

# **Machine Parameters and Lattice Design**

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		LER	HER	
Beam Energy	$E$	3.5	8.0	GeV
Luminosity	$\mathcal{L}$	$1.0 \times 10^{34}$		$\text{cm}^{-2}\text{s}^{-1}$
Luminosity Reduction Factor	$R_{\mathcal{L}}$	0.845		
Half crossing angle	$\theta_x$	11		mrاد
Tune shifts	$\xi_x/\xi_y$	0.039/0.052		
Tune shift reductions	$R_{\xi_x}/R_{\xi_y}$	0.737/0.885		
Beta functions	$\beta_x^*/\beta_y^*$	0.33/0.01		m
Beam current	$I$	2.6	1.1	A
Bunch spacing	$s_b$	0.59		ns
Particles/bunch	$N$	$3.3 \times 10^{10}$	$1.4 \times 10^{10}$	
Number of bunches/ring	$N_B$	5000		
Emittance	$\varepsilon_x/\varepsilon_y$	$1.8 \times 10^{-8}/3.6 \times 10^{-10}$		m
Bunch length	$\sigma_z$	4		mm
Momentum spread	$\sigma_\delta$	$7.1 \times 10^{-4}$	$6.7 \times 10^{-4}$	
Synchrotron tune	$\nu_s$	0.01~0.02		
Momentum compaction factor	$\alpha_p$	$1 \times 10^{-4} \sim 2 \times 10^{-4}$		
Betatron tunes	$\nu_x/\nu_y$	45.52/46.08	47.52/43.08	
Circumference	$C$	3016.26		m
Damping time	$\tau_E$	44.9	22.5	ms

Table 2.2: Machine Parameters of KEKB.

# Luminosity, tune shift, beam intensity

KEKB is a double-ring asymmetric e-e<sup>+</sup> collider at 3.5GeV×8GeV. Its target peak luminosity is 10<sup>34</sup>cm<sup>-2</sup>s<sup>-1</sup>.

Fundamental equations on the luminosity and the vertical beam-beam tune shift:

$$L = \frac{N_1 N_2 f}{4\pi\sigma_x^* \sigma_y^*} R_L(\theta_x, \beta_x^*, \beta_y^*, \epsilon_x, \epsilon_y, \sigma_z)$$

$$\xi_{yk} = \frac{N_{k-3} r_e \beta_{ky}^*}{2\pi\gamma_k (\sigma_x^* + \sigma_y^*) \sigma_y^*} R_{\xi_y}(\theta_x, \beta_x^*, \beta_y^*, \epsilon_x, \epsilon_y, \sigma_z)$$

$N_{1,2}$  the number of particles per bunch,

$f$  the bunch collision frequency,

$\theta_x$  the the half crossing angle.

The suffix k=1,2 specifies each beam in HER and LER.

$R_L$  and  $R_{\xi_y}$  represent reduction factors for the luminosity and the vertical tune shift due to the crossing angle and the hour-glass effect.

We simply set  $\xi_y, \beta_{x,y}^*, \epsilon_{x,y}$ , and  $\sigma_z$  equal for the two beams.

The possibility of a round-beam collision has been excluded.

By assuming equal beam-parameters and flat beams,

$$L = \frac{\gamma_k I_k \xi_y}{2e r_e \beta_y^*} \frac{R_L}{R_{\xi_y}}$$

$I_k$  the current for each beam.

If  $\beta_y^* \gg \sigma_z$ , the ratio of the reduction factors above becomes close to a unity:

$$\frac{R_L}{R_{\xi_y}} \approx 1$$

To a good approximation:

$$L \approx \frac{\gamma_k I_k \xi_y}{2e r_e \beta_y^*}$$

While the luminosity may be reduced due to the crossing angle, the beam-beam interaction is also reduced by approximately the same ratio. If the dynamics of the beam-beam interaction with the crossing angle allows the same value of  $\xi_y$  as the head-on collision, there is no loss of the luminosity for a fixed total beam current.

In the design of KEKB, we assume

$$\xi_y \approx 0.05 \text{ and } \beta_y^* = 1 \text{ cm.}$$

Beam intensities required for  $L = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$  are 2.6A for the LER and 1.1A for the HER.



## Crossing angle

Several beam separation schemes based on horizontal bend magnets, a finite beamcrossing angle, and their mixed combinations have been examined for KEKB.

A moderately large crossing angle has a big advantage:

*~ 10 mrad*

eliminate the need for separation bend magnets,  
reduce the critical energy of synchrotron radiation,  
avoid parasitic crossing effects,  
permit 0.6 m bunch spacing.

We have chosen 11 mrad.

It is close to the minimum that allows to eliminate IP separation bend magnets.

It is also nearly the maximum that allows final focusing both beams at IP with common quadrupole magnets.

The use of a crab-crossing scheme with superconducting cavities is also being considered.

## Bunch length

Shorter bunch length is preferred for  
reduce intrinsic synchrotron-betatron  
coupling in the beam-beam interaction,  
reduced hour-glass, and  
reduced crossing-angle effects.

The lower limit is given by the single-bunch longitudinal instability, Touschek life time, and the required accelerating voltage.

We choose  $\sigma_z \geq 4\text{mm}$ , which is close to the minimum possible value.

The bunch-lengthening due to the potential-well distortion needs to be taken into account. The target value includes the bunch-lengthening effect of 20% in the LER.

## Bunch spacing

The allowed bunch spacing will be integer multiple of 0.6m, because we reuse the TRISTAN RF resources which are based on 500MHz.

Since the total beam current has been determined, the number of particles per bunch is proportional to the bunch spacing.

The limit on the number of particles per bunch comes from the longitudinal single-bunch threshold for the LER.

At KEKB, the threshold bunch intensity is about 2.5 times the bunch intensity for 0.6m bunch spacing. Then either 0.6m or 1.2m is the possible choice.

The vertical beam-beam tune shift can be rewritten as

$$\xi_{yk} = \frac{N_{k-3} r_e}{2\pi\gamma_k} R_{\xi y} \sqrt{\frac{\beta_y^*}{\kappa\beta_x^*} \frac{1}{\epsilon_x}}$$

where  $\kappa$  is the ratio of the horizontal and vertical emittance. Thus we obtain

$$\epsilon_x \propto \frac{N}{\sqrt{\kappa\beta_x^*}}$$

If  $\kappa$  and  $\beta_x^*$  are kept constant, the required horizontal emittance is proportional to the bunch spacing.

The emittance must be increased to maintain the sufficient Touschek beam lifetime for an increased bunch intensity.

The maximum emittance is limited by design considerations on the IR. When the beam emittance is increased for the fixed  $\beta_x^*$  at IP, the angular divergence at IP is also increased and it results in an increased synchrotron radiation background to the detector.

We have chosen 0.6 m.



# Emittance

When the bunch spacing,  $\beta_x^*$  and  $\kappa$  are given, the horizontal emittance is determined.

Although a smaller  $\beta_x^*$  is preferred for larger  $\kappa$ , there is a limit given by the angular divergence limit at IP.

We have chosen the horizontal emittance so that the luminosity given by the strong-weak beam-beam simulation is maximized in the allowable range.

The lattice design incorporates quadrupole magnet 'knobs' so that it can vary the horizontal emittance in the range of

This is to manage possible changes of the bunch spacing, angular divergence, beam intensity, and the emittance ratio in actual operating conditions. For instance, a full current operation with 1.2 m instead of 0.6 m will be possible.



## Momentum spread

The momentum spread of the beam is set to be  $\sim 0.07\%$ , which is close to the upper limit value from the high energy physics experiment viewpoint.

It is hard to reduce the energy spread much below  $0.07\%$  without decreasing the radiation damping rate.

## Synchrotron tune

The last issue among the choice of the basic parameters is the synchrotron tune  $\nu_s$  and the momentum compaction factor  $\alpha_p$ .

Since the bunch length and the momentum spread have been determined,  $\nu_s$  and  $\alpha_p$  are not independent. They are related as:

$$\sigma_z = \frac{c \alpha_p}{\omega_s} \sigma_\delta$$

where  $\omega_s$  is the synchrotron frequency.

Generally speaking, a large synchrotron tune induces synchro-betatron resonances due to lattice nonlinearity effects and beam-beam interactions.

It also causes an anomalous emittance growth at synchro-betatron resonance lines.

A small value of  $\nu_s \leq 0.02$  is necessary to have a sufficiently large tune space as the possible operational parameter space.

On the other hand, a small  $v_s$  decreases the threshold for single-bunch instabilities. Also, the higher-order momentum compaction can be more harmful with small  $\alpha_p$  for synchrotron motions with large amplitudes.

The lattice design of KEKB can vary the momentum compaction in the range

$$-1 \times 10^{-4} \leq \sigma_p \leq 4 \times 10^{-4}$$

without affecting the horizontal beam emittance.

# Requirements for Lattice Design

1. Realize the beam parameters listed in the table.
2. Ensure large dynamic apertures for a high injection efficiency and a long beam lifetime, in particular the Touschek lifetime in LER.
3. Maintain a wide range to tunability for the beam parameters, especially for the horizontal emittance.
4. Allow a reasonable amount of tolerances for machine errors.



The main problem in designing low  $\beta^*$  lattice is how to correct the large chromaticity produced by final focus quadrupoles.

To solve this problem, we adopt a noninterleaved chromaticity correction scheme.

To cancel transverse nonlinearities of sextupoles, the best way is to make pairs of identical sextupoles, which are connected by the  $-I$  transformer in both horizontal and vertical planes.

If no other sextupoles exist between the paired ones, main transverse nonlinearities are canceled within a pair up to the third order in the Hamiltonian.

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ m_{21} & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & m_{43} & -1 \end{pmatrix}$$

$$m_{21} \neq 0$$

$$m_{43} \neq 0$$

pseudo  $-I$



	Injection	Touschek	$\nu_s$
Interleaved $\pi/3$ FODO Cell	bad	bad	bad
Noninterleaved $\pi/2$ FODO Cell	good	fair	bad
Noninterleaved $\pi$ Cell	excellent	fair	good
Noninterleaved $2.5\pi$ Cell	excellent	good	excellent
Noninterleaved $2.5\pi$ Cell +Local Chromaticity Correction	excellent	excellent	excellent

Table 6.1: Comparison of performances of cell structures and chromaticity correction schemes for the LER.

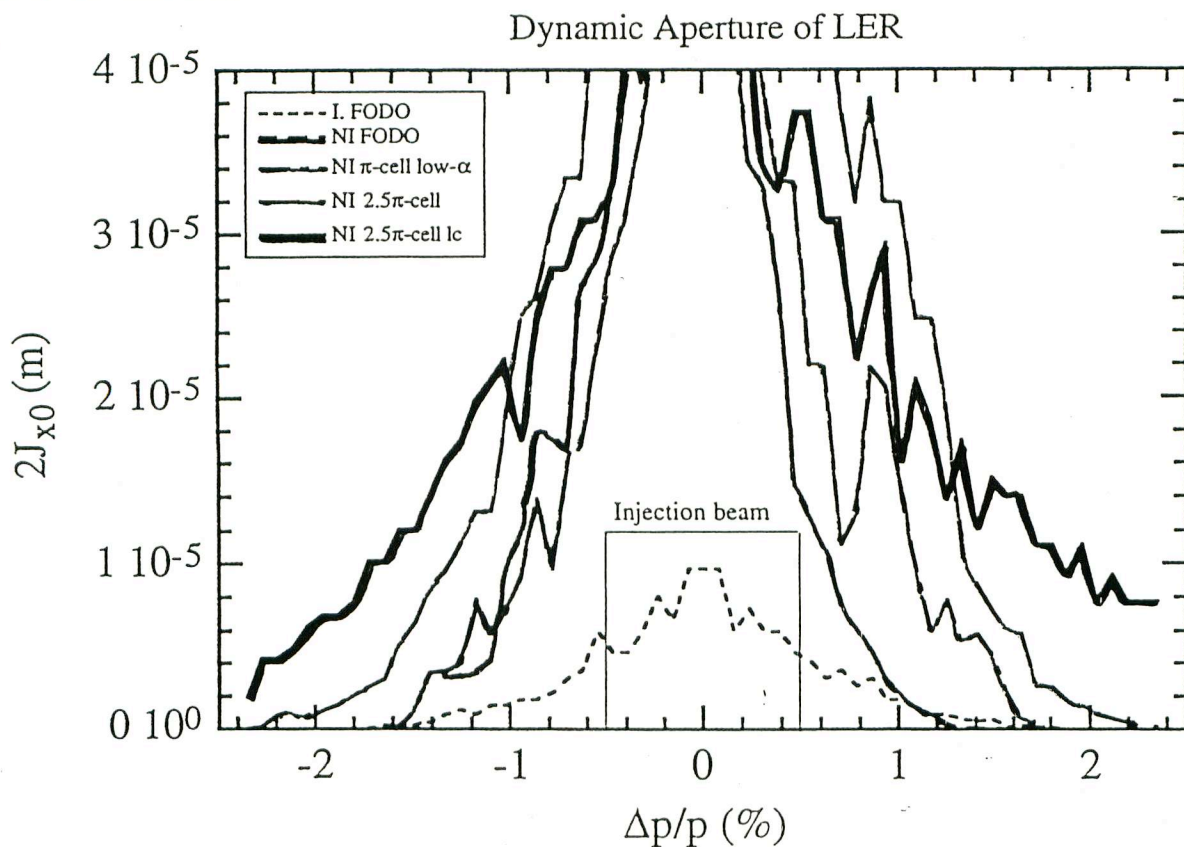
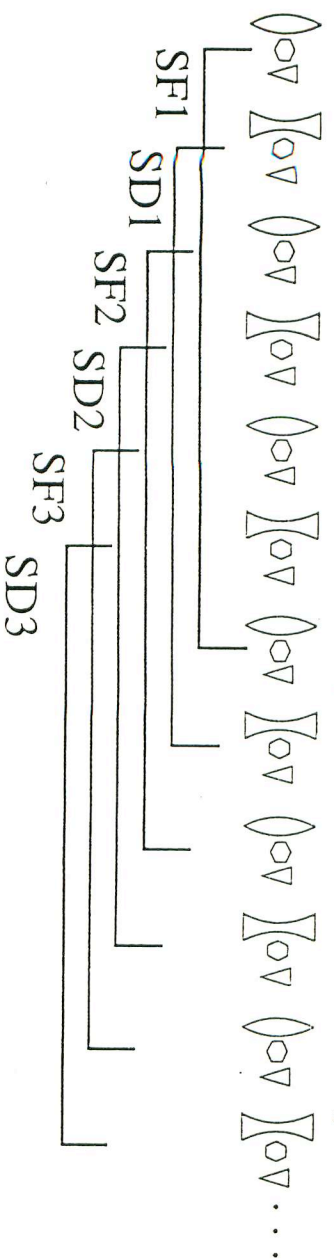


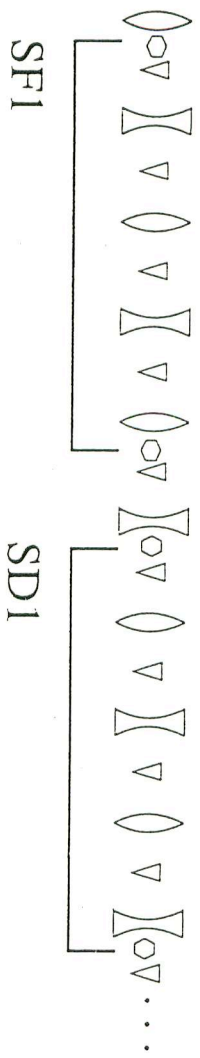
Figure 6.1: Dynamic aperture of the LER with five types of beam optics.

**Interleaved FODO ( $\pi/3$ )**

Heavy interference between sextupoles

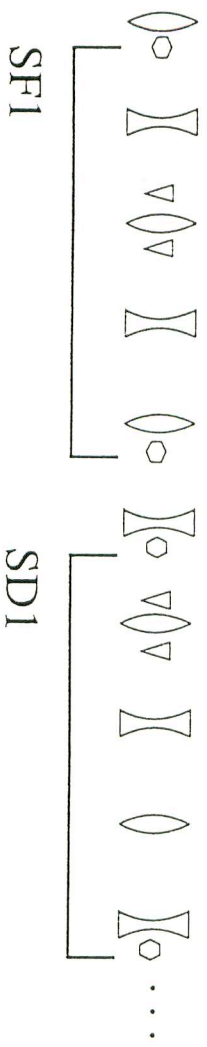


**Non-interleaved FODO ( $\pi/2$ ) High momentum compaction**



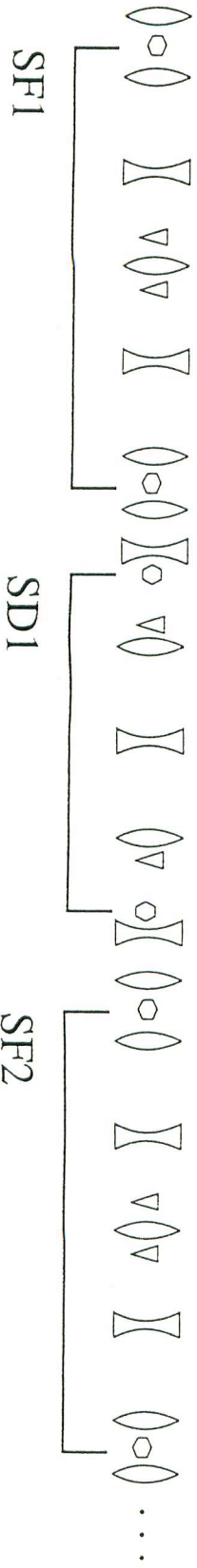
**Non-interleaved  $\pi$ -cell**

Can't put SFs at  $\pi/2$  phases



**Non-interleaved 2.5 $\pi$ -cell**

Efficient allocation of sexts + flexible emittance/momentum compaction



# Development of Beam Optics Design

We have studied five types of optics design:

1. **Interleaved  $\pi/3$  FODO Cell,**
2. **Non-interleaved  $\pi/2$  FODO Cell,**
3. **Non-interleaved  $\pi$  Cell,**
4. **Non-interleaved  $2.5 \pi$  Cell,**
5. **Non-interleaved  $2.5 \pi$  Cell  
+ Local Chromaticity Correction.**

These optics have different combinations of cell structures and chromaticity correction scheme.

We have compared the performance of these optics in the light of the following requirements:

1. To have a sufficiently large dynamic aperture for the beam injection,
2. To have a sufficiently large dynamic aperture for the Touschek lifetime,
3. To give small synchrotron tune.

## 1. Interleaved $\pi/3$ FODO Cell

$\pi/3$  phase advance per cell

Each quadrupole is associated with a sextupole.

Sextupoles with the  $\pi$  phase difference are paired to cancel the lowest order of the transverse nonlinearity.

Although we have tried chromaticity corrections with 6, 12, and 24 sextupole families, the dynamic aperture remained too small and did not satisfy the requirement.

## 2. Non-interleaved $\pi/2$ FODO Cell

$\pi/2$  phase advance per cell to install noninterleaved sextupole pairs.

The bending radius is determined from the requirement for the momentum spread.

Once the bending radius is fixed, we have only one free parameter: the horizontal tune of the arc, or the length of the cell.

If the arc is built with FODO cell to give the design emittance, the momentum compaction factor becomes 4 times bigger than the required.

This means  $\nu_s \sim 0.06$ , which gives difficult condition to find good operating points in the tune space.

The RF voltage becomes too high to achieve the design bunch length.



### 3. Non-interleaved $\pi$ Cell

To obtain small  $\alpha_p$ , we must reduce the horizontal dispersion at dipoles, while keeping the emittance.

This can be done by combining two  $\pi/2$  FODO cells, where four dipoles are merged into two.

In this scheme, we have two free parameters, the horizontal tune of the arc and the position of dipoles. Then we can achieve the desired  $\alpha_p$  and  $\epsilon_x$  simultaneously for LER.

In HER, since the dipoles are longer, the possible range of  $\alpha_p$  is narrower.

From a chromaticity correction view point, the arc built with  $\pi$  cell has a disadvantage that peaks of the horizontal dispersion appears only the phase step of  $N\pi$ .

If we place sextuples for horizontal correction only near the dispersion peaks, chromaticity correction at  $(N+1/2)\pi$  phase becomes difficult.

Consequently, the momentum aperture is unsatisfactory for the Touschek lifetime.

# $\pi$ cell

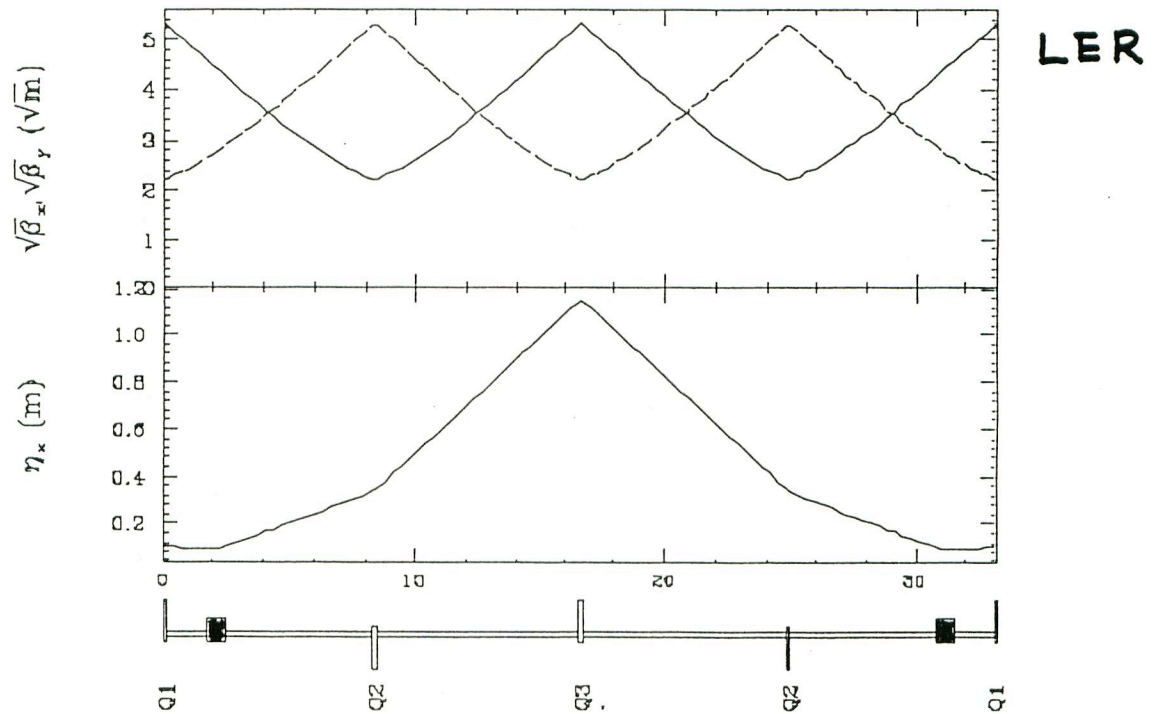
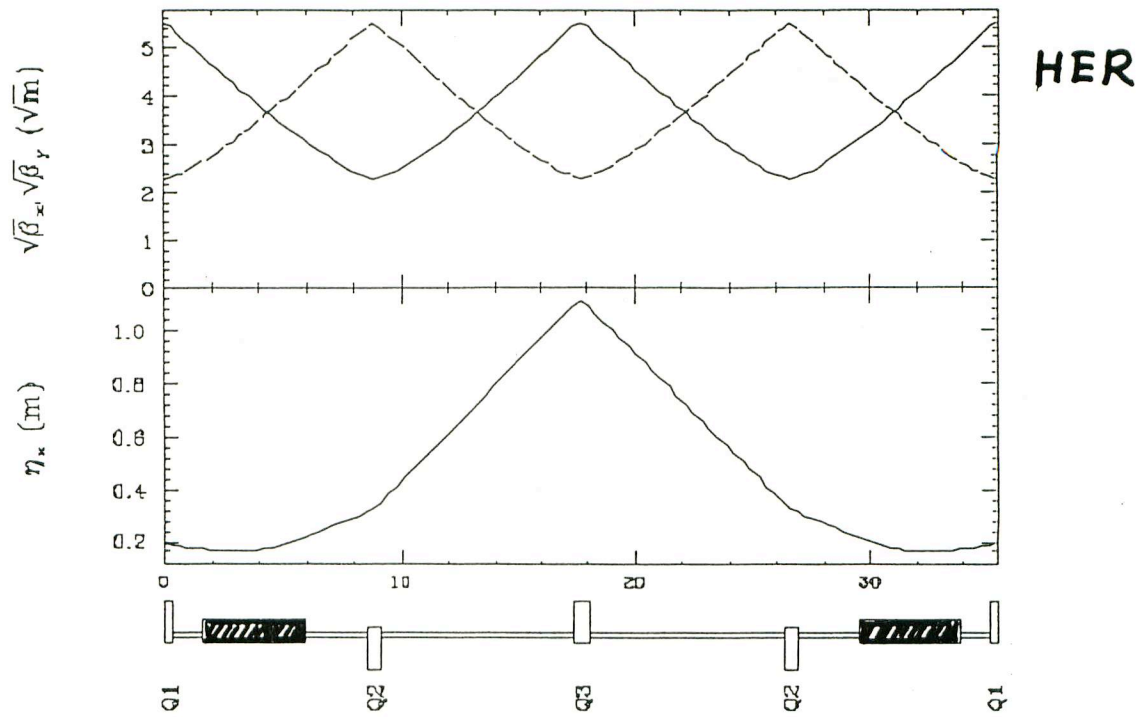


Figure 1.2: Structure of the  $\pi$  cell for the HER (above) and the LER (below).

## 4. Non-interleaved $2.5\pi$ Cell

The  $2.5\pi$  cell is made by combining five  $\pi/2$  FODO cells and by merging ten dipoles into four. In this cell, the dipoles are placed to form two dispersion bumps so that we can keep the dispersion small at the dipoles.

By adjusting the positions of dipoles and the dispersion there, we can achieve the required  $\alpha_p$  and  $\epsilon_x$  simultaneously both for HER and LER.

The  $2.5\pi$  cell enable us to install noninterleaved sextupole pairs efficiently. Successive SF(SD) pairs in the  $2.5\pi$  cell have the relative phase of  $3\pi/2$ . Thus chromatic kicks at  $N\pi$  and  $(N+1/2)\pi$  phases in both horizontal and vertical planes can be correct effectively. The dynamic aperture of the  $2.5\pi$  cell is significantly improved and satisfies all of the requirements.

Higher-order chromaticities still remain, because the sextuples are not sufficiently close to the main chromaticity source in the IR.

# 2.5 $\pi$ cell

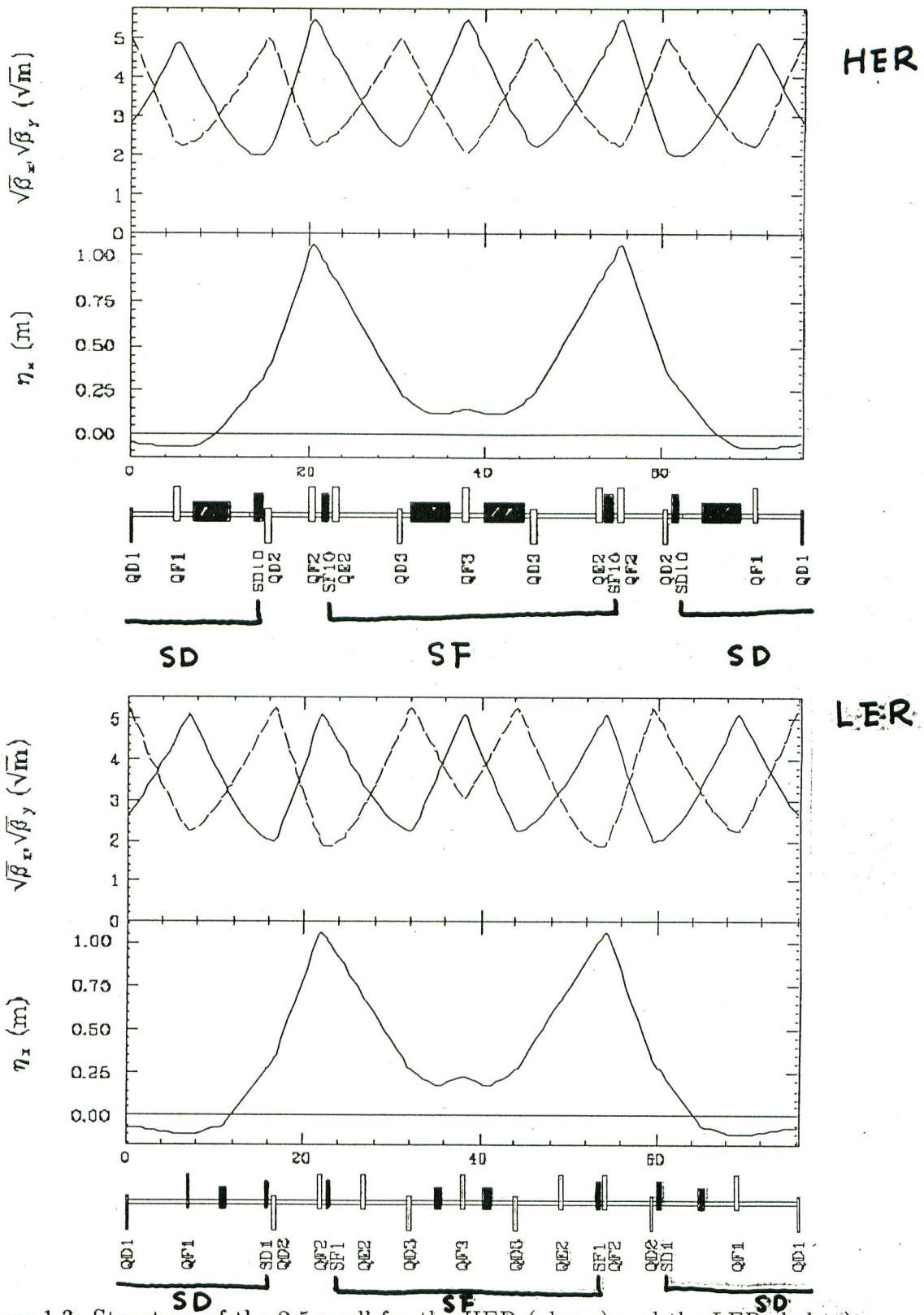


Figure 1.3: Structure of the 2.5π cell for the HER (above) and the LER (below).



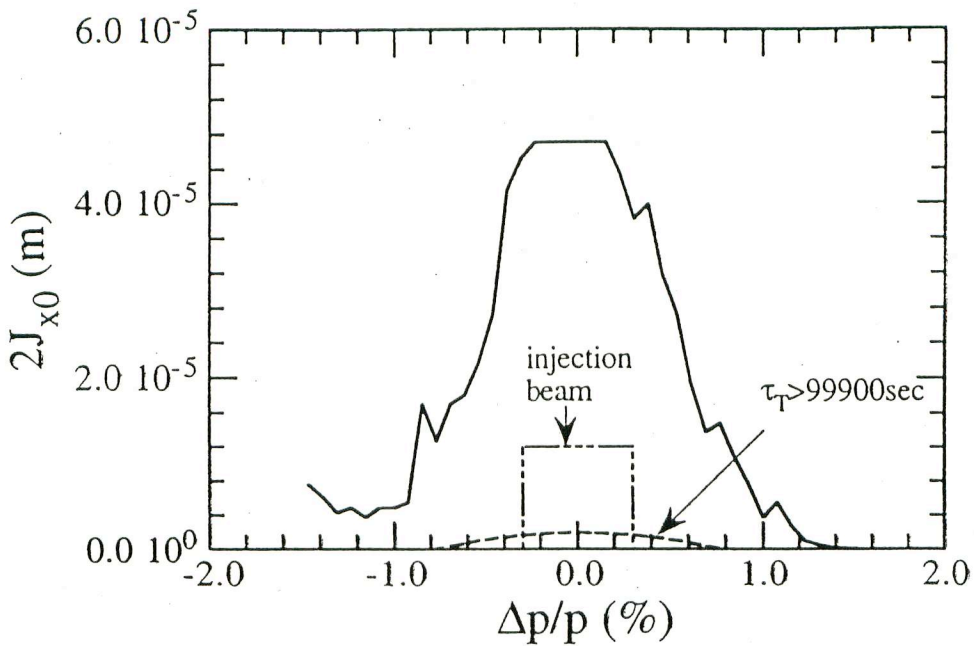


Figure 5.4: Dynamic apertures of HER with the  $2.5\pi$  cell.

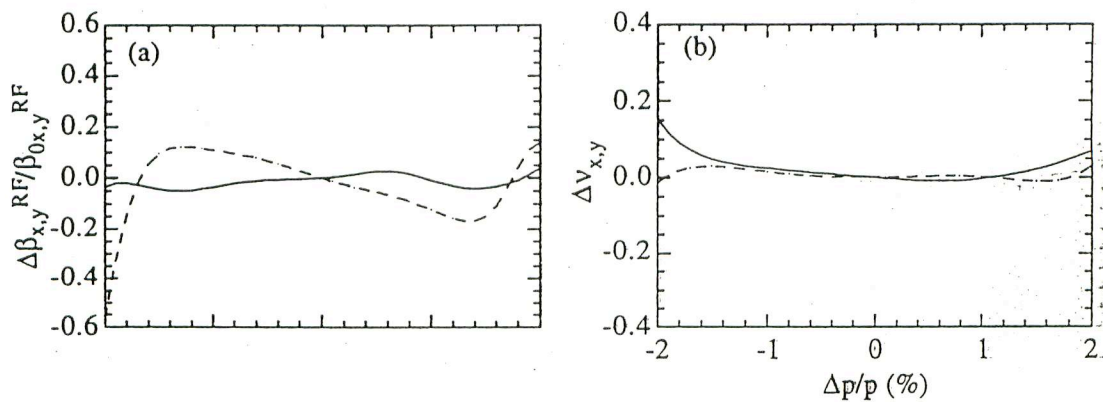


Figure 5.5: Chromaticity correction with the  $2.5\pi$  cell for LER.

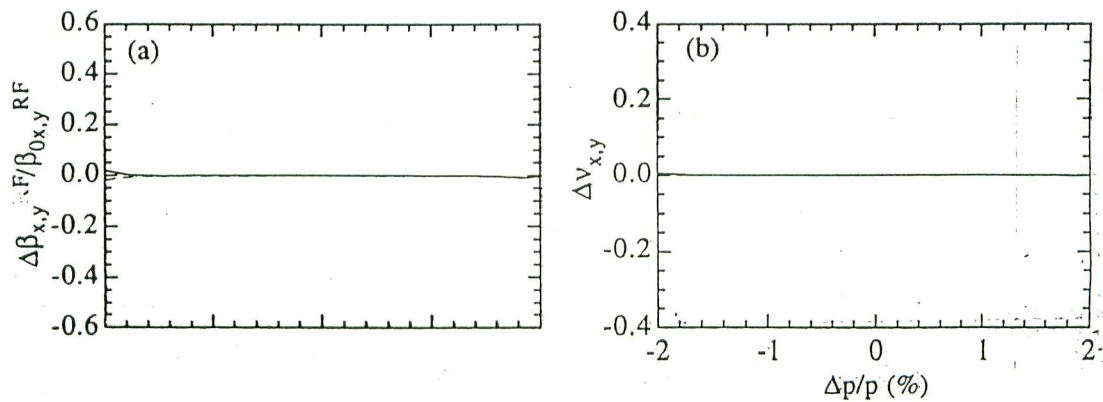


Figure 5.6: Chromaticity correction with the  $2.5\pi$  cell and the local chromaticity correction for LER.



## 5. Non-interleaved $2.5\pi$ Cell+Local Chromaticity Correction

The local chromaticity correction is a scheme that the large chromaticity produced by the final quadrupoles is corrected within the IR.

We can avoid creating higher-order chromaticities by the sextupoles placed as optically close to the final quadrupoles. ( The phase difference between the source and the sextupole pairs is  $\pi$ .)

It is difficult to install two sextupole pairs for correction of both horizontal and vertical places. We place only one pair for the vertical correction in each side of the IP in the IR.

The last pairs at the end of the arc are used for the horizontal correction.

We design the local chromaticity correction optics by minimizing the momentum dependence of optical functions in the bandwidth of 2~3%.

The local correction has significantly improved the chromaticity correction and also the dynamic aperture in the region of large momentum deviations.

# Local Chromaticity Correction

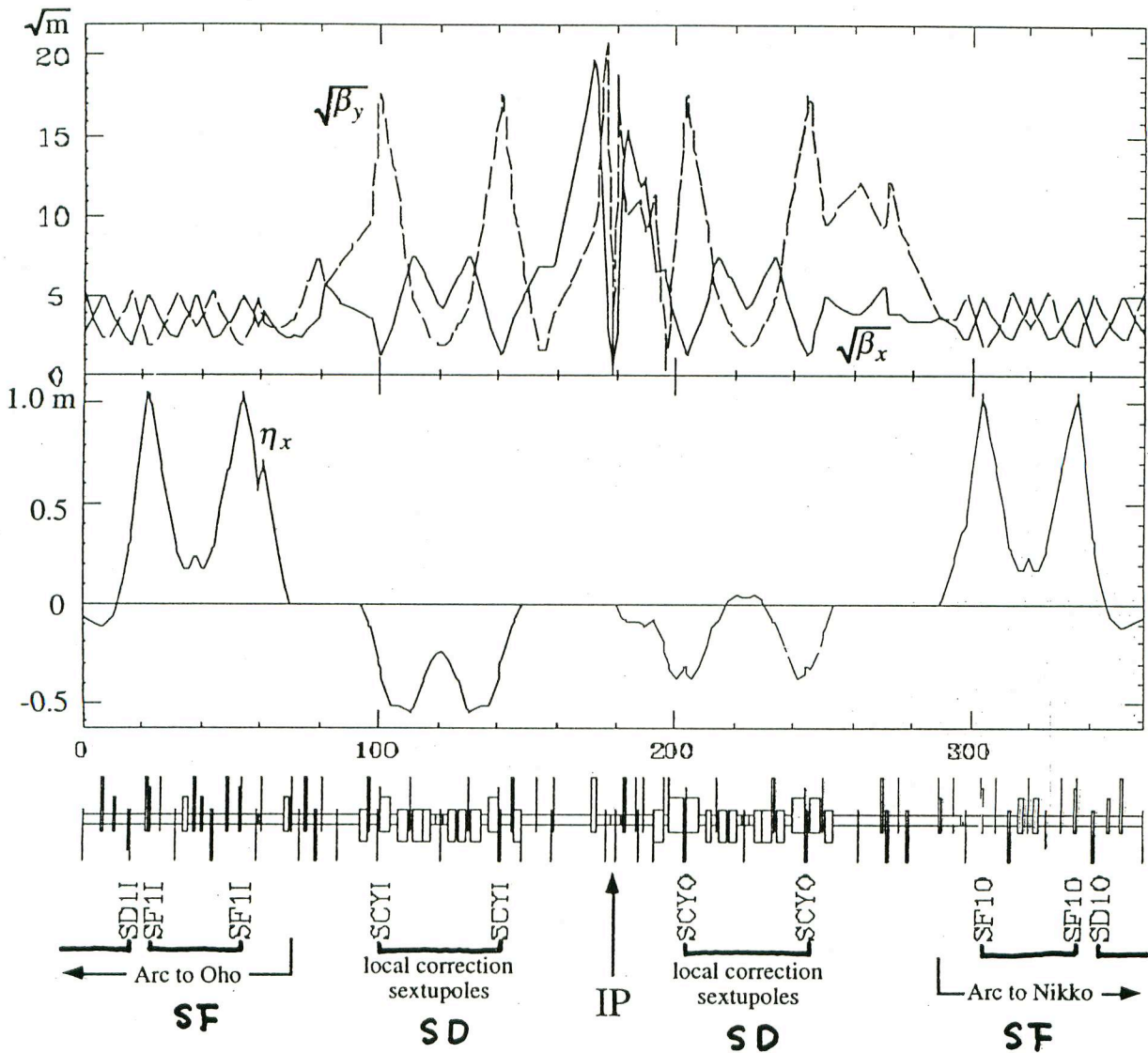


Figure 1.7: Optics of the local chromaticity correction for the LER. A pair of sextupoles for the vertical chromaticity correction is placed at each side of the IP.

## Tunability of Emittance and Momentum Compaction Factor

In the  $2.5\pi$  cell, we have two tunable knobs, the excitation of QF2's and QD2's, without breaking the pseudo -I condition.

We can change the momentum compaction factor in the range of

$$-1 \times 10^{-4} \leq \alpha_p \leq 4 \times 10^{-4}$$

by changing the strength of QF2's and QD2's by a few percent. ( $\epsilon_x$  is kept constant during this time.)

Since it is difficult to change the emittance in a wide range by only adjusting QF2's and QD2's, we introduce QE2's.

By using five free parameters (QF2, QF3, QD2, QD3, and QE2), we can obtain the required tunability

$$1.0 \times 10^{-8} \text{ m} \leq \epsilon_x \leq 3.6 \times 10^{-8} \text{ m}$$

while keeping  $\alpha_p$  constant and maintaining the pseudo -I condition between the SF's.

In summary, we can tune independently  $\alpha_p$  and  $\epsilon_x$  by adding QE2's.

# Emittance Control (LER)

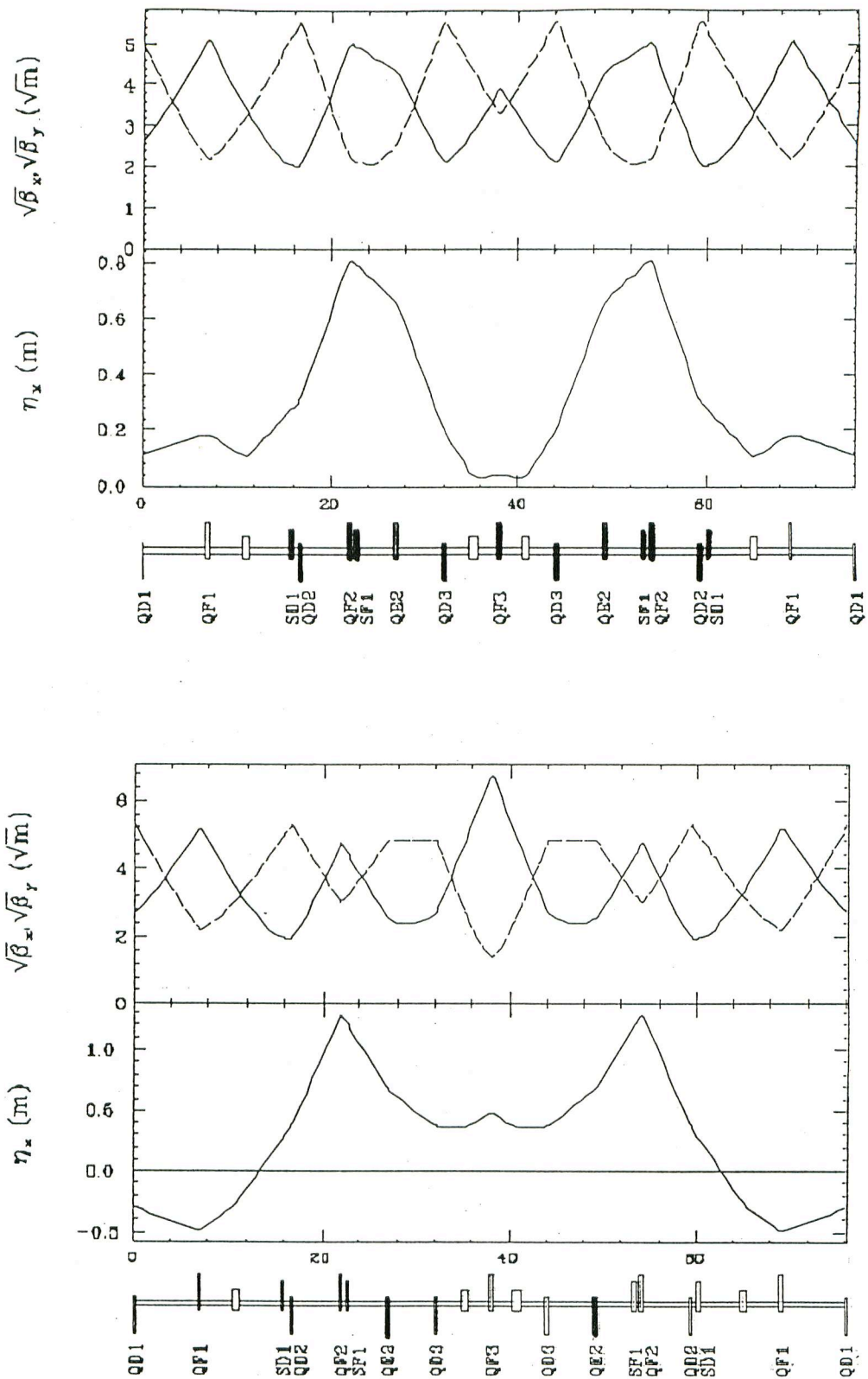


Figure 5.9: Examples of emittance control in LER;  $1.0 \times 10^{-8} \text{m}$  (above) and  $3.6 \times 10^{-8} \text{m}$  (below).



## Compensation of Detector Solenoid Field

One of the important issues in designing optics around the IP is compensation of the x-y coupling effects generated by the detector solenoid field.

Due to the finite crossing angle at the IP, the design beam orbits are not parallel to the detector solenoid axis. Thus dispersion is also generated by the solenoid.

Corrections have to be made for four coupling elements of the transfer matrix between the IP and the arc, and the dispersion near the IP.

A possible solution to this is to add four or more skew quadrupole magnets and several bend magnets on each side of the IP. This correction, however, is perfect only for on-momentum particles. The chromaticity of the x-y coupling remains uncorrected and it can significantly increase the "anomalous emittance" at

The best way to compensate the x-y coupling effects of the detector solenoid is to use counter solenoids so that the integrated field,

becomes zero within the drift space around the IP. This compensation works perfectly to particles with any momentum.

This is the solution that has been adopted in the design of KEKB.



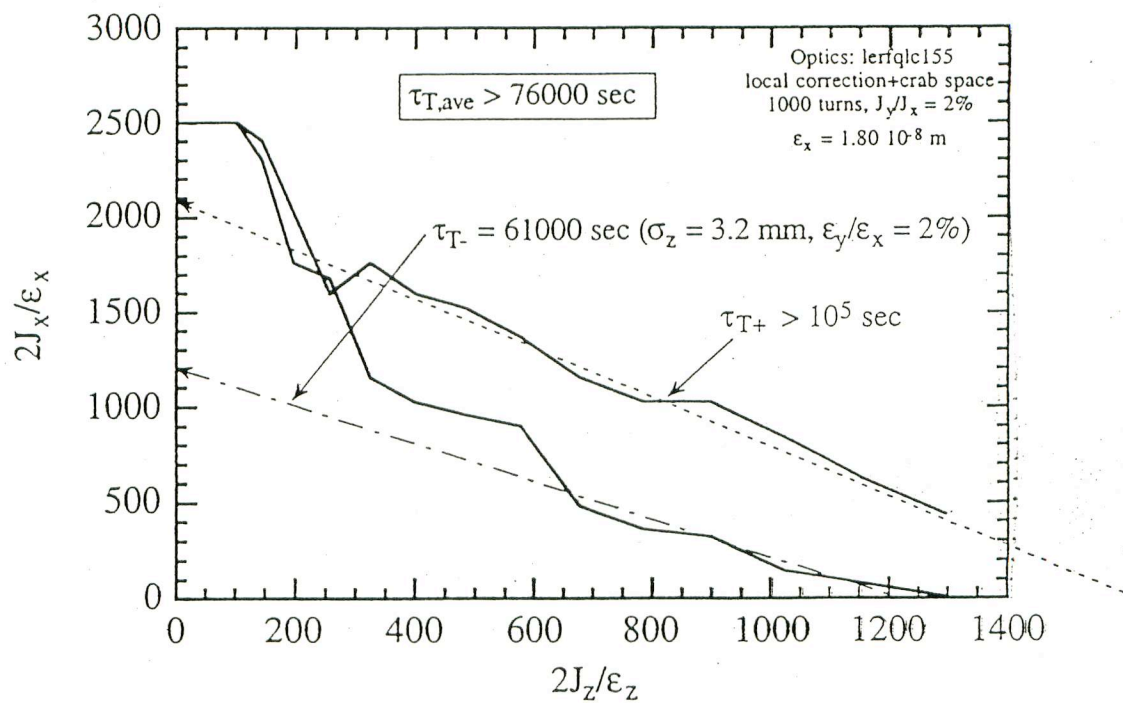


Figure 5.8: Dynamic aperture and Touschek lifetime of LER with the local chromaticity correction.

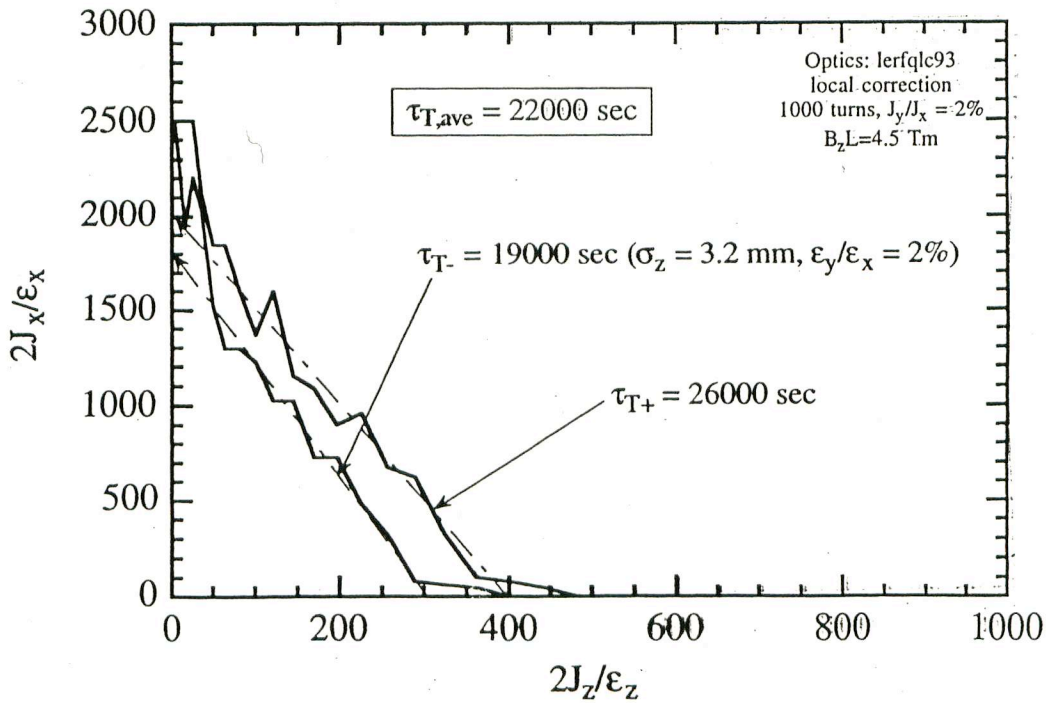
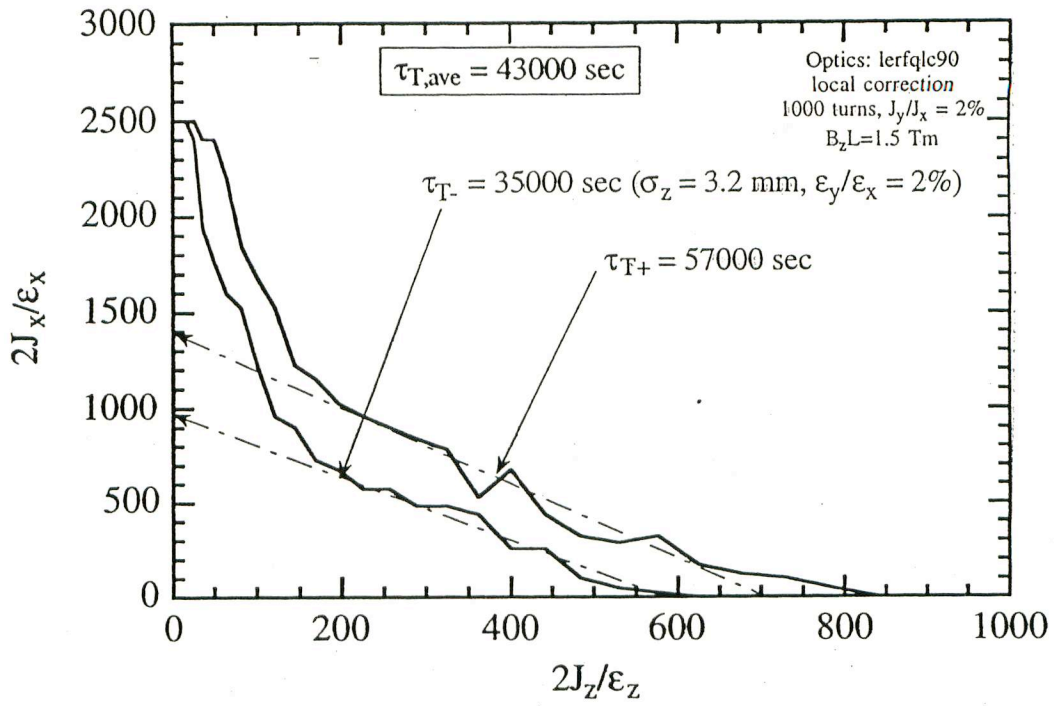


Figure 6.10: Dynamic apertures of the LER with the integrated solenoid field of 1.5 Tm (above) and 4.5 Tm (below).

## Crab Cavities

An optional use of a crab crossing scheme is currently being studied.

To minimize the required number of crab cavities, the transverse kicks given by the cavities should have a maximum effect on the bunch orientation at the IP.

This means that the horizontal phase distance of the crab cavities from the IP should be  $(N+1/2) \pi$ .

From the lattice design view point, it is possible to reserve such a dispersion-free drift space in the straight section near the arc.

The dynamic aperture has been also checked with the crab cavities.

The dynamic aperture is quite insensitive to the crab cavities, even with a big amount of errors in the amplitudes and phases of the crab-mode RF kicks.

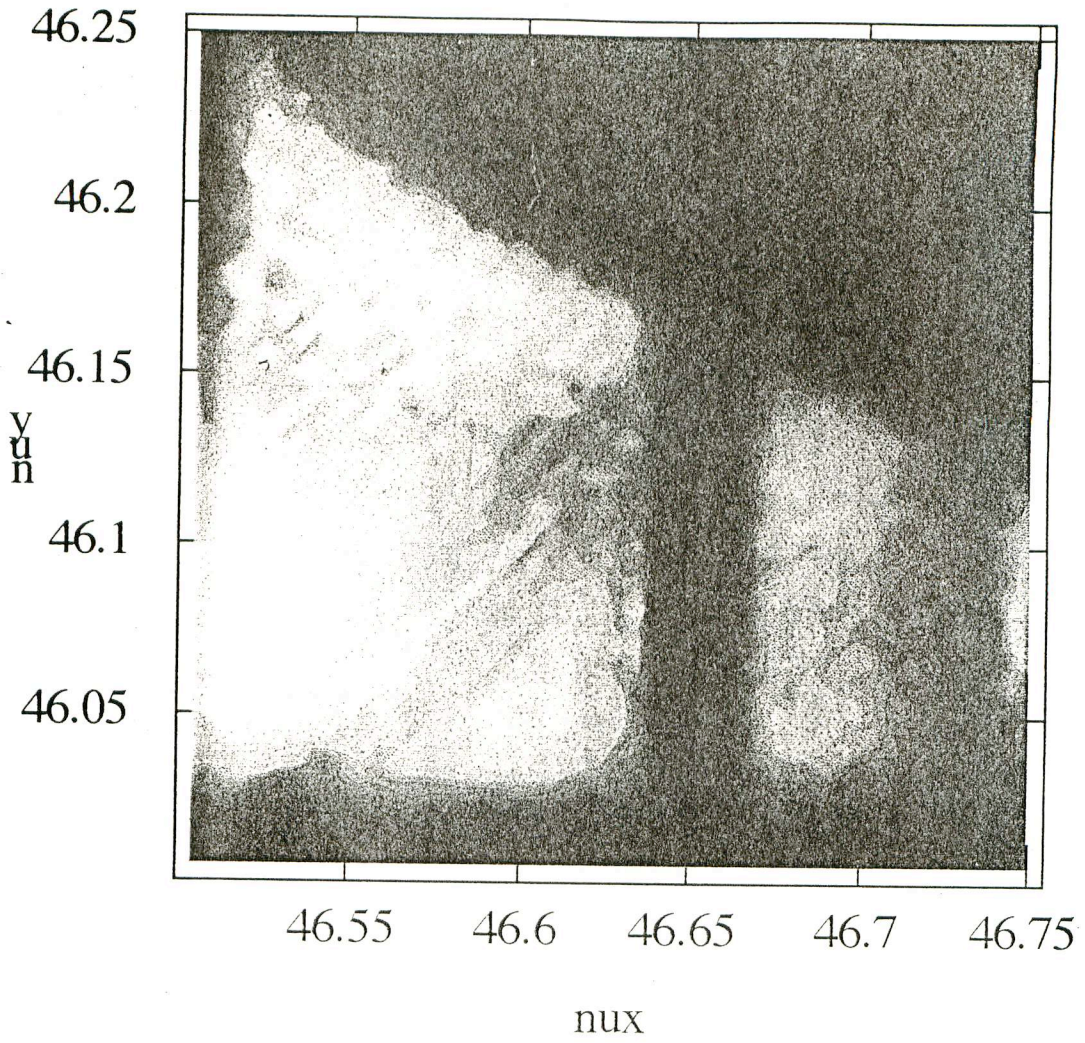
## Conclusions

We have designed the lattice consistent with the goal peak luminosity of  $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ .

The lattice based on the non-interleaved  $2.5\pi$  cells with the local chromaticity correction satisfies all of the requirements for the beam parameters and the dynamic apertures.

Detailed study of tolerances on the dynamic aperture and the emittance ratio  $\epsilon_y/\epsilon_x$  needs to be done.





# Dynamic Aperture

