Beam-beam limit in B factories

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2004 MAC for super KEKB
16-18 Feb. 2004

Introduction

- Beam-beam limit for head-on collision

1. Beam distribution
2. Diffusion couple to radiation
3. Coherent or incoherent

- Crossing angle and crab crossing

1. Arnold diffusion due to crossing angle
2. x-y coupling
3. Crab crossing

Beam-beam limit

$$L_{tot}(\mathrm{cm}^{-2}\mathrm{s}^{-1}) = 7.6\times 10^{34}\,\frac{I(A)\,\xi_y}{\beta_y(\mathrm{cm})}$$

- Super KEKB   I=10A, β_y=3 mm
- How is large ξ_y achieved?
- We study the limit value of ξ_y.
- Strong-strong and weak-strong simulations were used according to circumference.

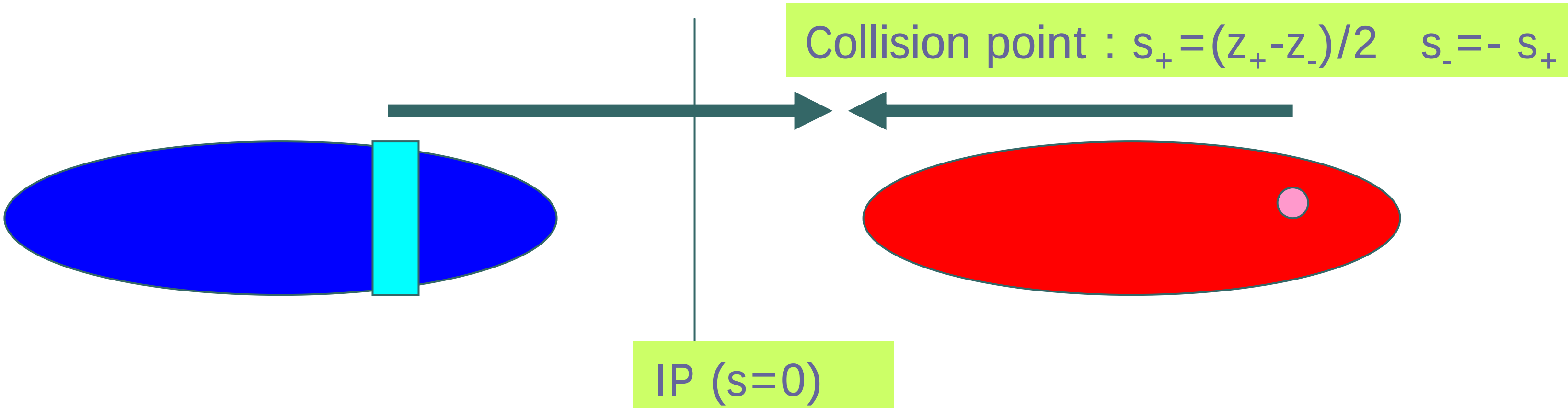
One turn map including the beam-beam interaction

$$\mathbf{x}_\pm(L) = V_0(L)\, S\exp\left[-:\int_{-IR}^{IR}\left(V_0^{-1}(s,0)\,\phi_\pm(s,\rho_\mp(-s))V_0(s,0)\right)ds:\right]\mathbf{x}_\pm(0)$$

$$V_0(L) = S\exp\left[-:\int_0^L H_0\,ds:\right]$$

Lattice one turn map

$V_0(s,0)$: map of IP to collision point of each particle



Beam-beam potential ϕ

- ϕ: potential given by solution of 2D Poisson equation.

$$\Delta\phi_{\pm}(x,y,z;s) = \frac{r_e}{\gamma}\rho_{\pm}(x,y,z;s)$$

Two methods are used to estimate ϕ.

1. Gaussian model: ρ is approximated to be transverse Gaussian distribution.
2. Particle In Cell method: Particle distribution is mapped on a transverse grid space. An arbitrary beam distribution can be treated.

Important parameters

- The map is represented by V_0, IR and ρ.
- Parameters which determine the beam-beam interaction are tunes, damping time and excitation, σ_z/β_y, and the beam-beam parameters, ξ_{x+}, ξ_{y+}, ξ_{x-}, ξ_{y-} for ideal case.
- Other optics parameters (including crossing angle)
- Nonlinearity of the lattice map.

Simulation

- Strong-strong simulation

Mesh 128x256, macro-particle 100,000

Z-slice 5

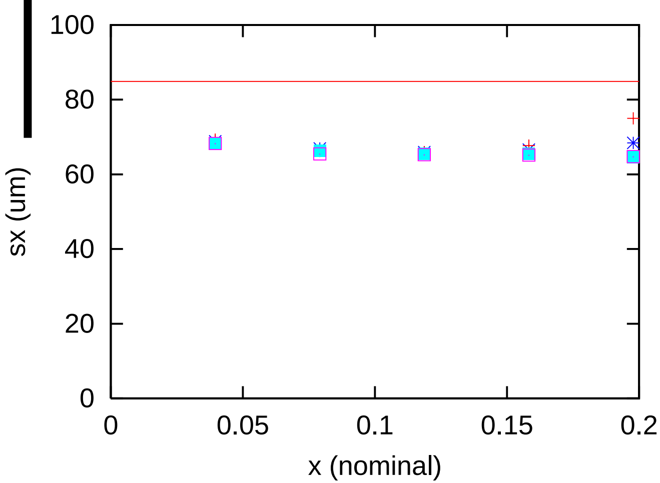
- Weak-strong simulation

Z-slice 5-10, macro-particle 100-10,000

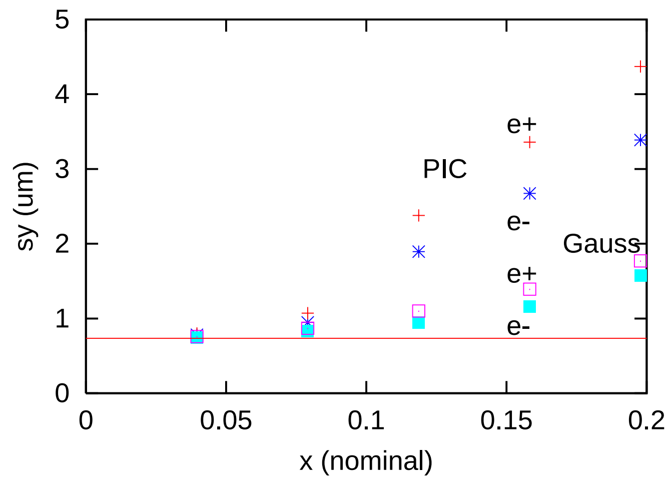
In both simulation, Gaussian and PIC models are used: namely we used 4 type of simulations according to circumference.

Strong-strong

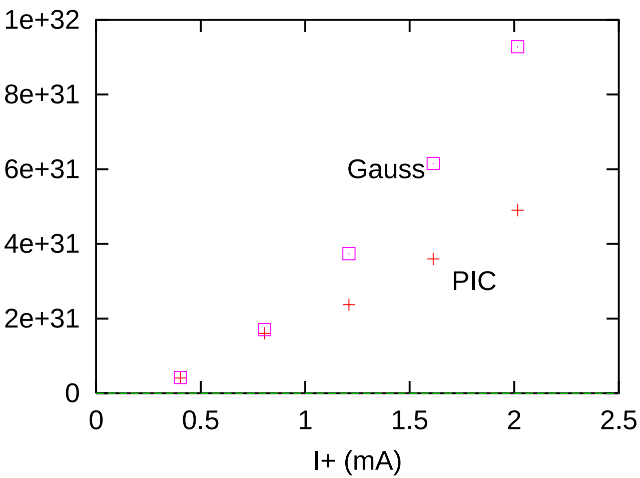
σ_x



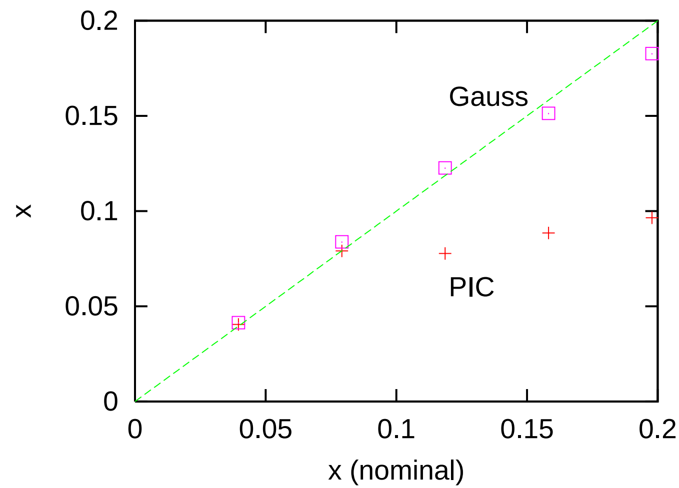
σ_y



L



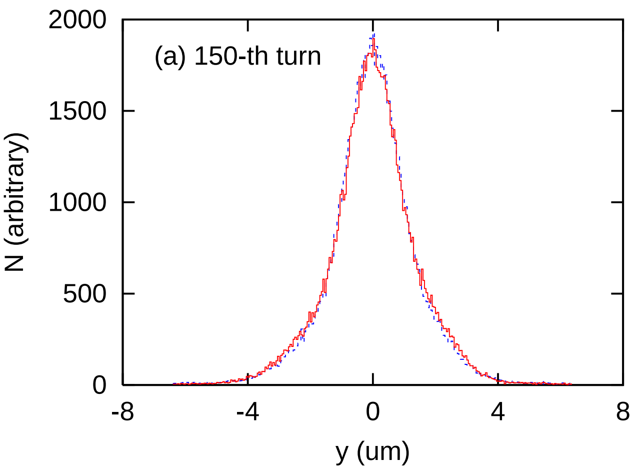
ξ_y



Change of particle distribution

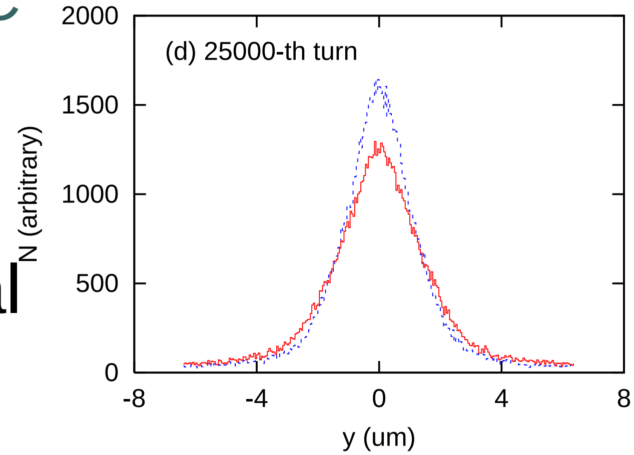
No coherent motion during the growth

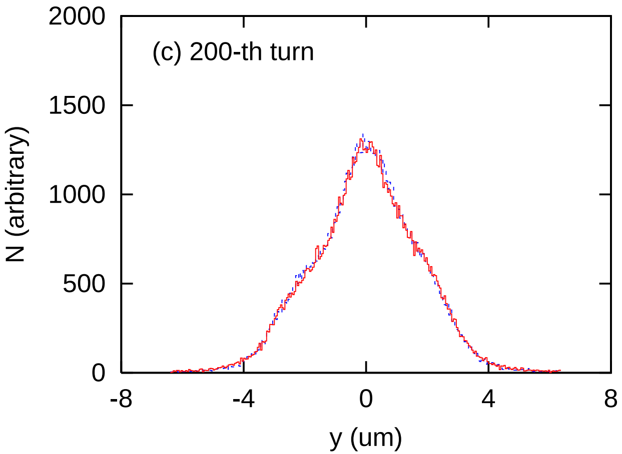
Strong-strong PIC



intermediate

final

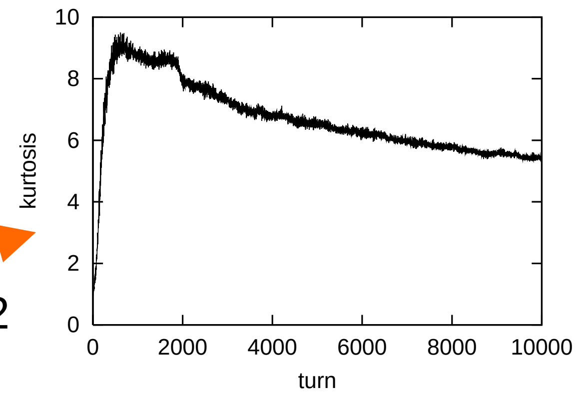




intermediate

Kurtosis

$k_y = \langle y^4 \rangle / 3 \langle y^2 \rangle^2$



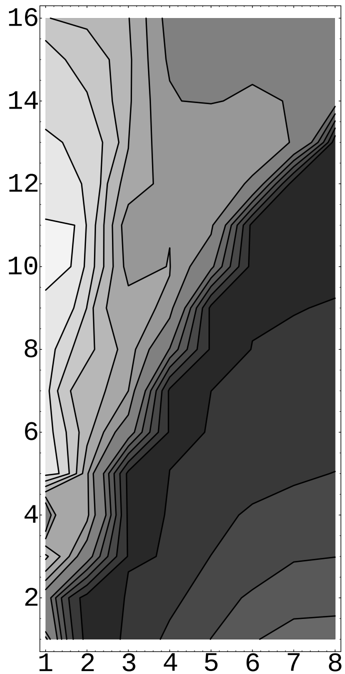
Tune scan

Strong-strong PIC

0.63

ν_y

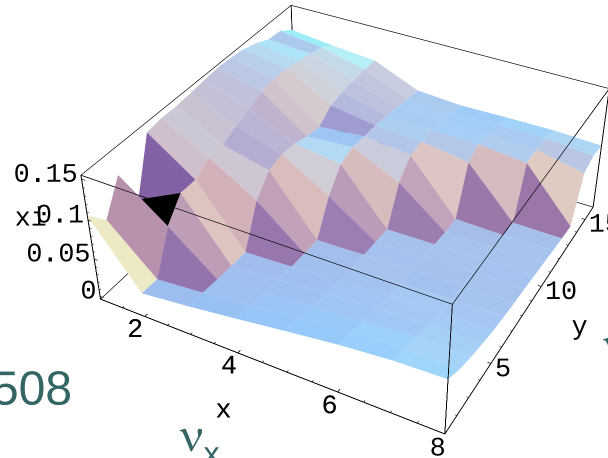
0.55



0.508 0.528

ν_x

ξ



0.63

ν_y

0.508

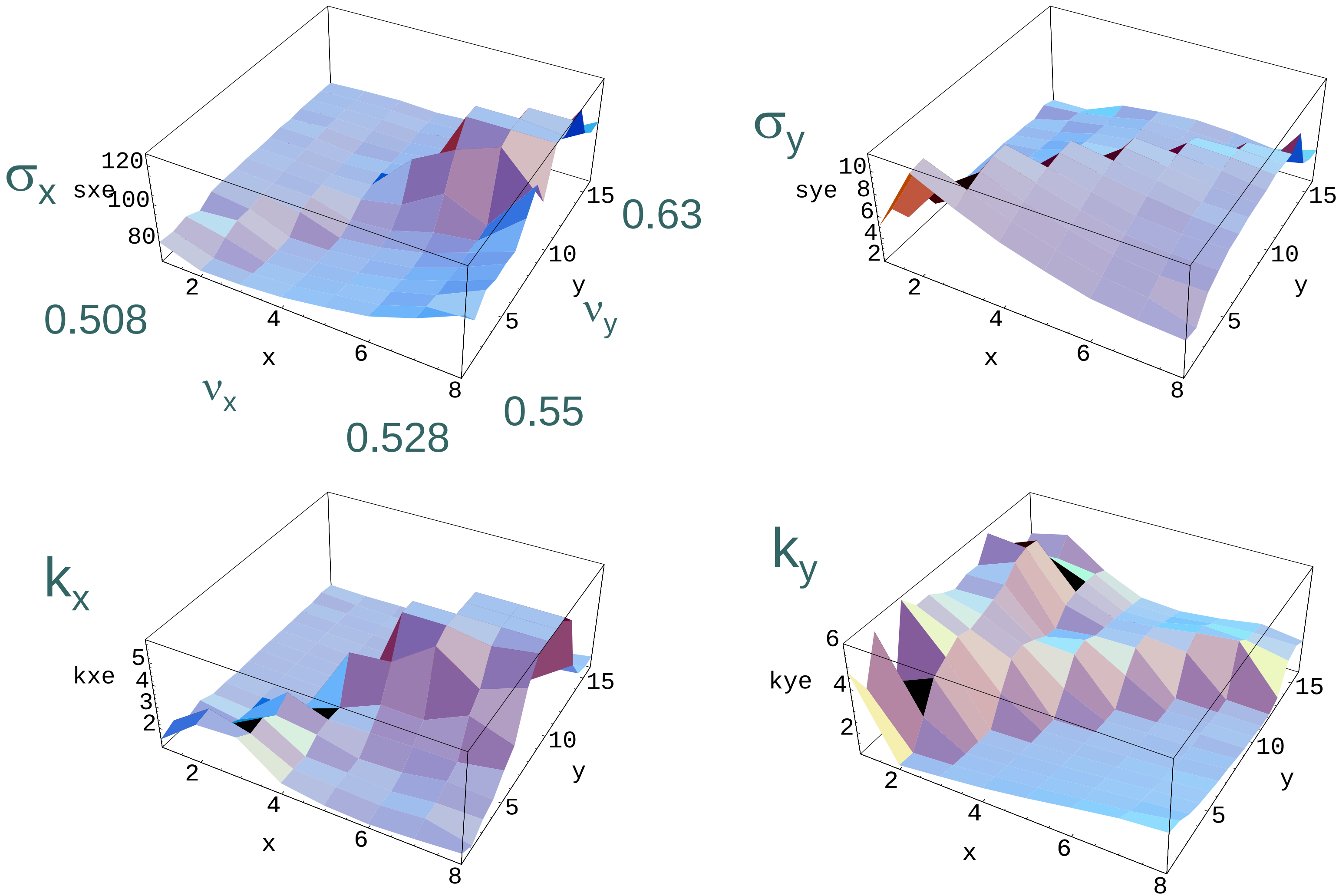
ν_x

0.55

0.528

Highest $\xi \sim 0.15$

Tune dependence of σ_{xy}, k_{xy}

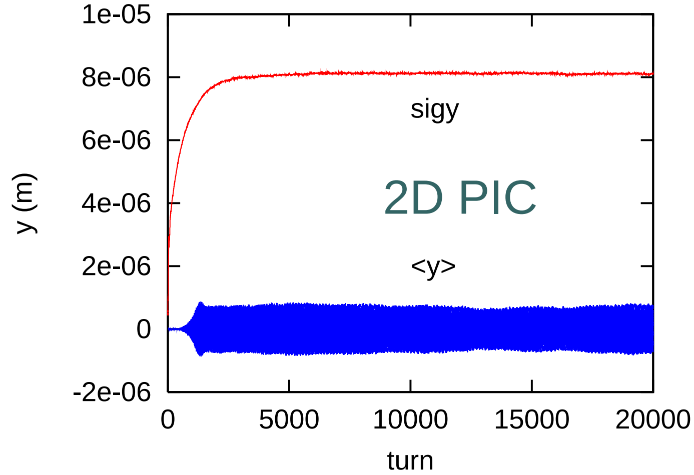


Summary of Tune scan

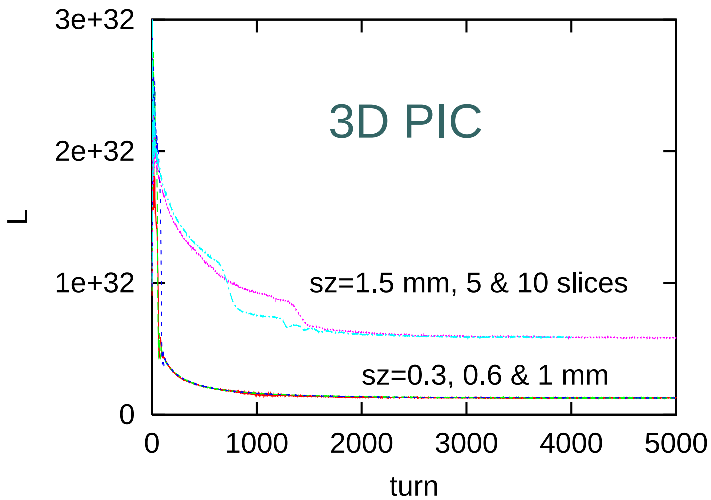
- Operating point with the best beam-beam parameter (ν_x, ν_y)~(0.51, 0.58).
- High k_y and low k.
- Tune survey
- ν_x~0.51 k_x~3-5 k_x~3-5
- ν_x~0.52 k_x~5-6 k_x~1-2
- ν_x>0.53 Horizontal coherent motion
- ν_x>0.55 Clear H motion → low luminosity
- Any vertical coherent motion is not seen at $0.5 < \nu_y < 0.65$.

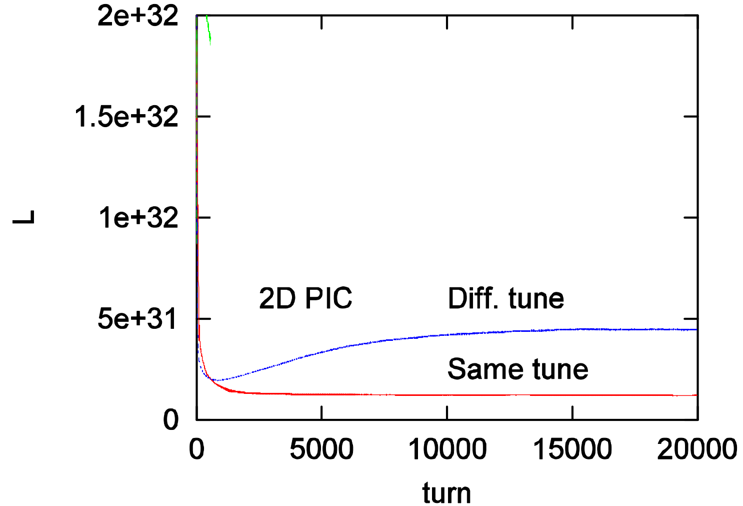
Beam-beam limit due to a coherent motion

- Coherent motion is seen in short bunch $\sigma_z<\beta_y/2$, but disappear for longer bunch.
- It also disappear for separating two tunes.



Strong-strong





Coherent or incoherent

- Coherent effects means instabilities related to the coherent betatron motion.

  $\omega = n\omega_{\beta+} + m\omega_{\beta-} + \alpha$

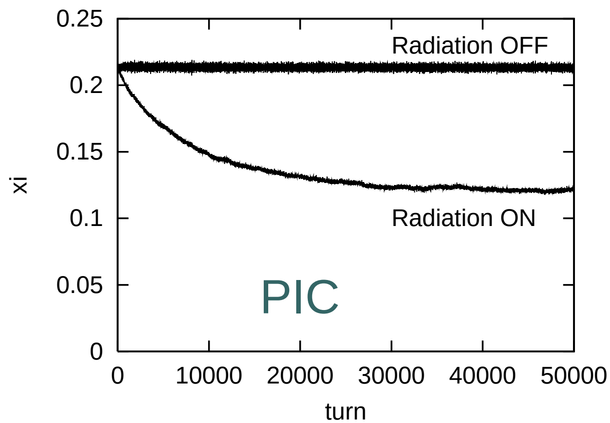
- A vertical coherent motion determined the beam-beam limit around $\xi \sim 0.05$ in 2D simulation.
- The vertical coherent motion disappear when the bunch length exceeds $\beta_y/2$ in 3D simulation.
- A horizontal coherent motion appears at $\nu_x > 0.53$.
- Tune difference, intensity and emittance unbalances contribute to suppress the coherent motion.

↓

- Incoherent effects determine the beam-beam limit.

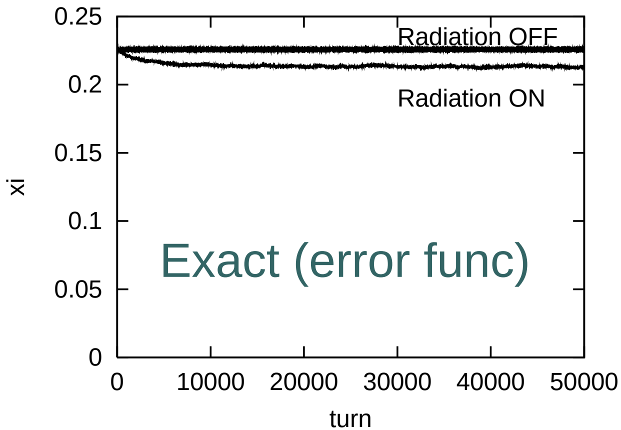
Diffusion in Head-on collision

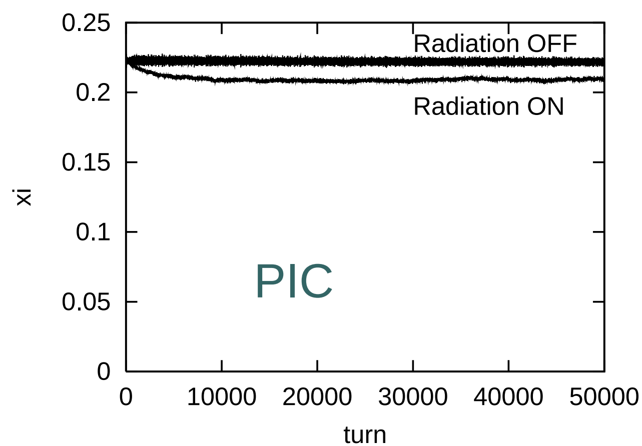
given by the weak-strong simulation



Strong beam：distorted beam

- Diffusion is very weak for no synchrotron radiation.
- If the strong beam is Gaussian, diffusion is weak even with radiation.
- The radiation excitation enhances diffusion when the strong beam is distorted.

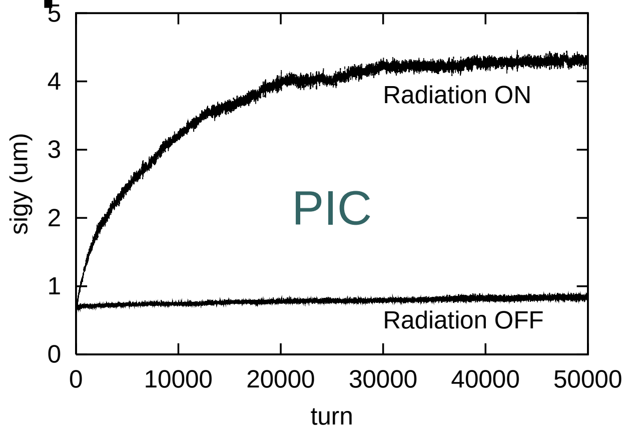




Strong beam: Gaussian

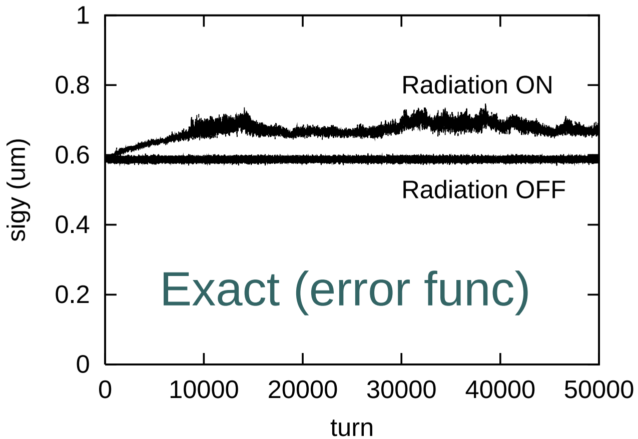
Solver: Error function & PIC

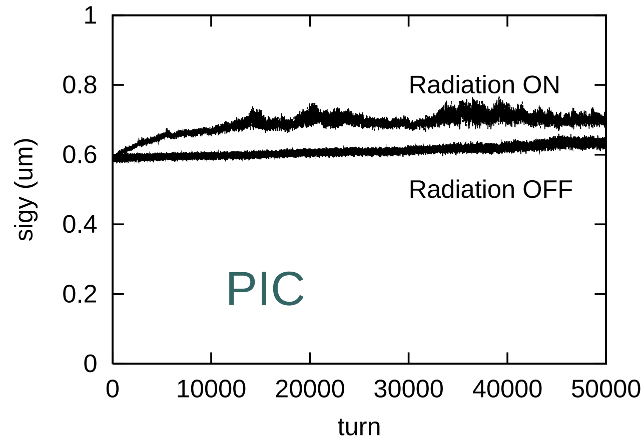
Diffusion seen in the vertical beam size



Strong beam：distorted beam

- These pictures explain the behaviors of ξ seen in previous slide.





Strong beam：Gaussian   Solver: Error function & PIC

Diffusion in the head-on collision

- Diffusion is investigated by the weak-strong simulations with/without radiation damping and excitation.
- In Head-on collision, symplectic diffusion was very weak.
- Radiation excitation enhances diffusion for the distorted beam in compared with Gaussian beam.
- Perhaps structure of the phase space is sensitive for the radiation excitation.

Crossing angle

- A kind of dispersion $\Delta x = \zeta \Delta z$ is introduced by the crossing angle in the arc transfer matrix V_0.
- Actually small nonlinear kinematical terms are included as follows,

$$x^* = \tan\phi\, z + [1 + h_x^* \sin\phi]\, x$$

$$p_x^* = (p_x - h\tan\phi)/\cos\phi$$

$$y^* = y + h_x^* \sin\phi\, x$$

$$p_y^* = p_y^* / \cos\phi$$

$$z^* = z/\cos\phi + h_z^* \sin\phi\, x$$

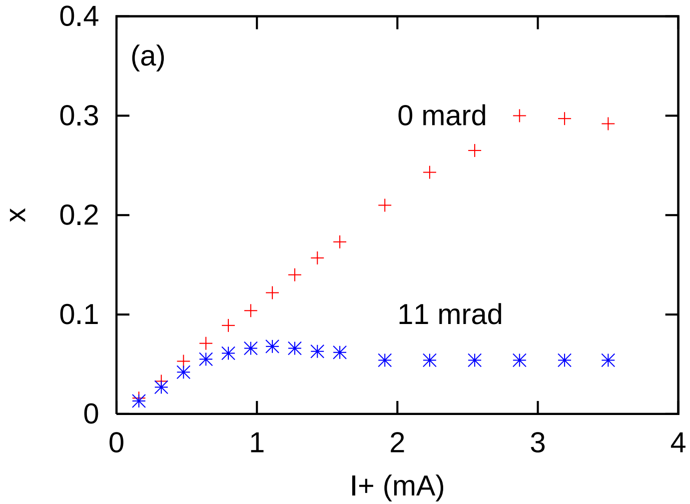
$$p_z^* = p_z - p_x \tan\phi + h\tan^2\phi$$

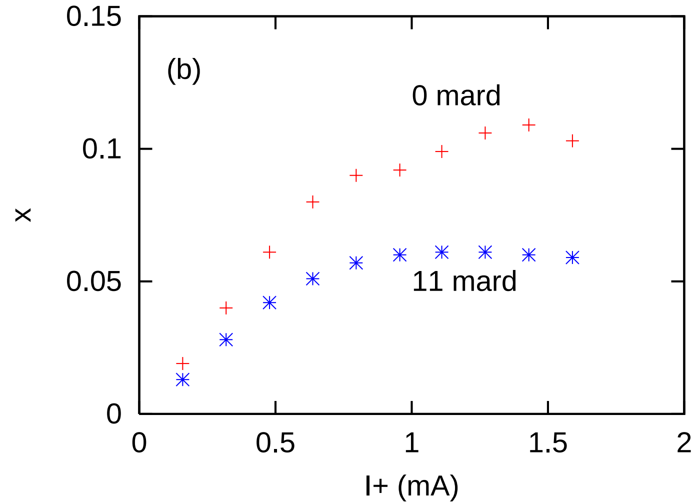
$$h = p_z + 1 - \sqrt{(p_z+1)^2 - p_x^2 - p_y^2}$$

Beam-beam parameter for zero and finite crossing angle

Strong-strong

Gauss model PIC



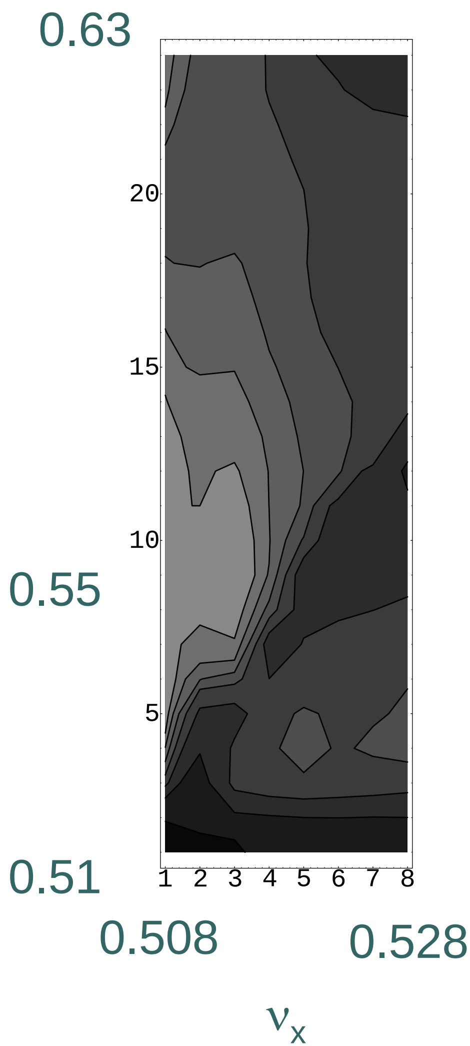


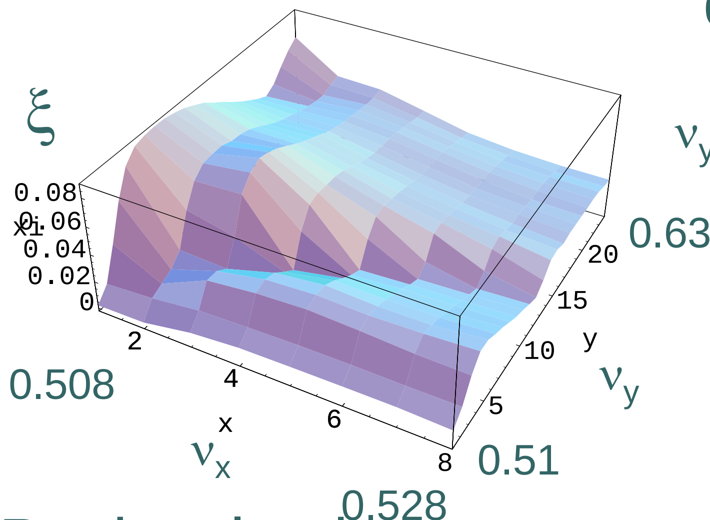
* Present KEKB parameter

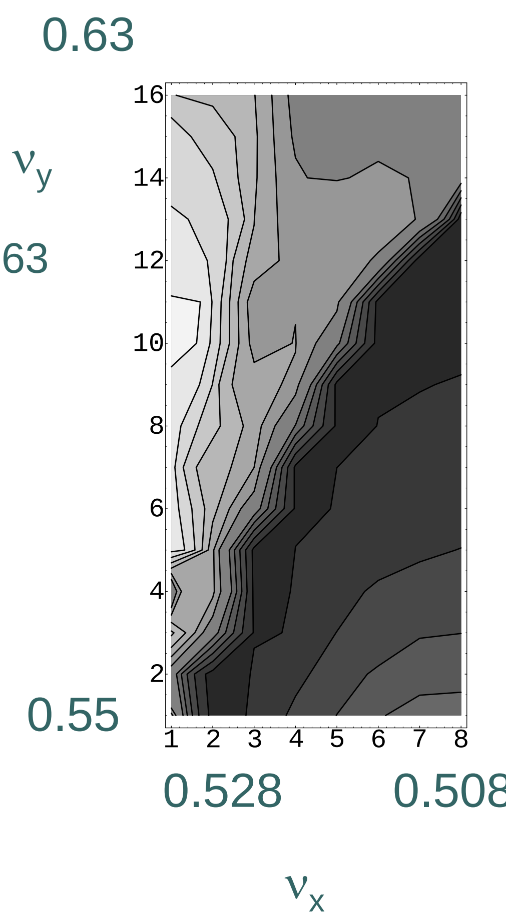
Tune scan

Strong-strong PIC

Head-on

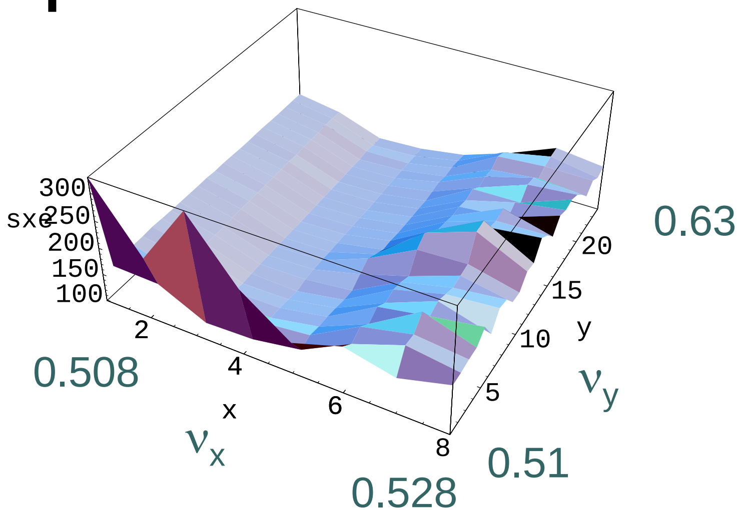


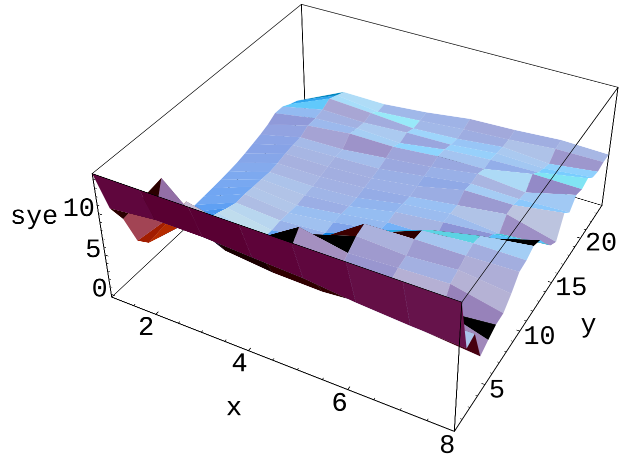


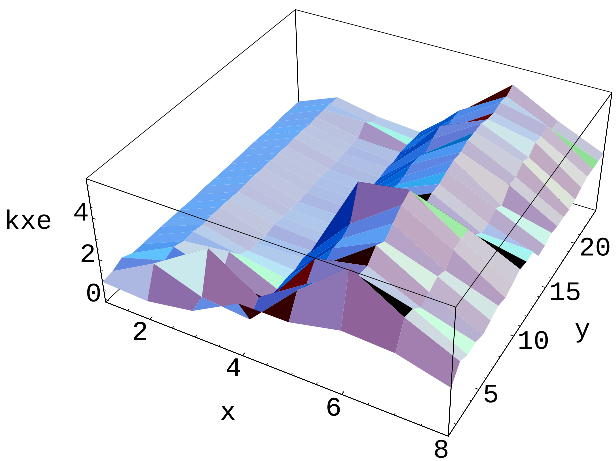


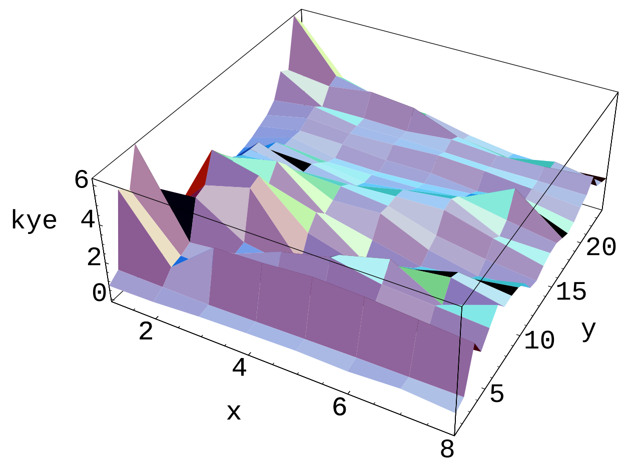
- Peak value is achieved to be ~0.08 at (0.51,0.55).
- Horizontal Coherent motion is seen for ν_x>0.55

Tune dependence of σ_{xy}, k_{xy}





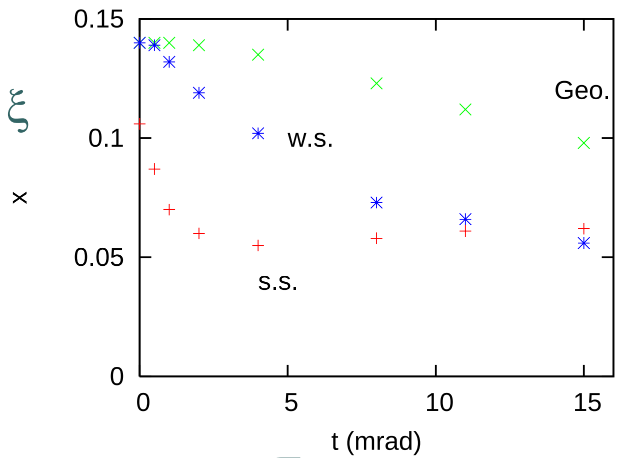




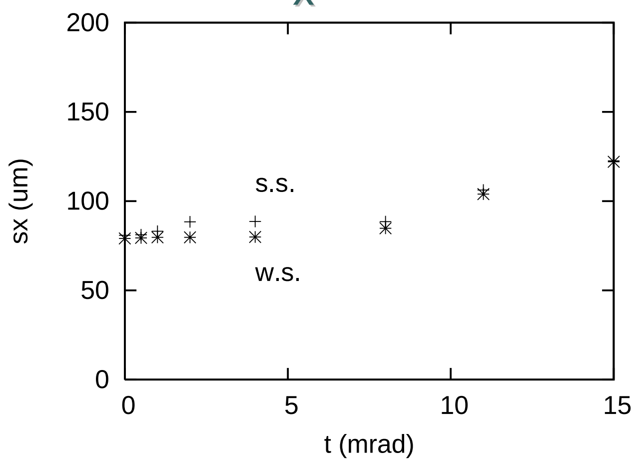
Crossing angle dependence

- Both simulations showed worse behavior than the geometrical values.
- Weak-strong and strong-strong simulations showed different tendency.

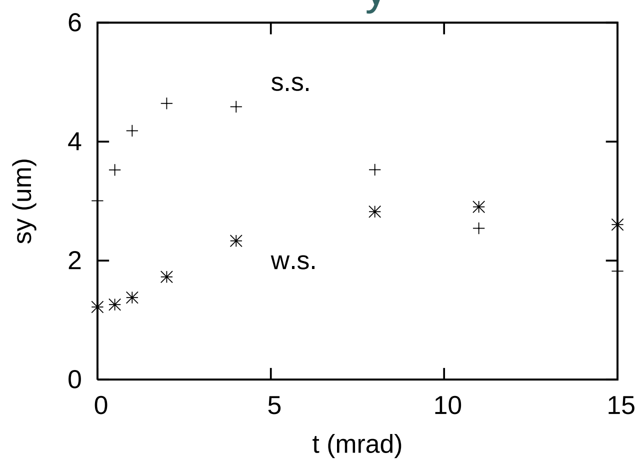
Strong-strong PIC & weak-strong Gauss



σ_x



σ_y



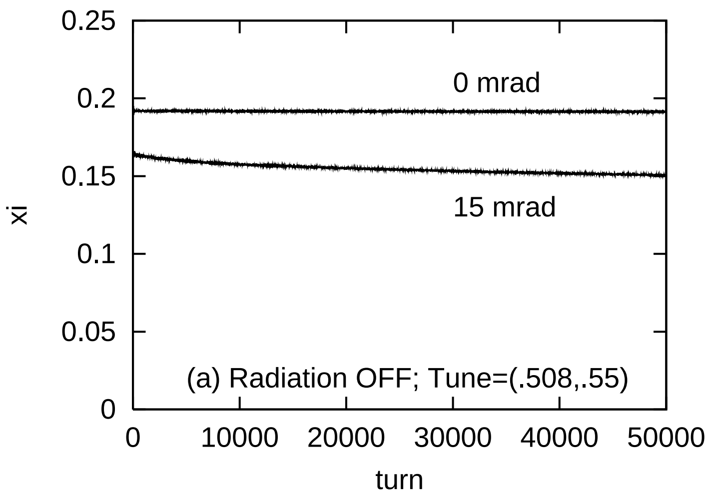
How do we understand the behavior?

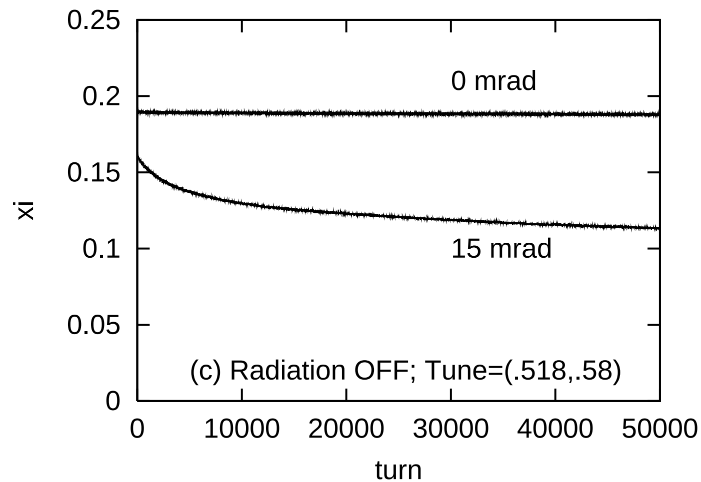
- Weak-strong and strong-strong simulations showed different tendencies.
- Weak-strong showed monotonically increase of the H and V beam sizes. This is natural, because the strong beam is fixed. Note $\theta=\sigma_x/\sigma_z$~14 mad.
- In strong-strong simulation, a vertical enlargement occurred even at small crossing angle ~1mad. Arnold diffusion due to the crossing angle (see next) may enhance the vertical diffusion seen in head-on collision.
- The enlargement becomes weak for large crossing angle >5 mad. Horizontal enlargement may make weaken in vertical one.

Diffusion due to crossing angle

given by the weak-strong simulation (Gauss)

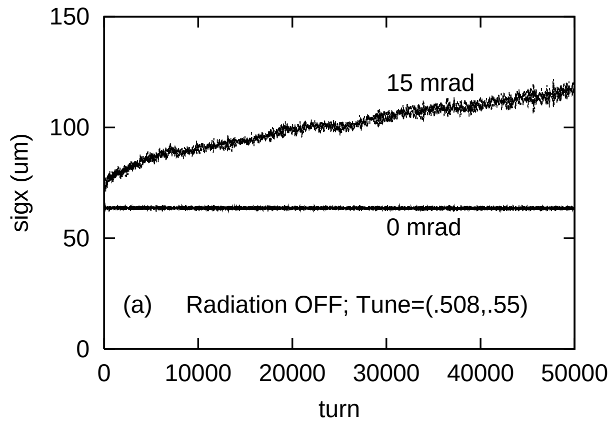
- There is diffusion even in symplectic system. For ϕ=15mrad, diffusion at (0.508,0.55) (present LER) is better than that at (0.518,0.58) (HER).

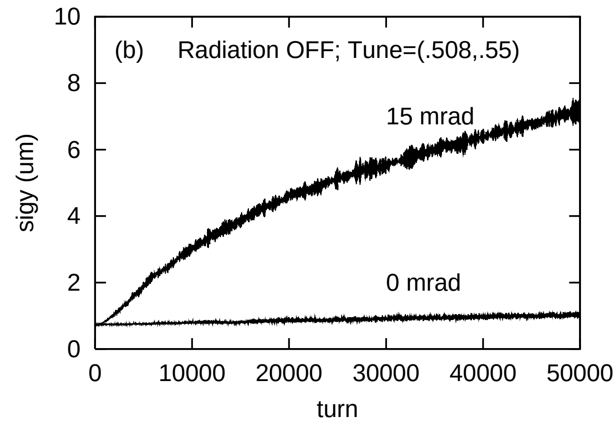


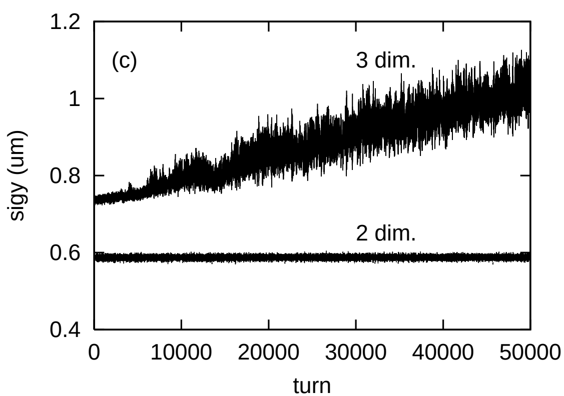


Strong beam：Gauss Solver：Error function

- Diffusion is seen in both of x-y.



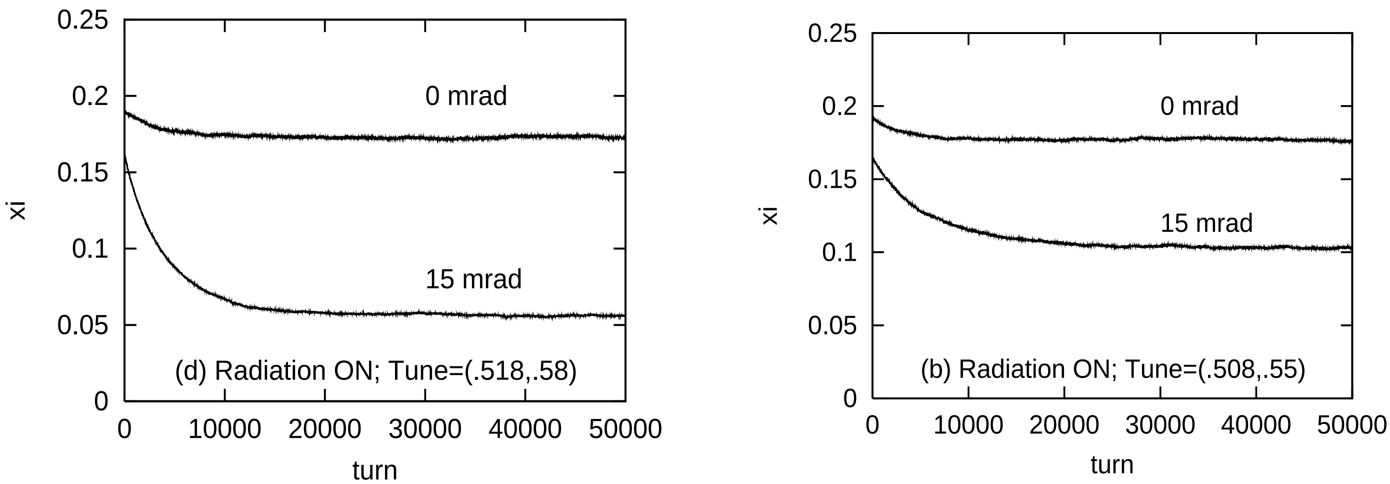




- Diffusion in 3 dim. calculation is stronger than that in 2 dim. one.

  (Head-on case)

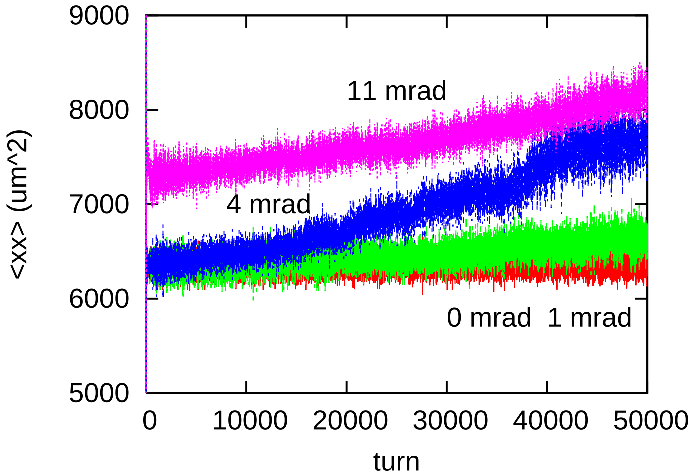
- Including the synchrotron radiation

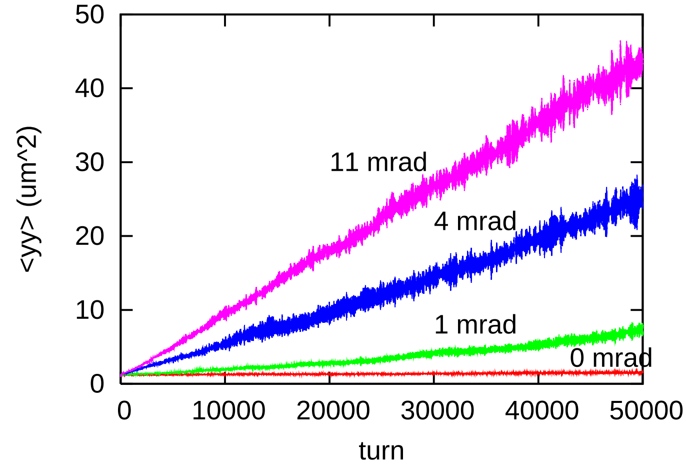


Strong beam：Gauss, Solver：Error function

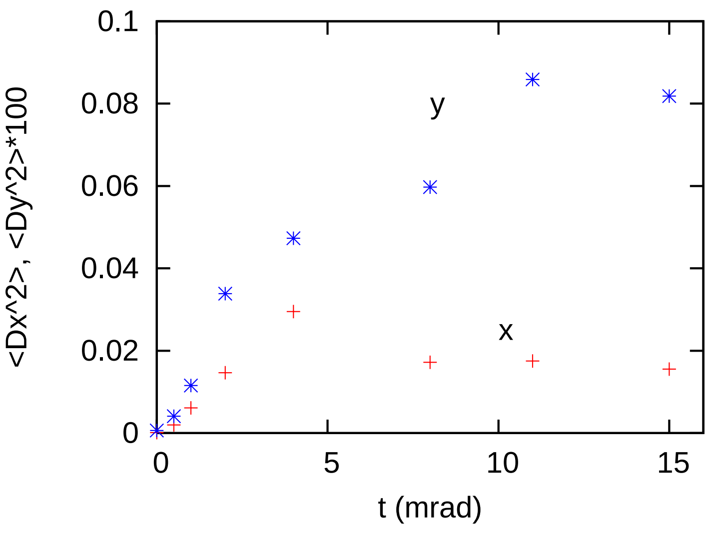
Diffusion for various crossing angle

given by the weak-strong simulation (Gauss)

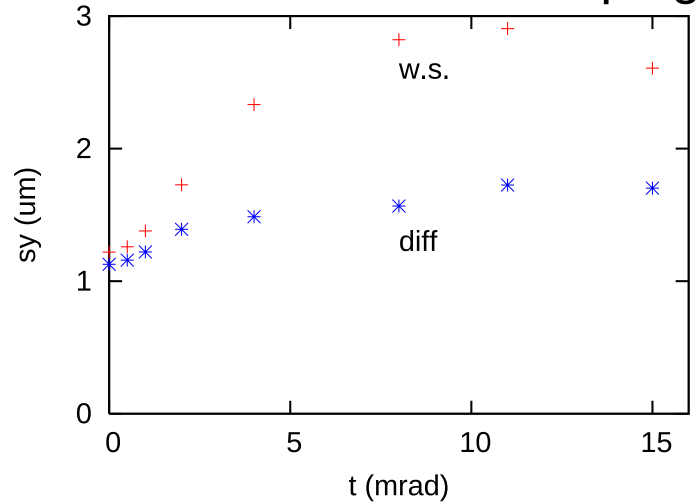




- Diffusion rate



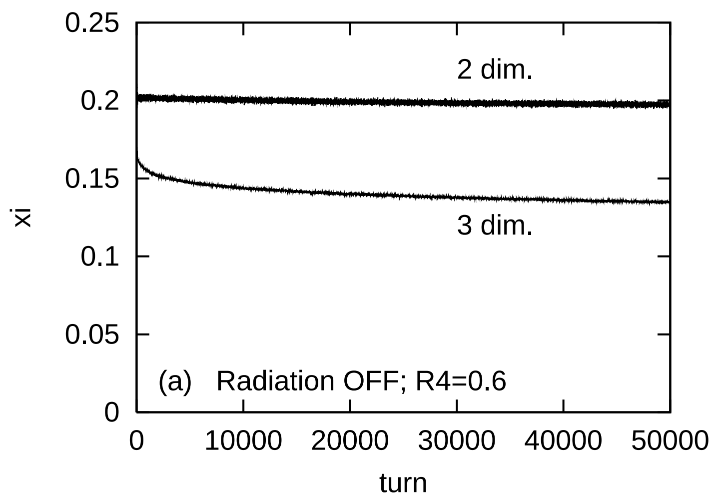
- Vertical equilibrium size obtained by the weak-strong simulation and the ratio of the diffusions for the rad. damping.

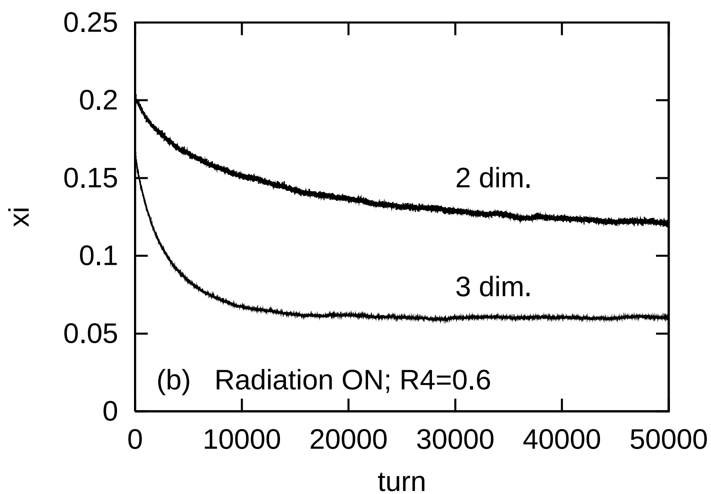


Diffusion due to x-y coupling

given by the weak-strong simulation (Gauss)

- X-y coupling induces symplectic diffusion.
- The diffusion in 3D simulation is stronger than that in 2D one.
- Stronger coupling induces stronger diffusion even in 2D.





Crossing angle and diffusion

- Crossing angle induces Arnold diffusion.
- Vertical beam size may be enlarged by the diffusion.
- The beam size is determined by $\sigma_y^2=(<\Delta y^2(rad)>+< \Delta y^2(diff)>)*\tau(rad)/2T_0$,

if two diffusions are independent.

- The beam size is somewhat larger than the evaluation.
- Interference between diffusions may exist.
- X-y coupling also induces the diffusion.

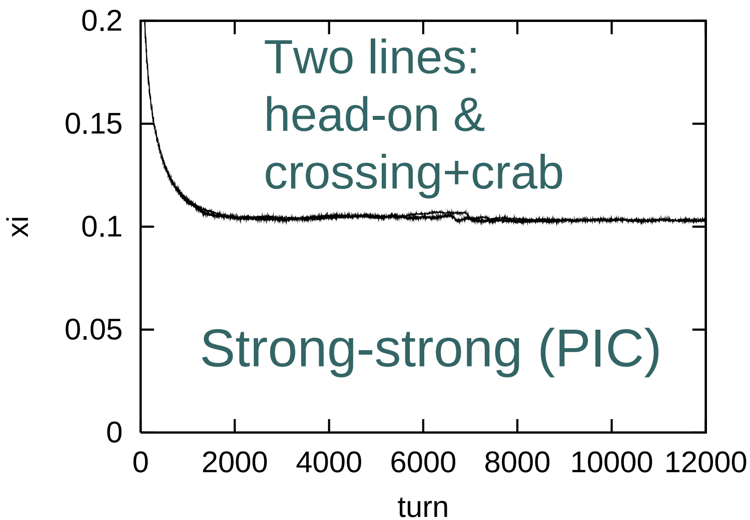
The diffusion in 3D simulation was stronger than that in 2D one.

Crab crossing

The transformation of finite crossing angle is cancelled by a dispersion $\Delta x = -\zeta \Delta z$ given by cab cavity.

We checked effects of the small kinematical term.

No difference: crab cavity works to cancel the crossing angle well.



Effects of dispersion $\Delta x' = \zeta_x' \Delta z$

ζ_x'=0	L/L_0=1
0.05	0.98
0.1	0.93

The dispersion can be set much less than these value.

Summary

- We studied the beam-beam limit using vaious simulations.
- The beam-beam limit was 0.1~0.15 for super KEKB with the crab crossing.
- Symplectic diffusion was very weak for head-on collision. How is proton beam?
- Radiation excitation played important role for the beam-beam limit.
- Crossing angle and x-y coupling induced symplectic (Arnold) diffusion.
- The beam-beam limit is degraded to be 0.06~0.08 due to the crossing angle.
- Other errors induce Arnold diffusion.
- To go to higher luminosity, the crab crossing and tuning are important.