# Fast Ion Instability

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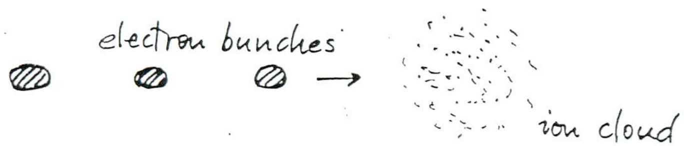
KEKB. MAC.

## Ion trapping

- Ion accumulation over many turns




  ⇒ two stream instability

- Usually cured by partil fill

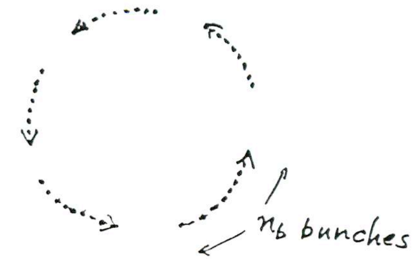
## Fast ion instability

- If high intensity / low emittance,
  serious growth without ion accumulation over many turns
- Number of ions $\propto$ electron bunch position in a train



# Linear Theory

## Basic Assumptions




- Ions disappear in the bunch gap.
  ⇒ no interaction between different trains
  ⇒ problem becomes "linac-like"
- Linearity assumptions
  - center-of-mass amplitude $\ll \sigma_y$
  - $\sigma_{xi}, \sigma_{yi}, \sigma_{xe}, \sigma_{ye}$ = const

  etc

# Basic Parameters

- Ion oscillation phase advance per electron bunch

$$\Theta \equiv \sqrt{\frac{2 z N m r_e L}{A M_N \Sigma_y (\Sigma_x + \Sigma_y)}}$$

$z, A$ : electrovalence and mass number of ion

$L$ : distance between bunches

$N$ : number of $e^-$ per bunch

$$\Sigma_{x(y)} = \sqrt{\sigma_{x(y),i}^2 + \sigma_{x(y),e}^2} \qquad \left( \sigma_{x(y),i} \approx \frac{1}{\sqrt{2}} \sigma_{x(y),e} \right)$$

$M_N, m$ : nucleon / electron mass

$r_e$ : classical electron radius

- Focusing of electrons by ion cloud

$\propto$ bunch index $n$ $\quad (n \leq n_b)$

$$\Delta \nu_y = R K n$$

$R$ : average ring radius

$$K = \frac{z n_i r_e \beta_y}{\gamma \Sigma_y (\Sigma_x + \Sigma_y)}$$

$n_i$ : average number of ions per $e^-$ bunch

$= N n_g \sigma_I$ ← ionization crosssection

$n_g$: residual gas density

$\beta_y$ : average $\beta$ function



# Amplitude growth factor

Unstable mode

$$y_n(s) \approx a_0 e^{i(\Theta n - k s)} \qquad \left(k \sim \frac{1}{\beta_y}\right)$$

↑ center of mass of $n$-th $e^-$ bunch

(mode $e^{i(-\Theta n - ks)}$ is damped)

Amplitude growth factor

$$G \equiv \left| \frac{a_n(s)}{a_0} \right| \approx 1 + \frac{1}{\Gamma} e^{\sqrt{\Gamma}} \qquad (\Gamma \gg 1)$$

$$\Gamma \equiv K s n \cdot \Theta n$$

$$= \sqrt{\frac{2m}{M_N}} \frac{\beta_y \sqrt{L}}{\gamma} \frac{n_g \sigma_I}{\sqrt{A}} \left[ \frac{r_e z N}{\Sigma_x \Sigma_y} \right]^{3/2} \cdot s \cdot n^2$$

(Effect of smearing due to non-linear incoherent oscillation

$$\sqrt{\Gamma} \Rightarrow \sqrt{\Gamma + (\alpha_0 \Theta n)^2} - \alpha_0 \Theta n$$

$$\alpha_0 \approx 0.077$$
)

Ion-species dependence

cross section $\sigma_I \propto Z$ (atomic number)

$$\Gamma \propto \frac{Z\, z^{3/2}}{\sqrt{A}}$$

Heavy ions are important if partial pressure is the same.

But no big difference between $N_2$, $O_2$, $CO$, $CO_2$ ....

except $H_2$. ←--- not important.

Take $CO^+$. partial pressure $1 \times 10^{-9}$ Torr.

$\Rightarrow$ $n_i \cong 100$ ions / $e^-$ bunch



with KEKB parameters (in the arc, $CO^+$, $10^{-9}$ Torr.)

$\Theta = 0.10 \sim 0.14$ rad

(Corresponds to the spread of factor 2 in A.)

$\Theta = 0.12$ rad $\longrightarrow$ one cycle of oscillation in 50 bunches

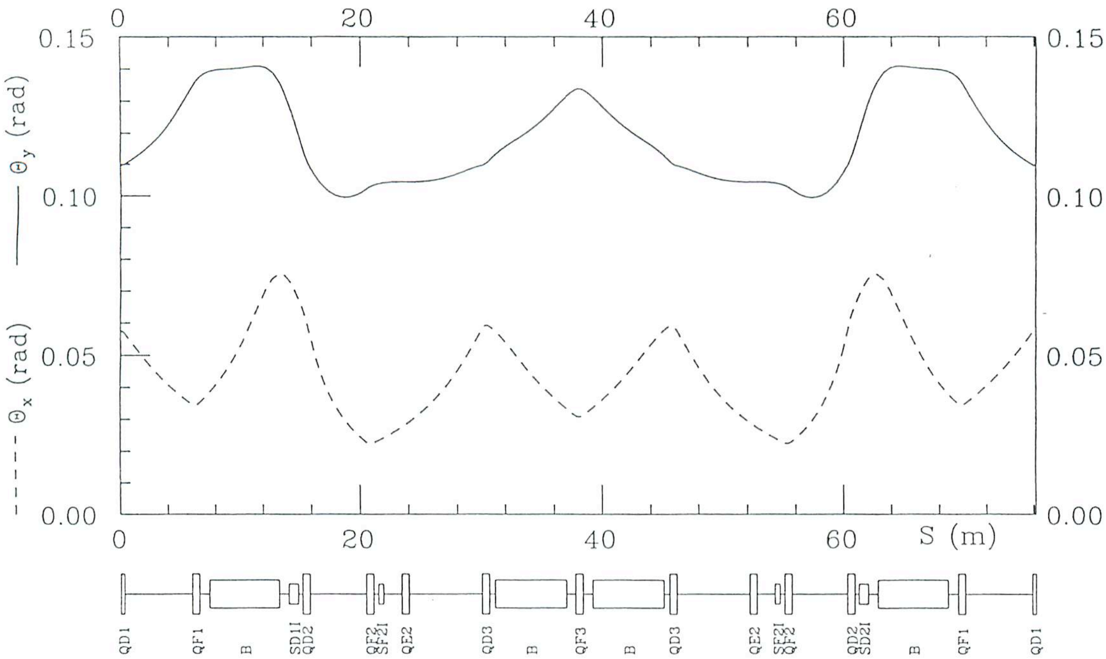
(multi-bunch mode number ~100)
(−100?)

Growth rate

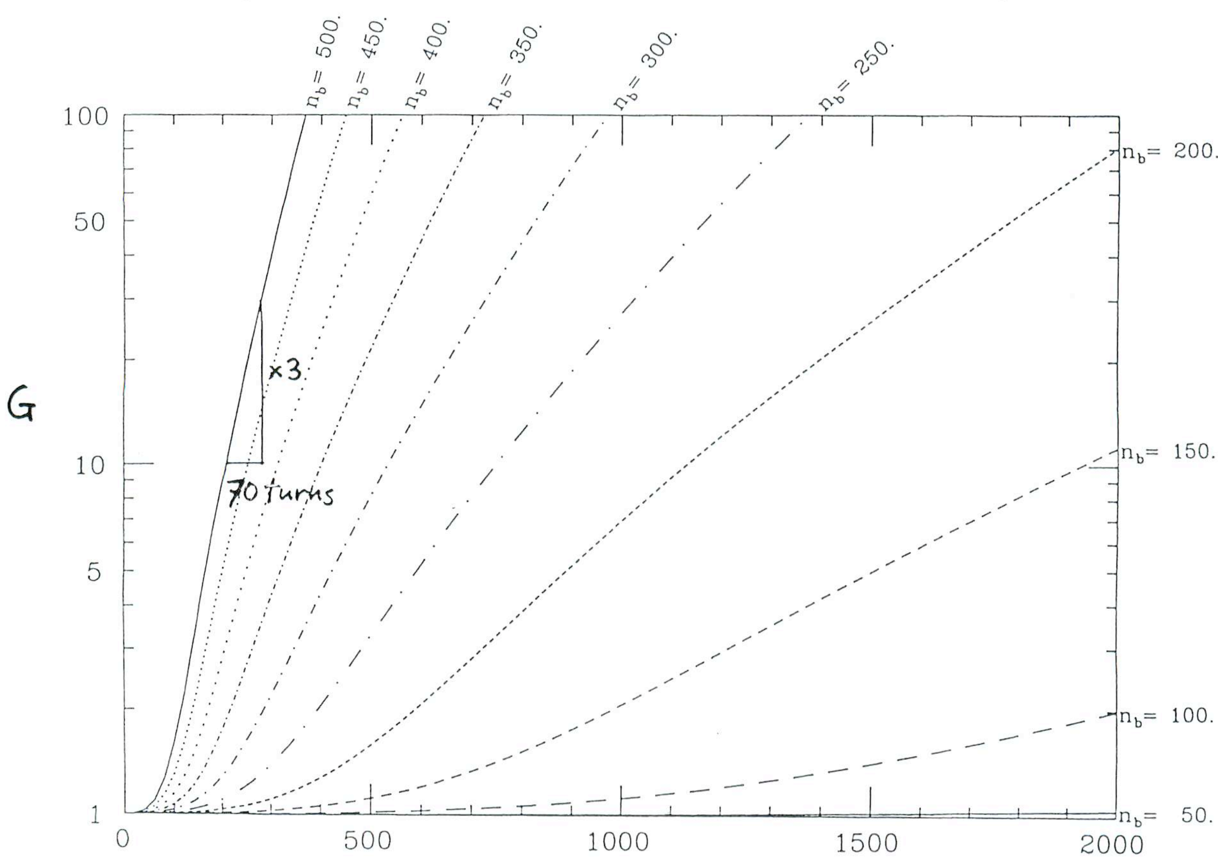
with $n_b = 500$

e-folding time $\approx$ 70 turns

# Ion Phase Advance in KEKB arc. $CO^+$



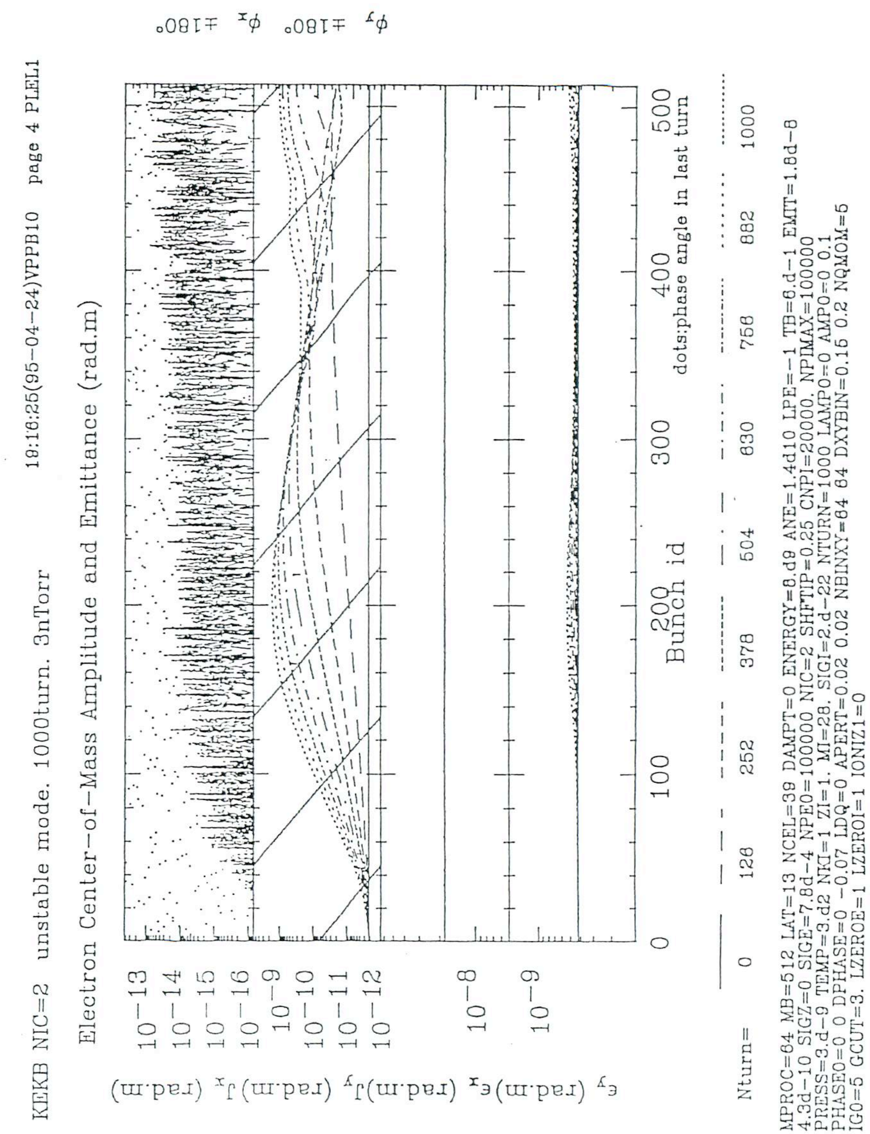

# Amplitude Growth Factor ($CO^+$, $10^{-9}$ Torr)



# Computer Simulation

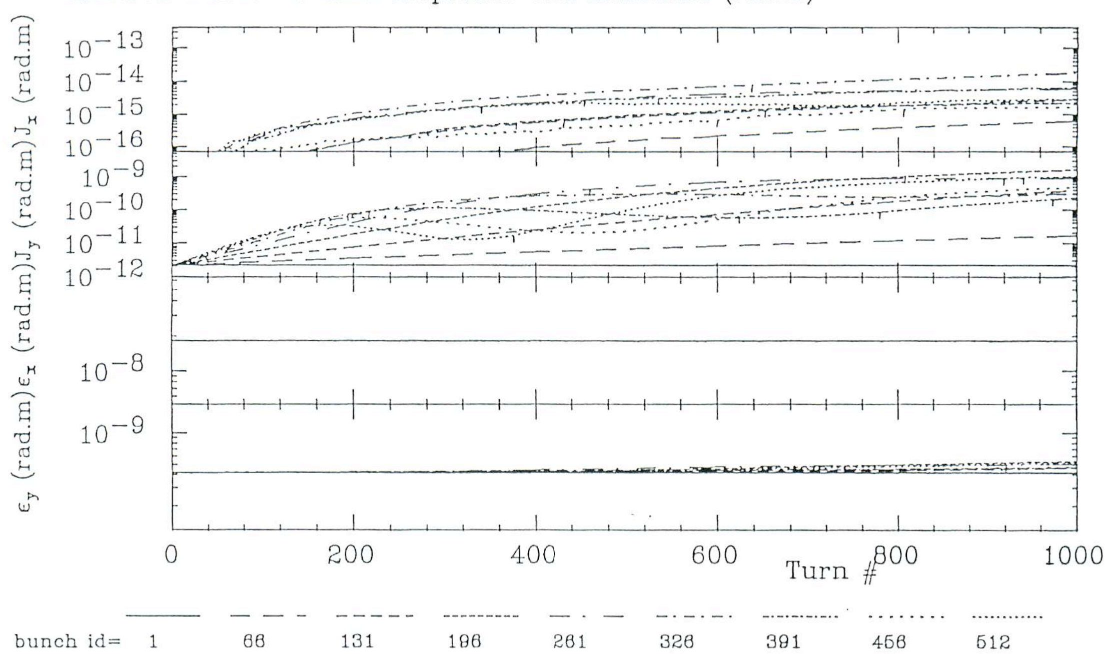
- electron bunches
  - $10^5$ macro-particles / bunch
  - up to 512 bunches
- ion
  - limited number of ionization points
    (instead of continous distribution over the ring)
    - 2 ~ 20 points per ring
      .... no essential difference.
  - $\lesssim 10^5$ macro ions / ionization point
- interaction
  - strong-strong. transverse mesh 64 × 64
    (upto 64 × 256)
  - upto $10^6$ times interaction.
    - e.g. 2 ioniz. points × 512 bunches × 1000 turns





KEKB NIC=2 unstable mode. 1000turn. 3nTorr 19:16:25(95-04-24)VPPB10 page 5 PLEL2

Electron Center-of-Mass Amplitude and Emittance (rad.m)




MPROC=64 MB=512 LAT=13 NCEL=39 DAMPT=0 ENERGY=8.d9 ANE=1.4d10 LPE=−1 TB=6.d−1 EMIT=1.8d−8
4.3d−10 SIGZ=0 SIGE=7.8d−4 NPE0=100000 NIC=2 SHFTIP=0.25 CNPI=20000. NPIMAX=100000
PRESS=3.d−9 TEMP=3.d2 NKI=1 ZI=1. MI=28. SIGI=2.d−22 NTURN=1000 LAMP0=0 AMP0=0 0.1
PHASE0=0 0 DPHASE=0 −0.07 LDQ=0 APERT=0.02 0.02 NBINXY=64 64 DXYBIN=0.15 0.2 NQMOM=5
IG0=5 GCUT=3. LZEROE=1 LZEROI=1 IONIZ1=0

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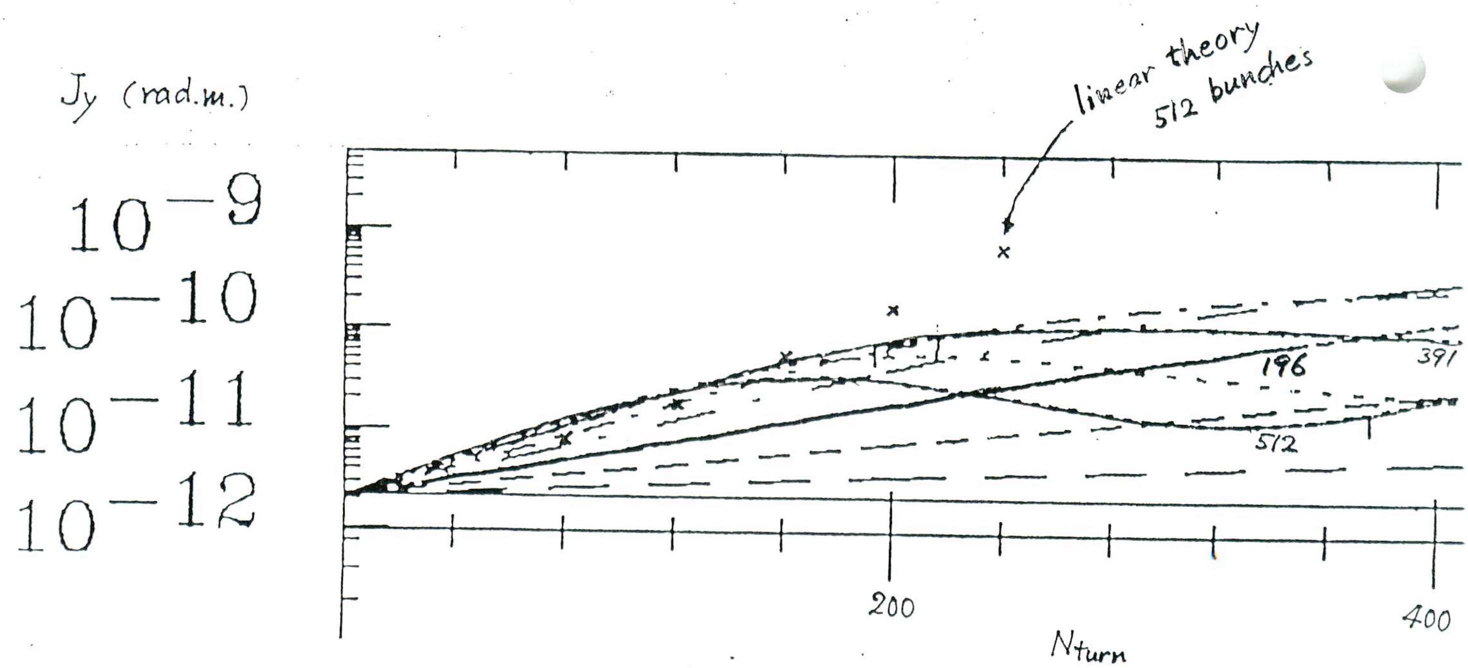
Comparison with the linear theory



# How many empty bunches are needed between trains ?

After $n$ empty bunches, the ion beam size becomes

$$\sigma_{x(y)} \approx \sigma_{x(y)}(0) \times \sqrt{1 + (\Theta_{x(y)} \cdot n)^2}$$

e.g. after 50 bunches, $\Theta_y \cdot n \approx 6$

$$\Rightarrow \sigma_y \approx 6\sigma_y(0) \sim \sigma_x(0)$$

Simulation

Results of 512 consecutive bunches compared with

256 + [25] + 256

256 + [50] + 256

256 + [100] + 256

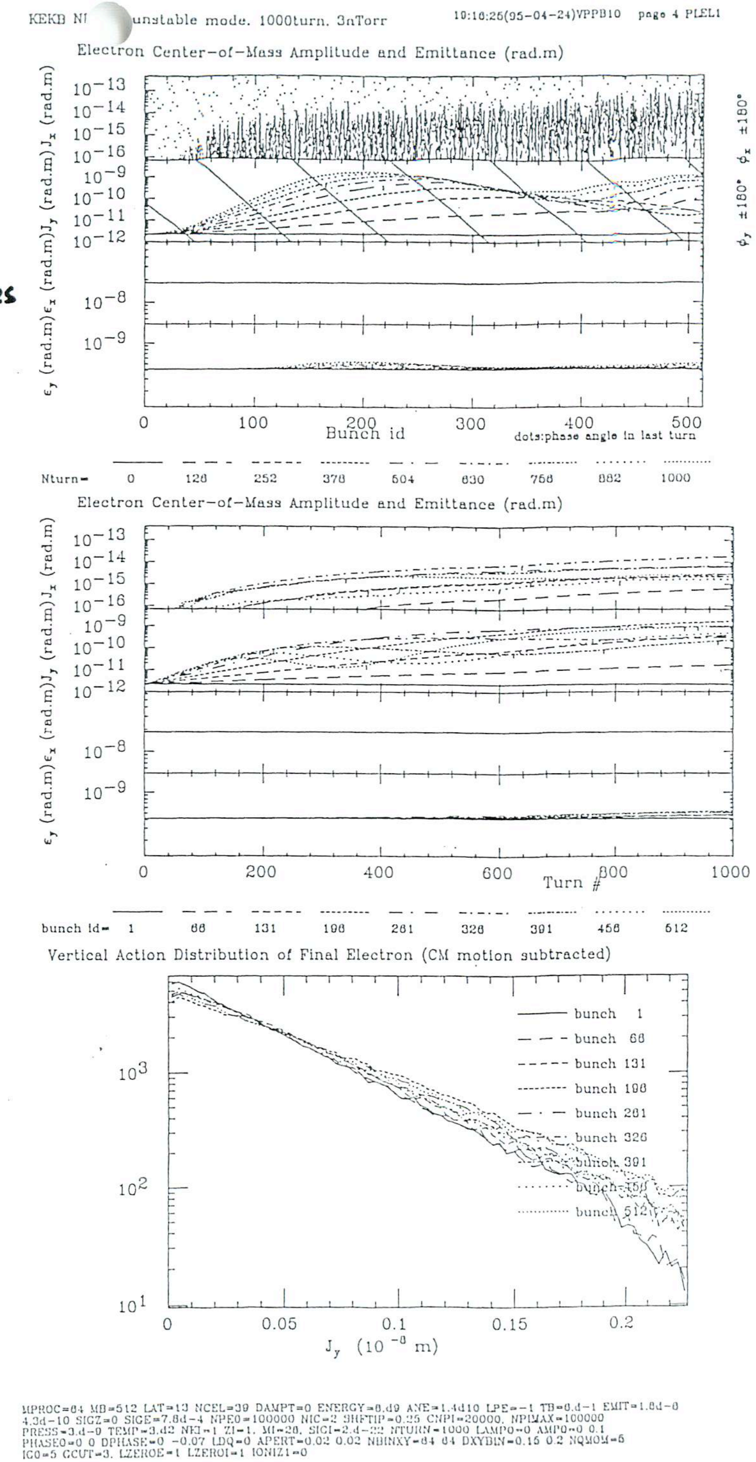
128 + [25] + 128 + [25] + 128 + [25] + 128

⇒ No significant improvement upto 50 empty bunches

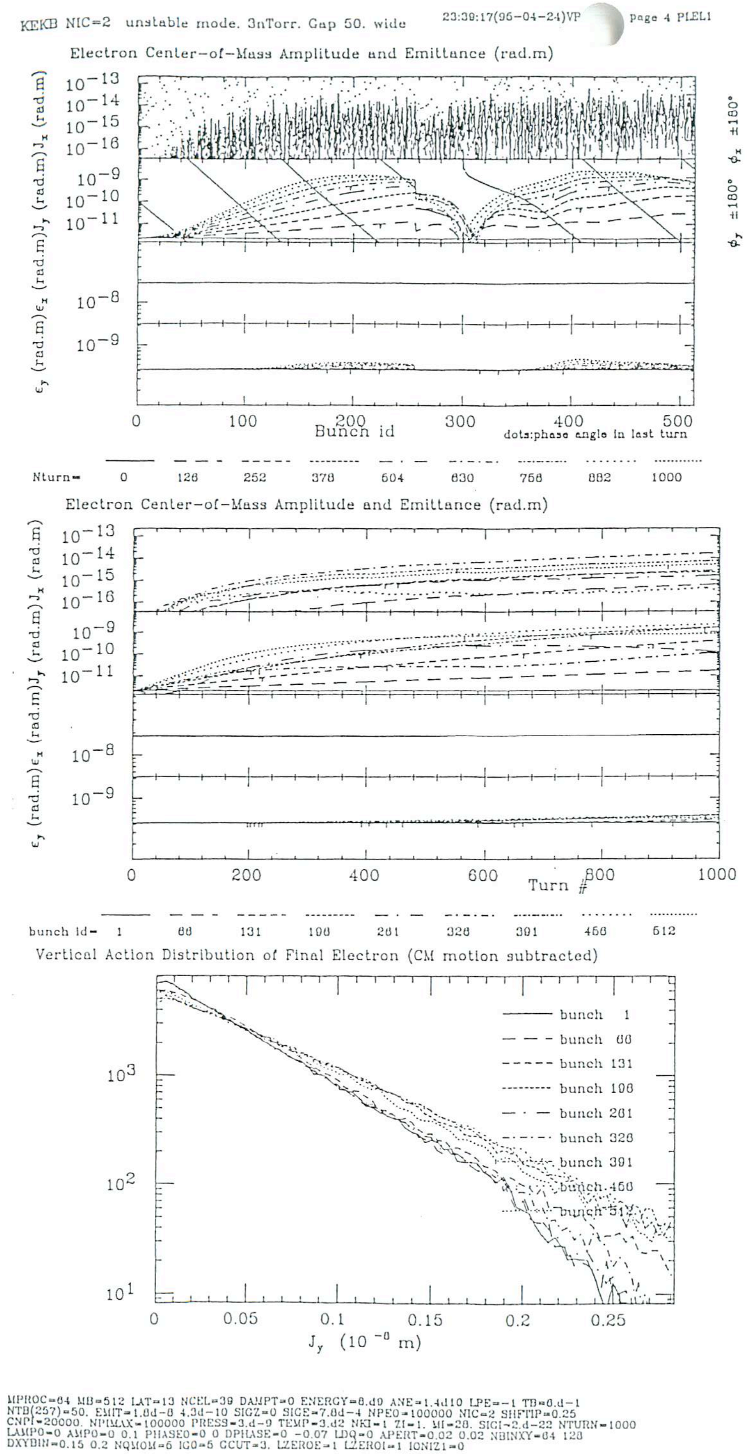
( Results for 100 empty bunches cannot be relied on. )



512 bunches



256
+ [50]
+ 256
bunches






# Conclusion

[1] Linear theory predicts the growth time about 70 turns for 500-th bunch.
( $CO^+$. $1\times10^{-9}$ Torr. )
Unstable mode ····· one cycle with ~50 bunches

[2] Linear theory is not a bad approximation compared with simulation.

[3] Simulation shows ~30% increase of emittance with 500 bunches in 1000 turns

[4] Empty buckets of 50 is not yet very effective

# To be studied yet

[5] How many empty bunches needed after 500 bunches ? Program must be improved

[6] Feedback system as fast as 1ms can prevent the amplitude (and emittance) growth ?
( if not ····· 300 bunches + gap etc. )

[7] What happens with more turns ?
New equilibrium ? ···· May be an academic question.

[8] What can be tested in AR ?