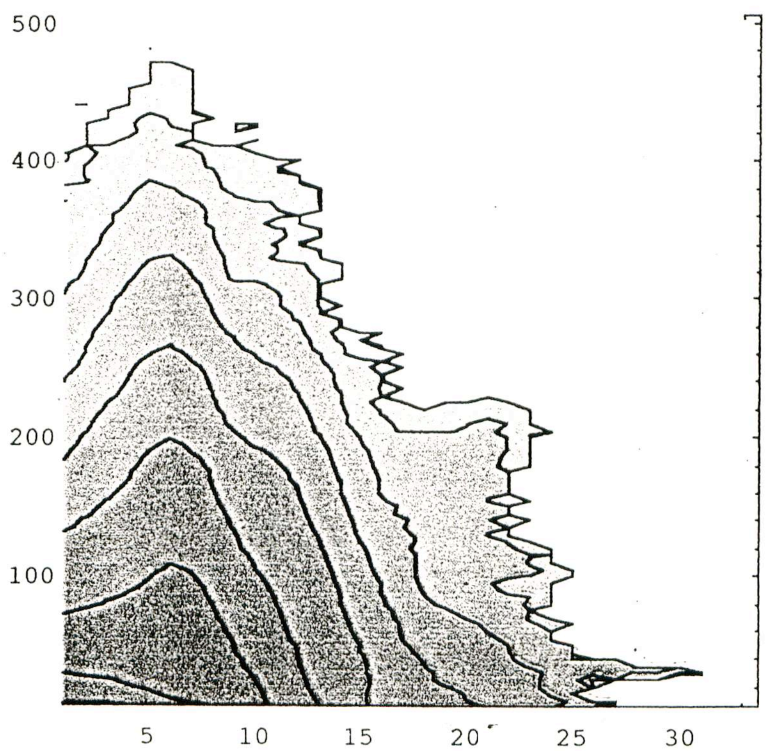
The probability of finding a particle whose amplitude A_y exceeds 30 is $\sim 10^{-12}$. calculated as

21

(0.52,0.08)

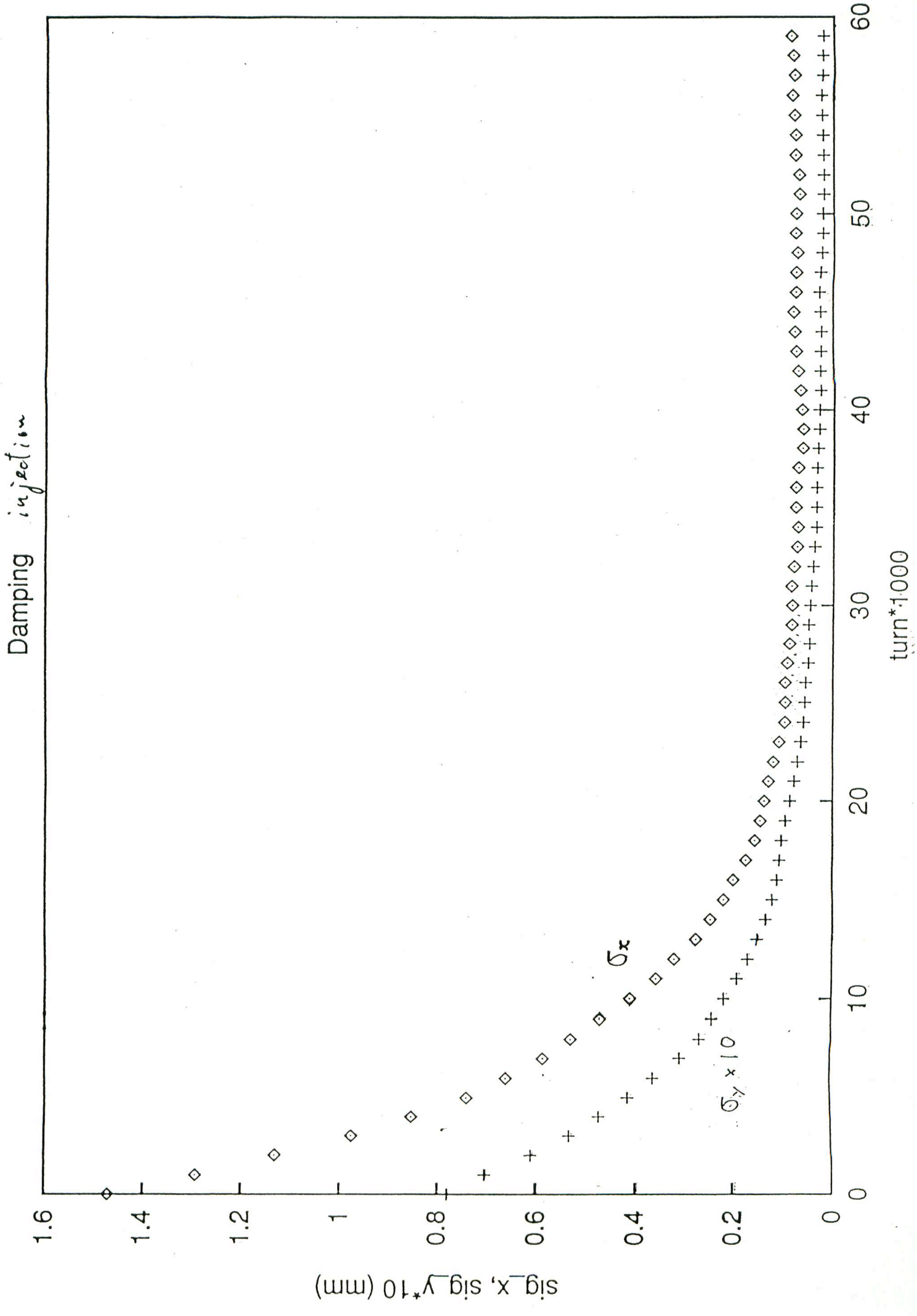


(0.64,0.2)



← Ay max is 15.4 σ_y

Figure 7:


Damping injection
turn*1000
sig_x, sig_y*10 (mm)
0
10
20
30
40
50
60
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
σ_x
$\sigma_y \times 10$

3 Discussions

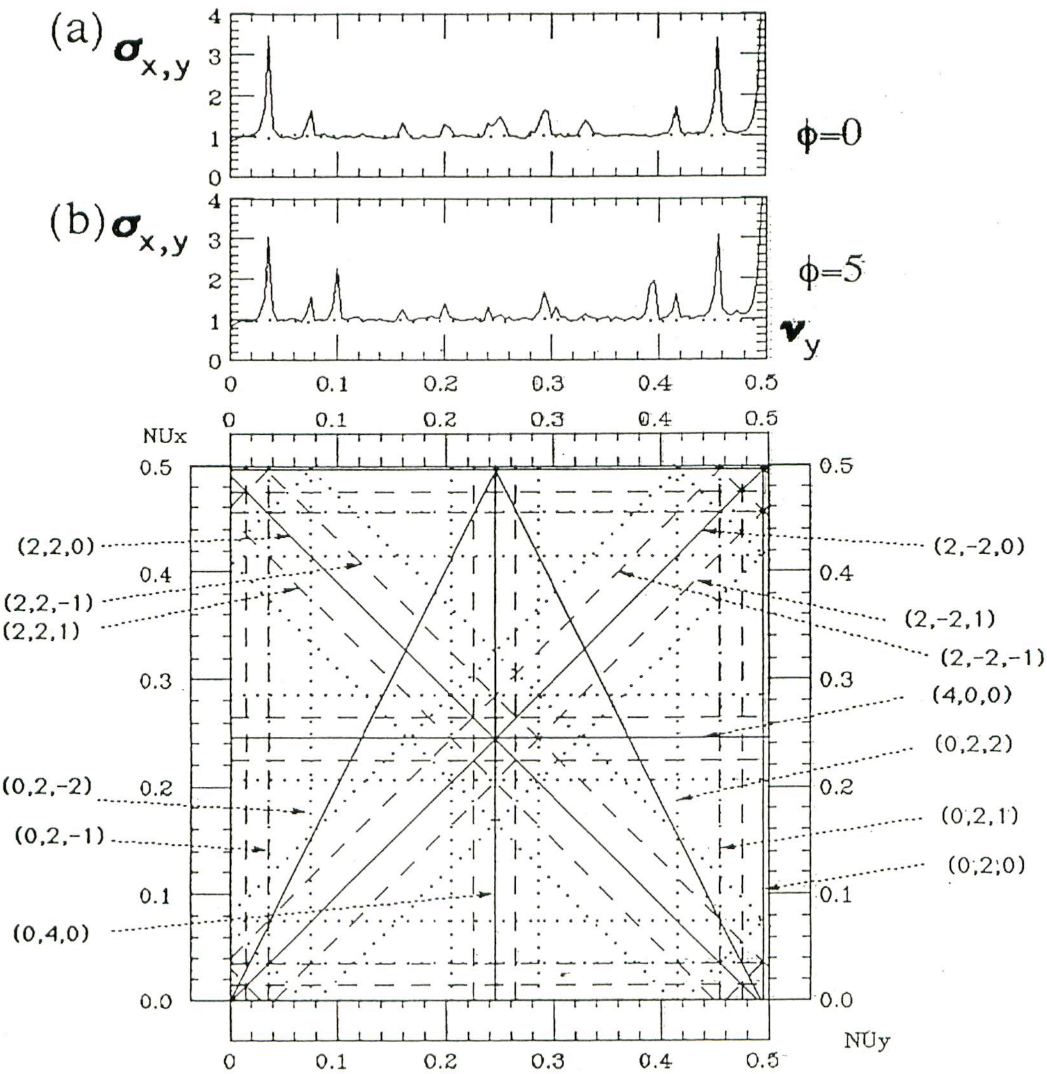
3.1 Crossing Angle

Map of Piwinski[4]:

$$f(x,y) \longrightarrow f(x+\phi z, y), \qquad \sigma_x^2 \longrightarrow \sigma_x^2 + \phi^2 \sigma_z^2.$$

$$\delta y' = f, \qquad \epsilon \longrightarrow \epsilon + \phi \delta y'.$$

No SB coupling when $\phi = 0$.



[4]A. Piwinski, *Limitation of the Luminosity by Satellite Resonances*, DESY 77/18 (1977).

- **50 particles on (6,1,1)**

■ bad region

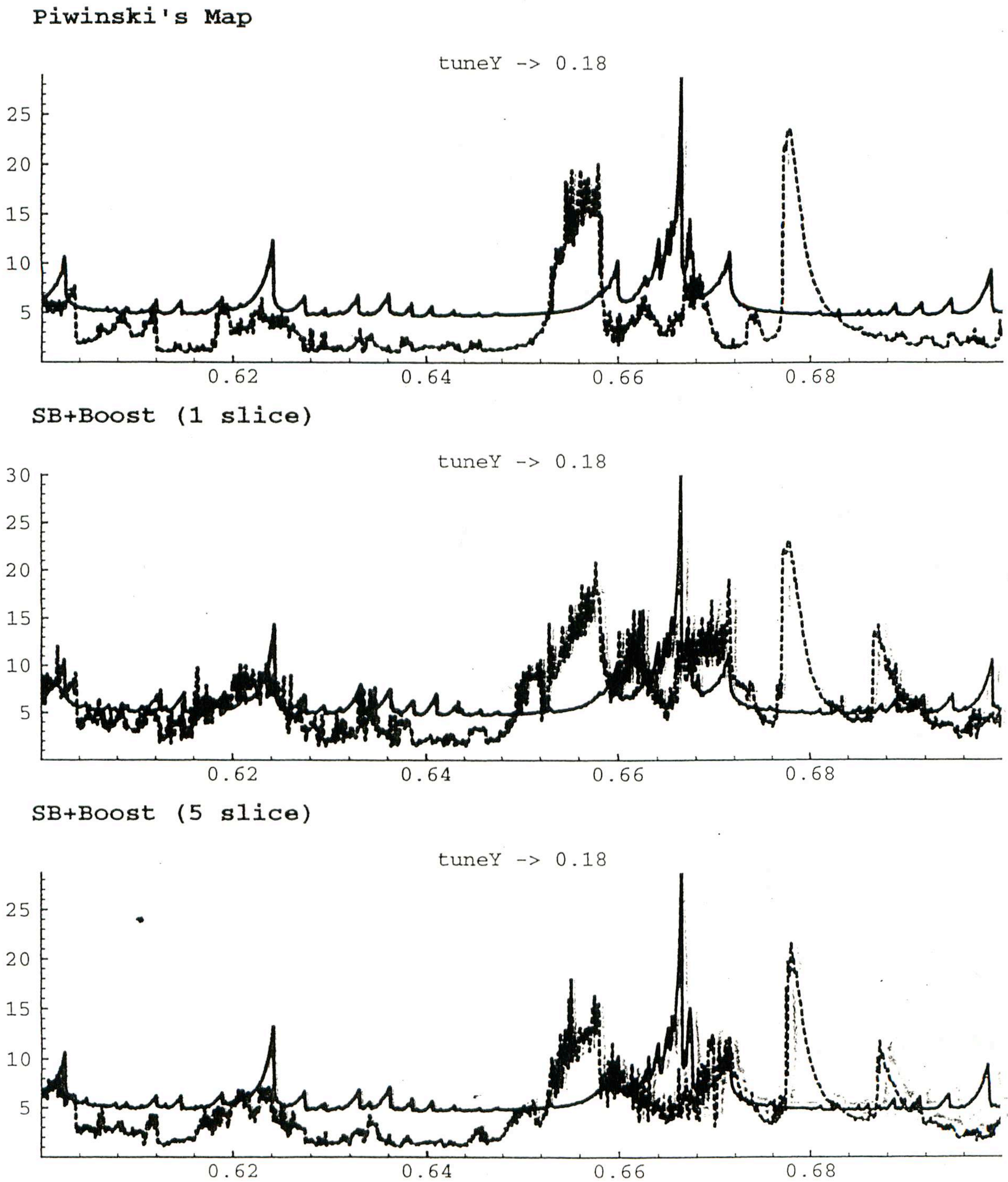


Figure 8: Tracking without radiation for $T^3 = (6, 1, 1)$.

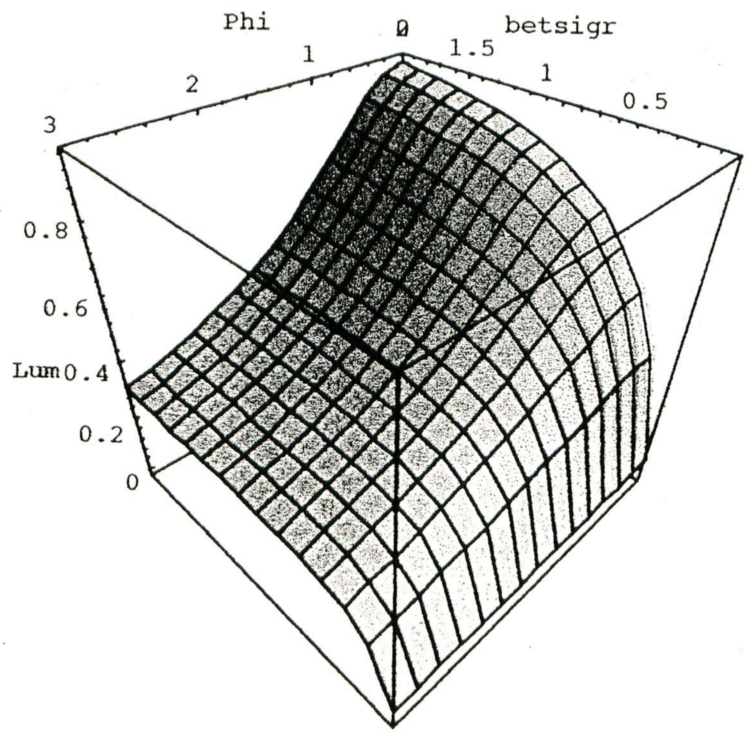
3.2 A Little Physics

Luminosity Reduction Factor by Hourglass and Crossing Effects

$L_0 = L$ with $\sigma_z = 0$ (no hourglass effect) and $\phi = 0$.
$L_g = L$ without beam-beam interaction (geometrical).

$$R_L = \frac{L_g}{L_0} = \sqrt{\frac{2}{\pi}} a e^b K_0(b), \qquad (1)$$

$$a = \frac{\beta_y^*}{\sqrt{2}\sigma_z^*}, \qquad b = a^2 \left[1 + \left(\frac{\sigma_z^*}{\sigma_x^*} \tan\phi\right)^2\right].$$



$Phi = \phi\sigma_z/\sigma_x$, $betsigr = \beta_y/\sigma_z$

Beam-Beam Reduction Factor by Hourglass and Crossing Effects

For $\phi = 0$ and $\sigma_z = 0$, a collision with offset x has

$$\xi_{x,y} = F_{x,y}(x, \sigma_x, \sigma_y)\eta_{x,y},$$

where $F(x, \sigma_x, \sigma_y) \equiv F(x/\sigma_x, \sigma_y/\sigma_x)$'s are Montague's reduction factor[5] of ξ and η is for central conllision.

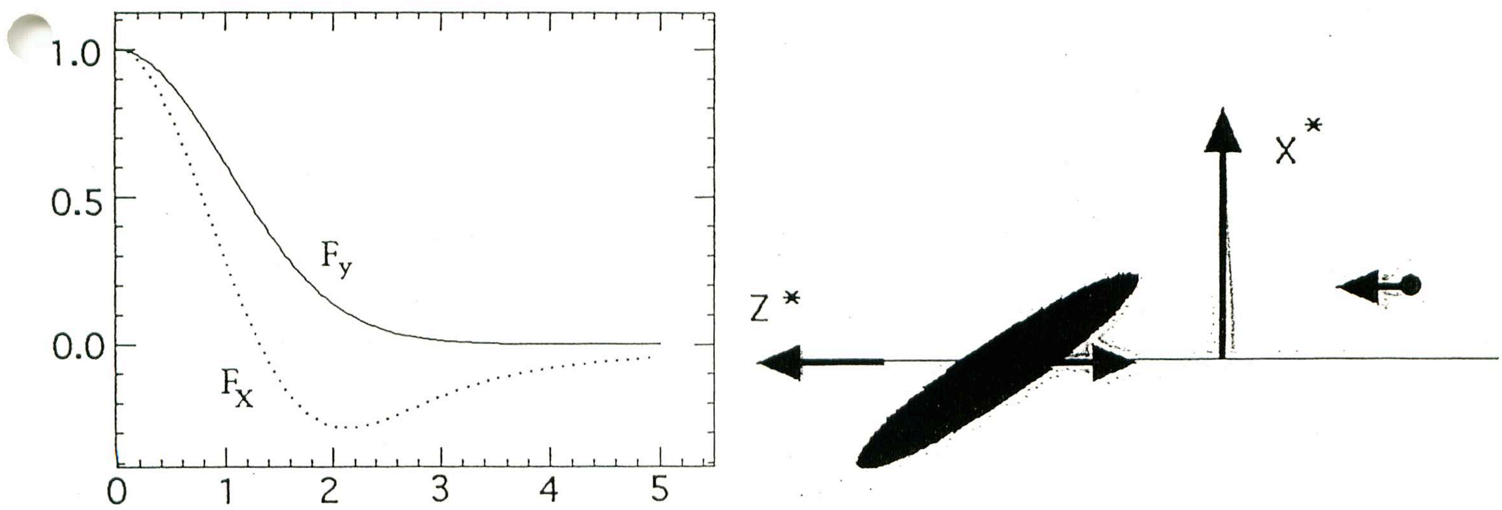


Figure 9: Montague's reduction factor and neam-beam collision in the headon frame.

$$\begin{aligned} R_\xi^{x,y}(z) &= \xi_{x,y}/\eta_{x,y} \qquad (3)\\ &= \int dz^\dagger \rho(z^\dagger) \frac{\beta_{x,y}(S)}{\beta^*_{x,y}} \frac{\sigma^*_{x,y}(\sigma^*_x + \sigma^*_y)}{\sigma_{x,y}(S)(\sigma_x(S) + \sigma_y(S))} \\ &\quad \times F_{x,y}(z^\dagger \tan\phi, \sigma^*_x(S), \sigma^*_y(S)) \\ &\simeq \int dz^\dagger \rho(z^\dagger) \sqrt{\frac{\beta_{x,y}(S)}{\beta^*_{x,y}}} F_{x,y}(z^\dagger \tan\phi, \sigma^*_x(S), \sigma^*_y(S)), \end{aligned}$$

$$S = (z - z^\dagger)/2.$$

[5] B. W. Montague, CERN report CERN/ISR-GS/75-36 (1975).

The beam-beam interaction falls off more rapidly than the luminosity as ϕ increases.

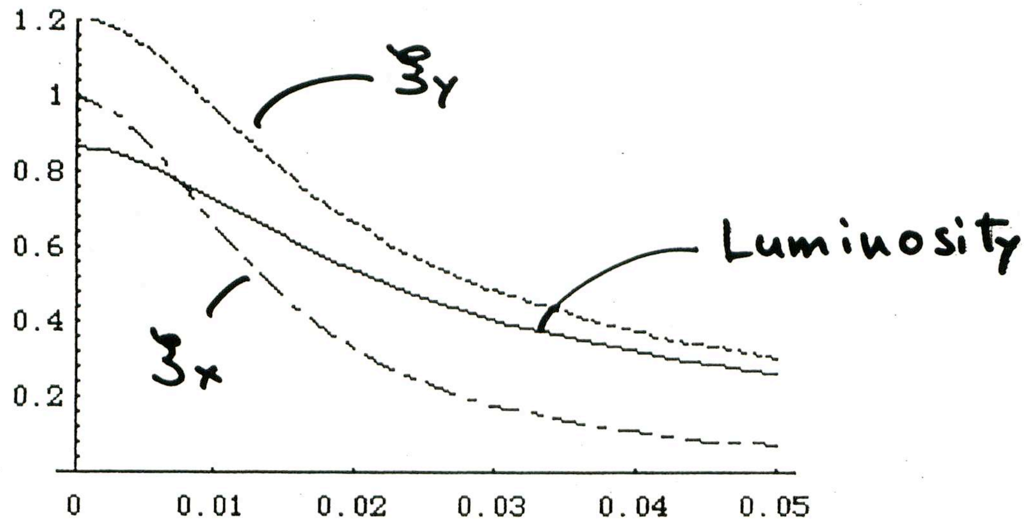


Figure 10: The (geometrical) luminosity reduction factor R_L (solid) and tune shift reduction factors R_ξ^y (dotted) and R_ξ^x (dot-dash).

Dynamic Beta and Dynamic Emittance[6].

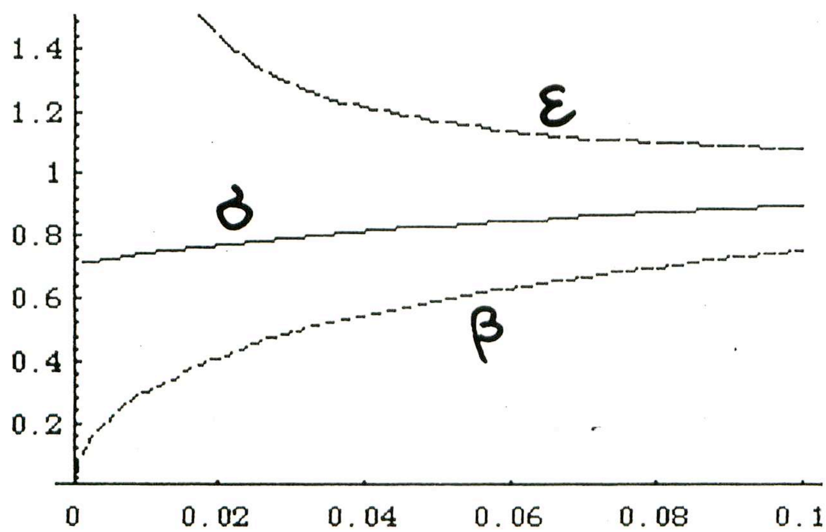


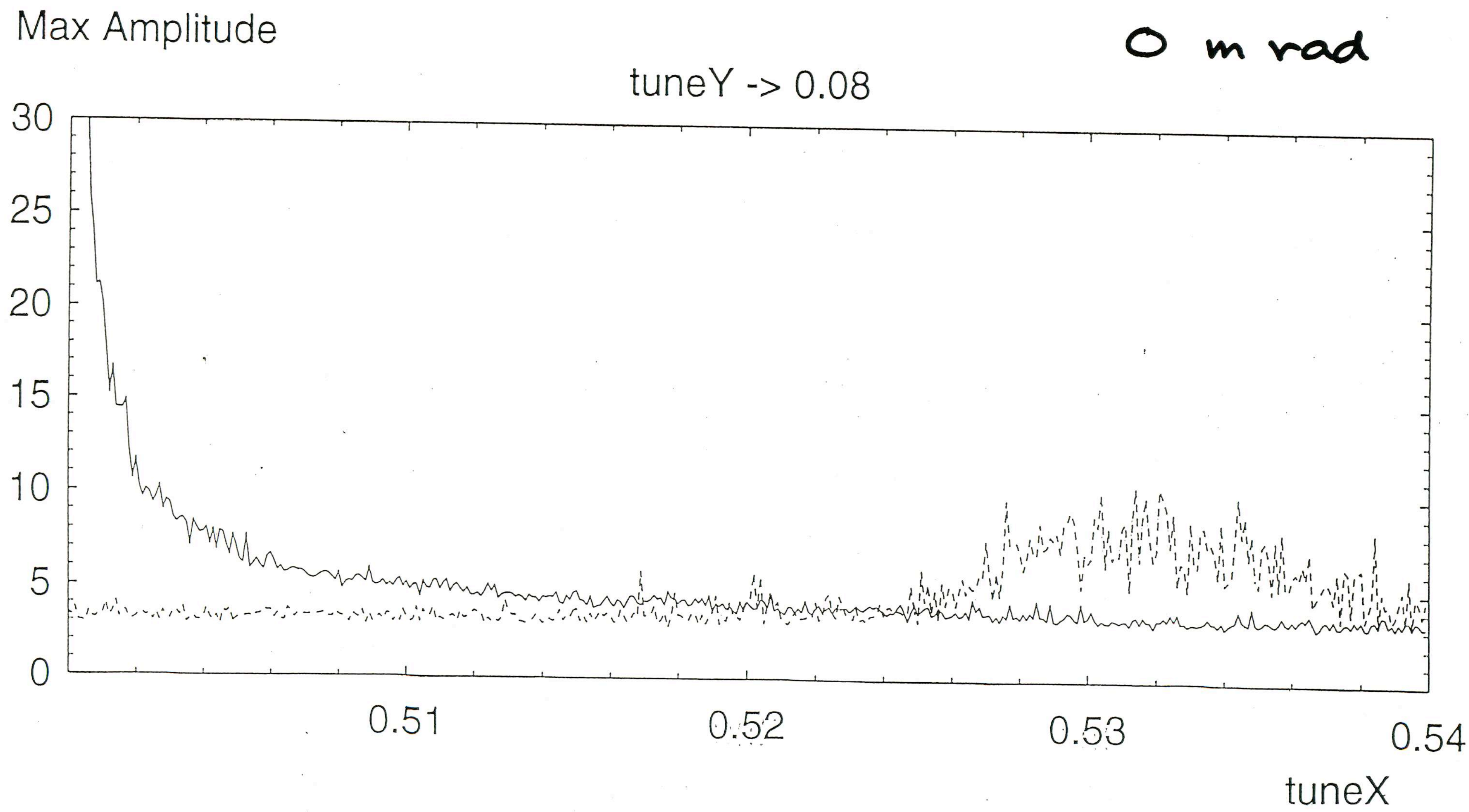
Figure 11: beam-size (solid) beta (dotted) and emittance (dashed) for $\eta = 0.05$.

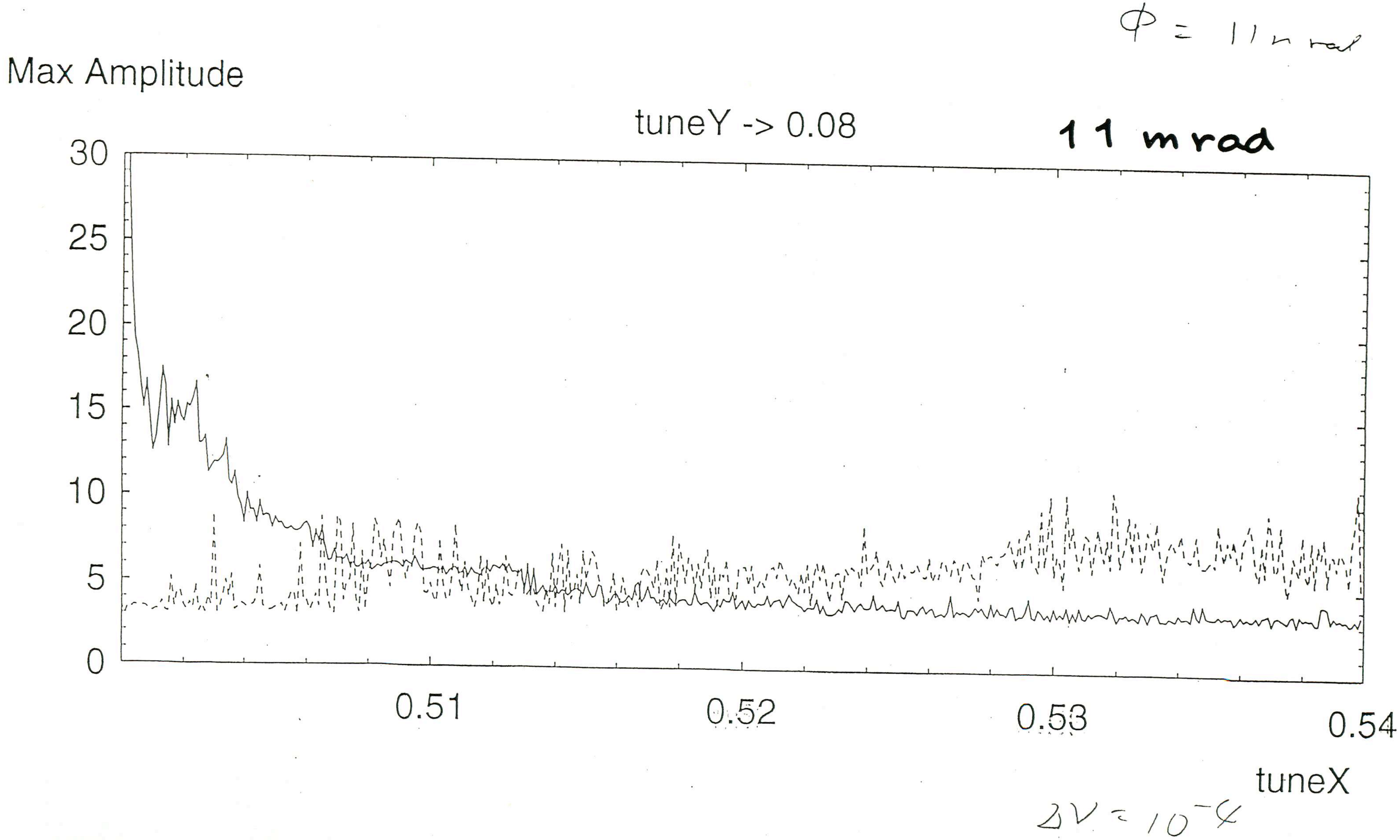
[6]K. Hirata and F. Ruggiero, Part. Accel. 28, 137 (1990).

3.3 Very Fine Tune Survey

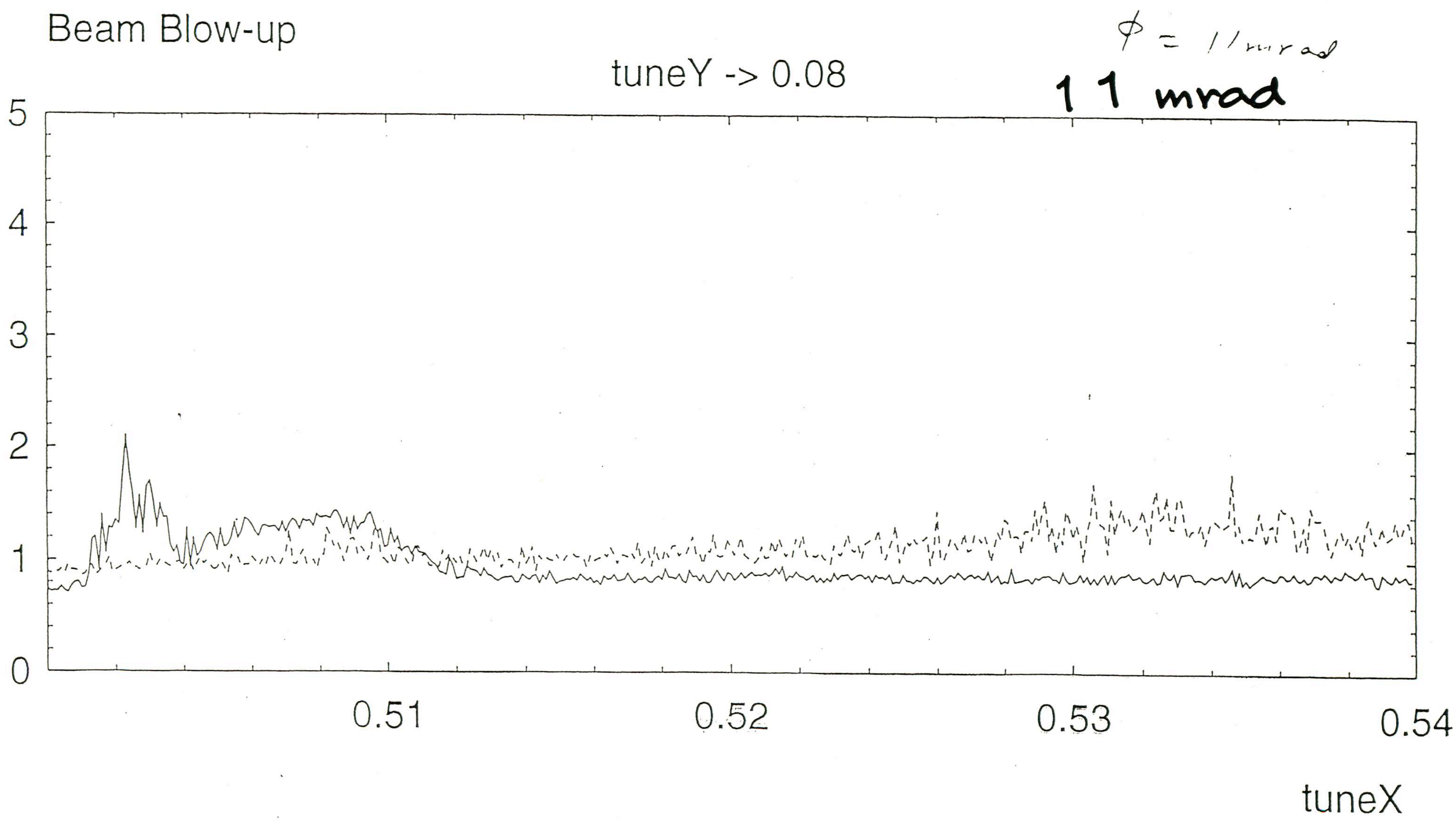
$\Phi = 0$

0 m rad



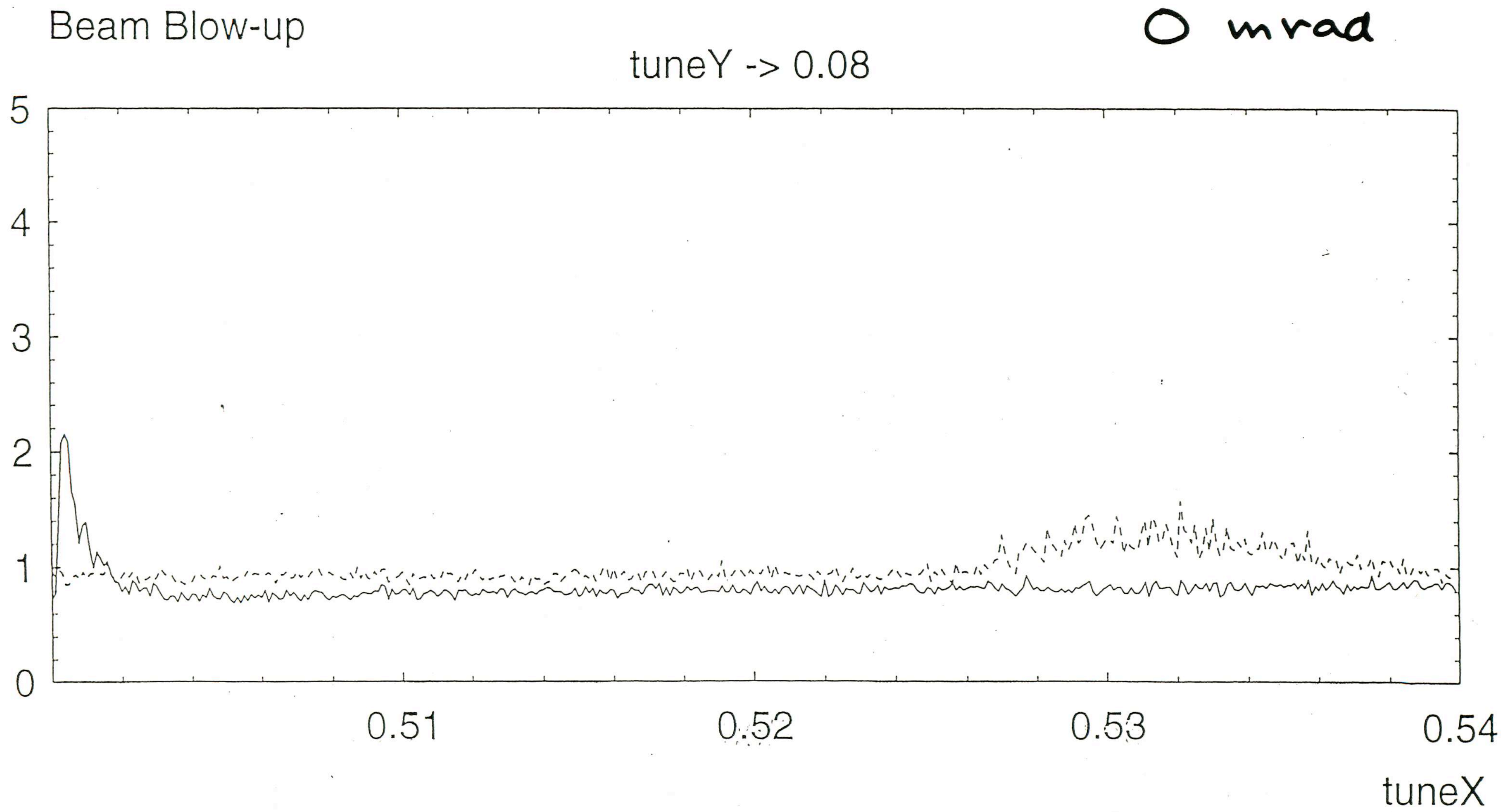


φ = 11 mrad
Max Amplitude
tuneY -> 0.08
11 mrad
30
25
20
15
10
5
0
0.51
0.52
0.53
0.54
tuneX
Δν = 10⁻⁴



Beam Blow-up
tuneY -> 0.08
ϕ = 11 mrad
11 mrad
5
4
3
2
1
0
0.51
0.52
0.53
0.54
tuneX

$\phi = 0$



4 ~~Discussion~~ Conclusion

Within the simulation studies performed so far, **the luminosity in the design goal can be achieved with the finite angle crossing of 2×11 mrad** without significant difficulties concerning the background and the life time.

Unfortunately, this performance cannot be fully experimentally tested until operating KEKB.