Beam Based Gain Calibration Method for KEKB Beam Position Monitors

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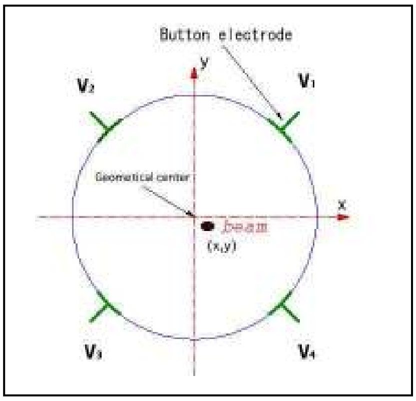
Introduction

In KEKB, Beam Position Monitor (BPM) has been calibrated the geometrical center of each head before the commissioning. However, it is afraid that variations of performance of these calibrations break the balance of the output of each head and make its electric center wander.



1 Beam based gain calibration

1.1 Modeling of output data

The output data from four electrodes are given by

$$V_i = g_i \cdot q \cdot F_i(x,y), \quad i = 1,2,3,4$$

Where

q: beam charge

(x,y): relative displacement to the geometrical center.

$F_i(x,y)$: response function of four electrodes.

g_i: gains of each electrode.

The g_i includes the impedance imbalance which is caused by cables, connectors, attenuators, switches and etc.

$F_i(x,y)$ is given by polynomials

$$\begin{aligned} F_i(x,y) = 1 & \\ & + a_{1i}x + b_{1i}y \\ & + a_{2i}\left(x^2 - y^2\right) + b_{2i}(2xy) \\ & + a_{3i}\left(x^3 - 3xy^2\right) + b_{3i}\left(3x^2y - y^3\right) \\ & + a_{4i}\left(x^4 - 6x^2y^2 + y^4\right) + b_{4i}\left(x^3y - xy^3\right) \end{aligned}$$

$$\begin{cases} F_i(0,0) = 1 \quad \cdots \quad \textit{normalized} \\ F_2(x,y) = F_1(-x,y),\ F_3(x,y) = F_1(-x,-y),\ F_4(x,y) = F_1(x,-y) \quad \cdots \quad \textit{symmetry} \end{cases}$$

These coefficients were decided with model mapping calculated by boundary element method.

1.2 Gain estimation

Measurement of m times beam mapping

Output voltage at each position $x_j, y_j \ (j = 1,,,m)$ are given by

$$V_{i,j} = g_i \cdot q_j \cdot F_i(x_j, y_j), \quad (i = 1,2,3,4, \quad j = 1,,,m)$$

Analysis by Non-linear least square method

- Unknown parameter (3+3m) …………… g_i, q_j, x_j, y_j

  but $g_1 = 1, g_2 = G_2/G_1, g_3 = G_3/G_1, g_4 = G_4/G_1$

- Measurements (4m)……………………… $v_{1,j}, v_{2,j}, v_{3,j}, v_{4,j}$

→ Least square method (m>3)

$$\sum_{i=1}^{4}\sum_{j=1}^{m}(V_{ij} - v_{ij})^2 \Rightarrow \text{minimum}$$

2 Consistency check method between four ways of beam positions

Usually the beam position is calculated from the output of four electrodes (A, B, C and D) of a BPM.

$$\begin{cases} h = (V_A - V_B - V_C + V_D)/(V_A + V_B + V_C + V_D) \\ v = (V_A + V_B - V_C - V_D)/(V_A + V_B + V_C + V_D) \end{cases} \qquad \begin{cases} x = F_x^{ABCD}(h, v) \\ y = F_y^{ABCD}(h, v) \end{cases}$$

Then four beam positions are also obtainable from the output voltage of any three electrodes chosen out of four electrodes.

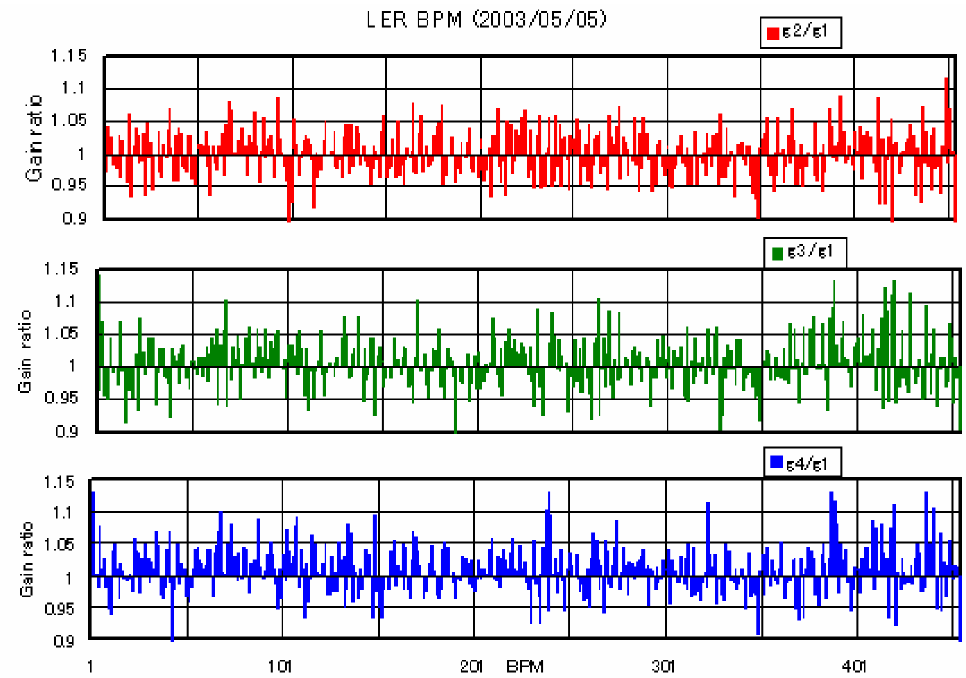
$$\begin{cases} h_1 = \dfrac{V_A - V_B}{V_A + V_B}, \quad h_2 = \dfrac{V_D - V_C}{V_D + V_C} \\ v_1 = \dfrac{V_B - V_C}{V_B + V_C}, \quad V_2 = \dfrac{V_A - V_D}{V_A + V_D} \end{cases} \qquad \begin{cases} x_1 = F_x^{ABC}(h_1, v_1), \quad y_1 = F_y^{ABC}(h_1, v_1) \\ x_2 = F_x^{BCD}(h_2, v_1), \quad y_2 = F_y^{BCD}(h_2, v_1) \\ x_3 = F_x^{ACD}(h_2, v_2), \quad y_3 = F_y^{ACD}(h_2, v_2) \\ x_4 = F_x^{ABD}(h_1, v_2), \quad y_4 = F_y^{ABD}(h_1, v_2) \end{cases}$$

If the four outputs have ideal correlation, these four beam positions { (A, B, C), (B, C, D), (C, D, A), (D, A, B) } should coincide with each other

$$\begin{cases} Consistency\_x = std.dv.(x_1, x_2, x_3, x_4) \\ Consistency\_y = std.dv.(y_1, y_2, y_3, y_4) \end{cases}$$

3 Calibration results

Beam based gain calibration was performed for the first time in 2003.



As a result of having corrected the output voltage with a new gain, the consistency became very small.