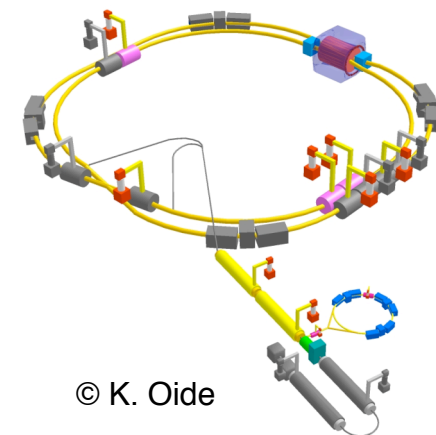


# SuperKEKB Lattice and Dynamic Aperture

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Feb/21-23 2005



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# Introduction

- Concept of lattice design in Super KEKB is the same as the KEKB lattice.
- Non-interleaved sextupole scheme is successfully working in KEKB.
  - Chromaticity correction with reducing nonlinearities
  - Sextupole bumps are used for optics corrections.
- Lattice of interaction region(IR) is different from KEKB.
  - Very small beta function at IP ( $\beta_x^*/\beta_y^* = 20 \text{ cm} / 3 \text{ mm}$ )
  - Finite crossing angle is 30 mrad.
- Half of wigglers are replaced with RF cavities in LER.
  - Damping time becomes longer. (43 msec  $\rightarrow$  ~56 msec)
- We need to check lattice performance of SuperKEKB.
  - Tracking simulations includes machine errors and optics correction
  - Dynamic aperture (injection, lifetime)

# Machine parameters of SuperKEKB

	Symbol	LER	HER	Unit
Luminosity	L	2.5x10 <sup>35</sup>		cm <sup>-2</sup> s <sup>-1</sup>
Energy	E	3.5	8.0	GeV
Beam current	I	9.4	4.1	A
No. bunches	n <sub>B</sub>	5018		
Emittance	ε <sub>x</sub>	24		nm
Beta at IP	β <sub>x</sub> <sup>*</sup>	20		cm
	β <sub>y</sub> <sup>*</sup>	3		mm
Bunch length	σ <sub>z</sub>	3		mm
Momentum compaction	α <sub>p</sub>	3x10 <sup>-4</sup>		
RF voltage	V <sub>c</sub>	14	23	MV
Betatron tune	ν <sub>x</sub>	45.506	44.515	
	ν <sub>y</sub>	43.545	41.580	
Synchrotron tune	ν <sub>s</sub>	0.031	0.019	
Beam-Beam	ξ <sub>y</sub>	0.14		
Crossing angle	θ	30→0 (crab crossing)		mrad

# Lattice Parameters and Beam-Beam Effect

	Symbol	bare lattice	with beam-beam	unit
Beam current (LER/HER)	I	9.4/4.1		A
Beam energy (LER/HER)	E	3.5/8.0		GeV
Emittance	$\epsilon_x$	24	77	nm
Horizontal beta at IP	$\beta_x^*$	20	4.5	cm
Vertical beta at IP	$\beta_y^*$	3	2.3	mm
Horizontal beam size	$\sigma_x^*$	69	59	$\mu\text{m}$
Vertical beam size	$\sigma_y^*$	0.7	1.4	$\mu\text{m}$
Beam size ratio	$r = \sigma_y^*/\sigma_x^*$	1	2.4	%
Crossing angle	$\theta_x$	0	0	mrad
Luminosity reduction	$R_L$	0.86	0.81	
$\xi_x$ reduction	$R_{\xi x}$	0.99	0.97	
$\xi_y$ reduction	$R_{\xi y}$	1.11	1.17	
Reduction ratio	$R_L/R_{\xi y}$	0.78	0.70	
Horizontal beam-beam (estimated with S-S simulation)	$\xi_x$	0.08	0.05	
Vertical beam-beam (estimated with S-S simulation)	$\xi_y$	0.14	0.12	
Luminosity	L	$2.5 \times 10^{35}$		$\text{cm}^{-2}\text{s}^{-1}$

# Non-Interleaved $2.5\pi$ Sextupole Scheme (NISS)

- Emittance and momentum compaction factor can be adjusted independently.
  - 7 families of quadrupole in a cell. (4 for -I, 2 for emittance and momentum compaction factor, 1 for phase advance)
- Sextupole pair is connected by -I'.

$$-I' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ m_{21} & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & m_{43} & -1 \end{pmatrix}$$

- Chromaticity can be corrected with canceling nonlinearities in each sextupole pair.

-I' transformation with sextupole pair

$$\begin{pmatrix} x_{\beta 1} + \eta_{x1}\delta \\ x'_{\beta 1} + \eta'_{x1}\delta \\ y_{\beta 1} \\ y'_{\beta 1} \end{pmatrix} \xrightarrow{\text{NISS}} \begin{pmatrix} -x_{\beta 1} + \eta_{x1}\delta \\ m_{21}x_{\beta 1} - x'_{\beta 1} - \eta'_{x1}\delta + \frac{K_2^{(1)} - K_2^{(2)}}{2}(x_{\beta 1}^2 - y_{\beta 1}^2) + (K_2^{(1)} + K_2^{(2)})\eta_{x1}\delta x_{\beta 1} \\ -y_{\beta 1} \\ m_{43}y_{\beta 1} - y'_{\beta 1} - (K_2^{(1)} - K_2^{(2)})x_{\beta 1}y_{\beta 1} - (K_2^{(1)} + K_2^{(2)})\eta_{x1}\delta y_{\beta 1} \end{pmatrix}$$

$$K_2^{(1)} = K_2^{(2)}$$

- Large dynamic aperture is expected.

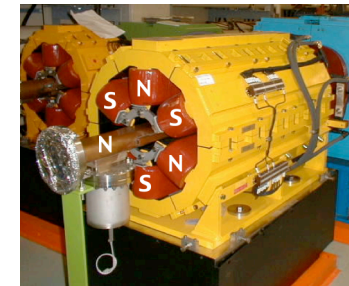
# Non-Interleaved $2.5\pi$ Sextupole Scheme (NISS)

- Lattice errors can be corrected by sextupole displacement.
  - Horizontal displacement of sextupole generates quadrupole element and vertical displacement generates xy-coupling.

$$\begin{cases} \Delta x' = -\frac{K_2}{2}(x^2 - y^2) - (K_2 \Delta x_{\text{sext}})x - (K_2 \Delta y_{\text{sext}})y \\ \Delta y' = K_2 xy + (K_2 \Delta x_{\text{sext}})y + (K_2 \Delta y_{\text{sext}})x \end{cases}$$

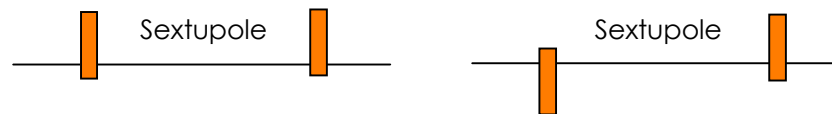
Quad. element      Skew element

$\Delta x_{\text{sext}}$  : Horizontal displacement  
 $\Delta y_{\text{sext}}$  : Vertical displacement



- Sextupole magnet has a mover which can transversely move the magnet.

Orthogonal knobs:



displacement	same direction	opposite direction	
$\Delta x_{\text{sext}}$	$\beta$ -function	$\eta_x$	SF
	localized dispersion	localized $\beta$ -beat	
$\Delta y_{\text{sext}}$	xy-coupling	$\eta_y$	SD
	localized dispersion-coupling	localized xy-coupling	

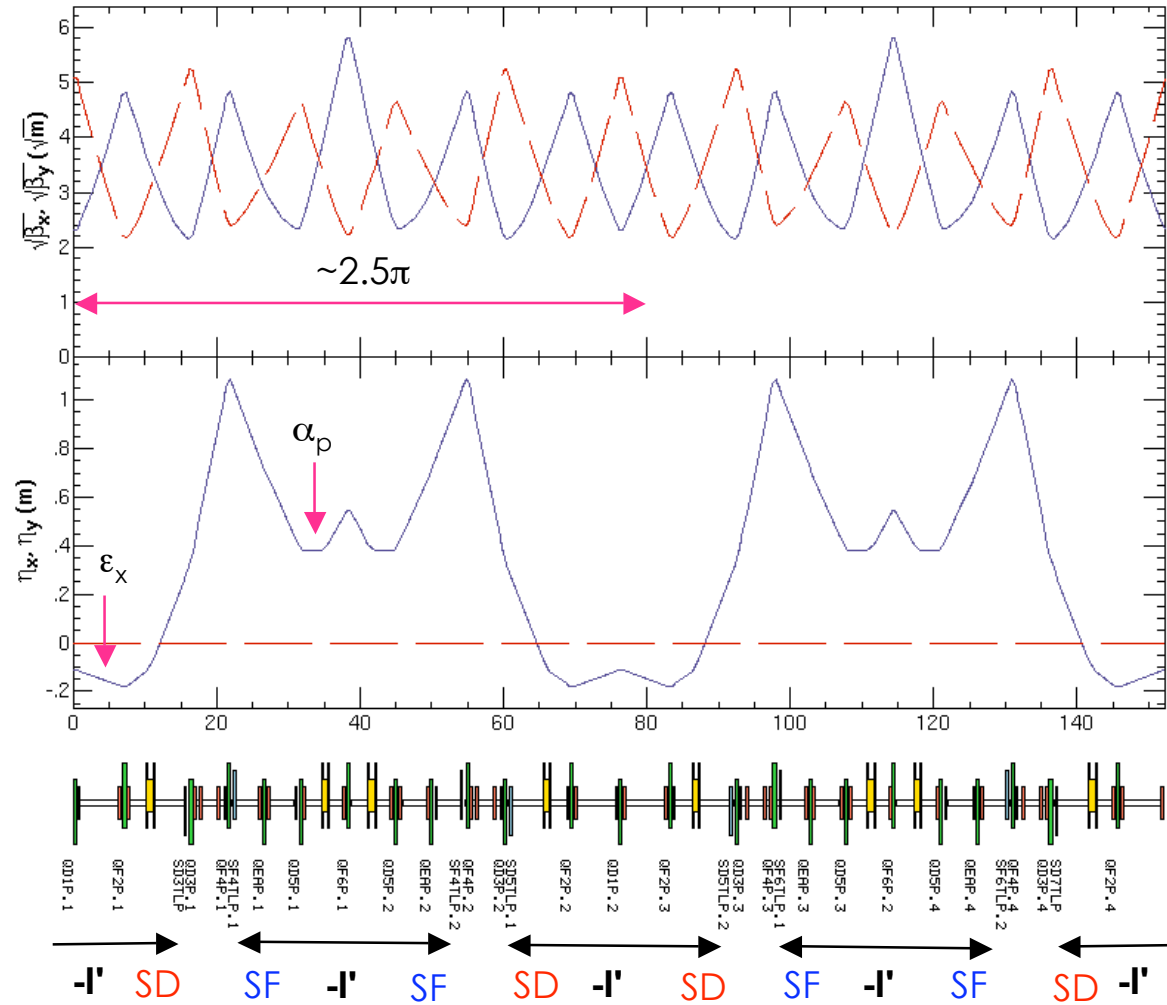
# NISS Arc Lattice

LER

$$\alpha_p = 4.27 \times 10^{-4}$$

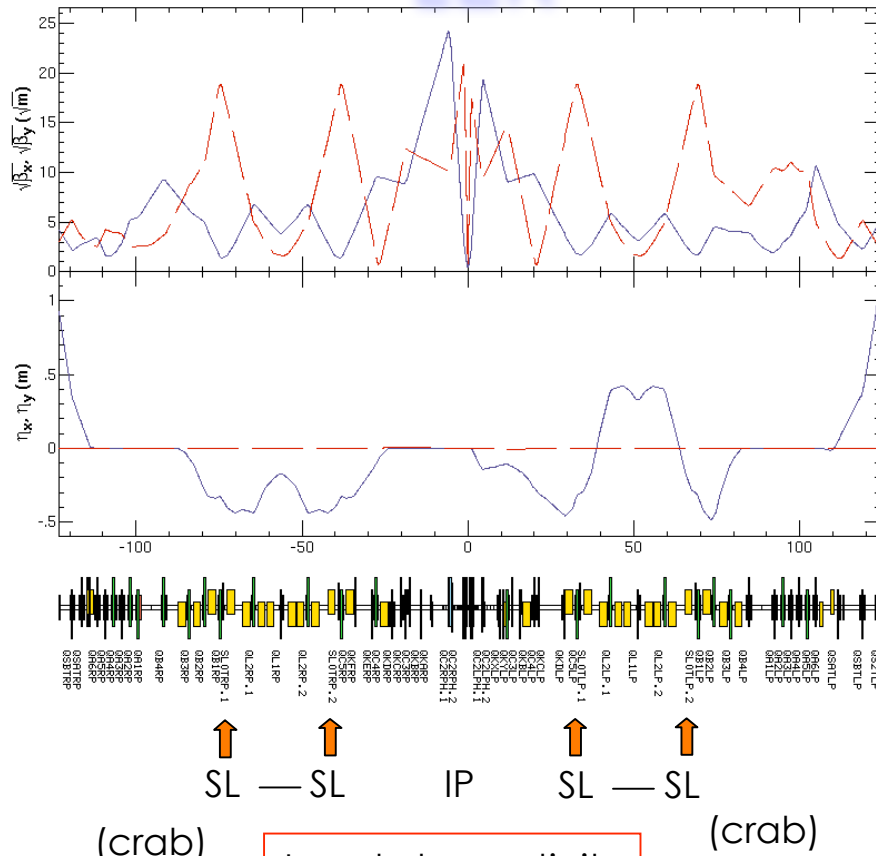
$$\epsilon_x = 24 \text{ nm}$$

Flexibility:  
 $10 \text{ nm} < \epsilon_x < 36 \text{ nm}$   
 $-4 \times 10^{-4} < \alpha_p < +4 \times 10^{-4}$



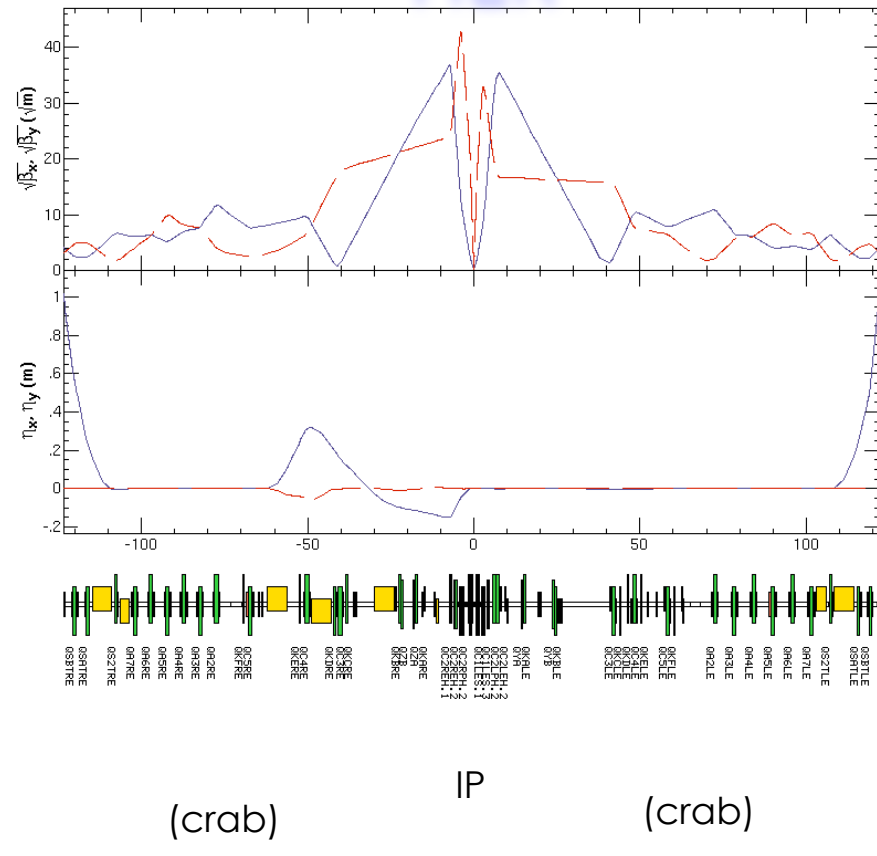
# Interaction Region

LER



Local chromaticity correction

HER





# Computer Simulations



- **SAD:** SAD is a computer program complex for accelerator design. It has been developed in KEK and used at KEKB. (<http://acc-physics.kek.jp/SAD/sad.html>)
- **Machine errors:**
  - The errors are assumed to be Gaussian distribution with a cut-off of  $3\sigma$ .
  - LER is considered to estimate tolerance of machine errors.

	alignment error		rotation error	gradient error
	$\Delta x$ ( $\mu\text{m}$ )	$\Delta y$ ( $\mu\text{m}$ )	$\Delta\theta$ (mrad)	$\Delta k/k$
Bend	100	100	0.1	$10^{-4}$
Quadrupole	100	100	0.1	$10^{-4}$
Sextupole	100	100	0.1	$10^{-4}$
Steering	100	100	0.1	$10^{-3}$

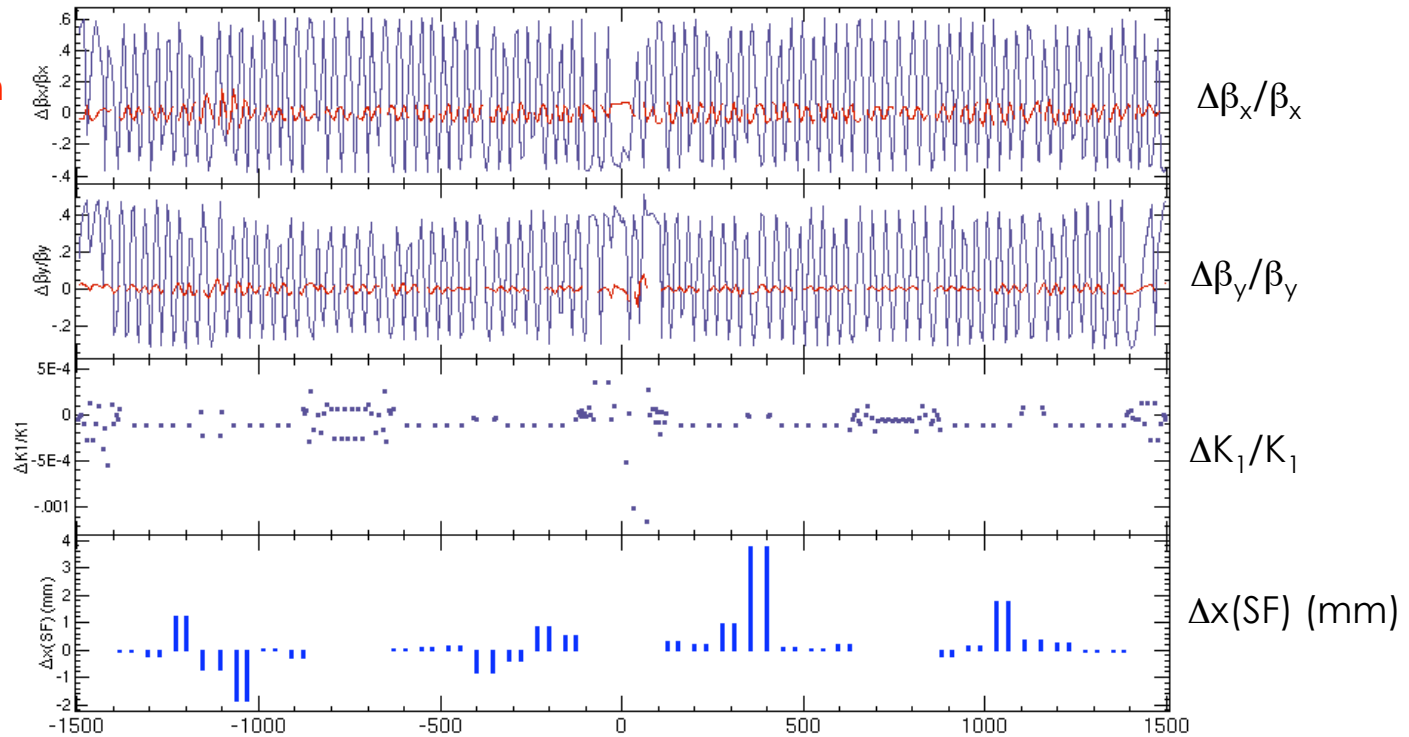
- Lattice errors are corrected by sextupole movers, skew quadrupoles and field gradient of quadrupoles.

# Beta Correction

Blue: before  
Red: after correction

correction  
of field gradient  
(fudge factor)

sextupole (SF)  
mover  
(horizontal)



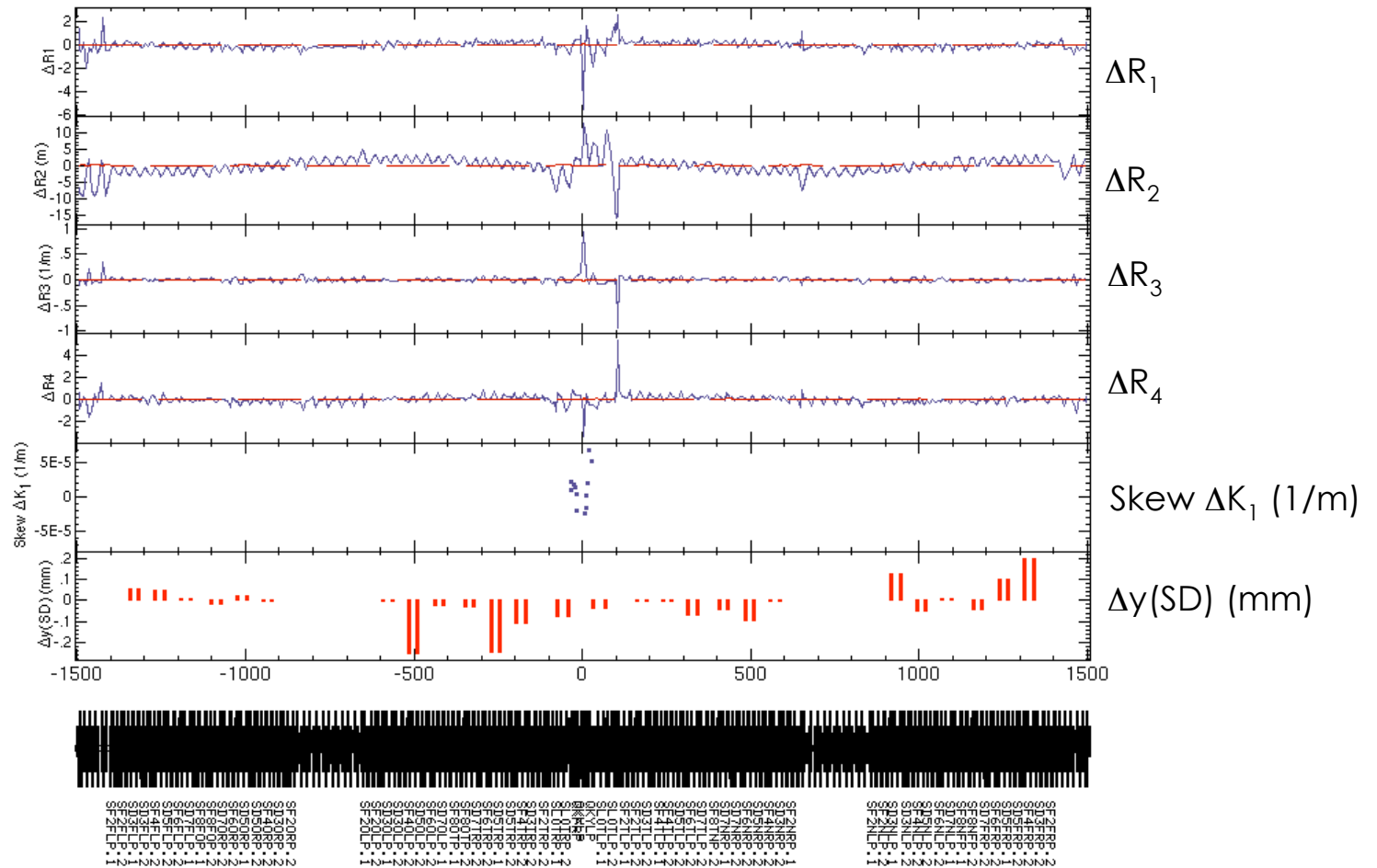
IP

# xy-coupling Correction

Blue: before  
Red: after correction

skew quad.  
correction

sextupole(SD)  
mover  
(vertical)



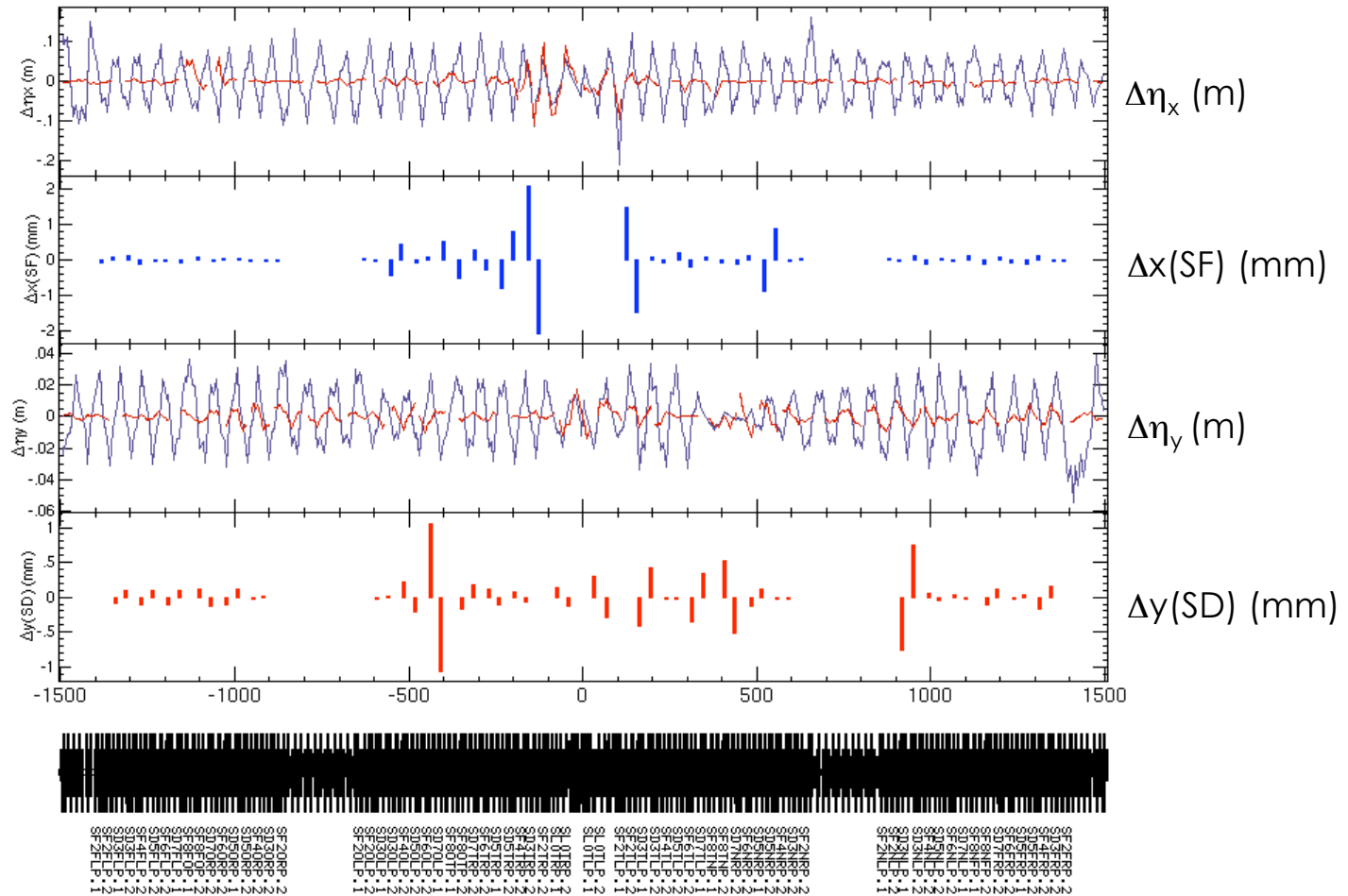
IP

# Dispersion Correction

Blue: before  
Red: after correction

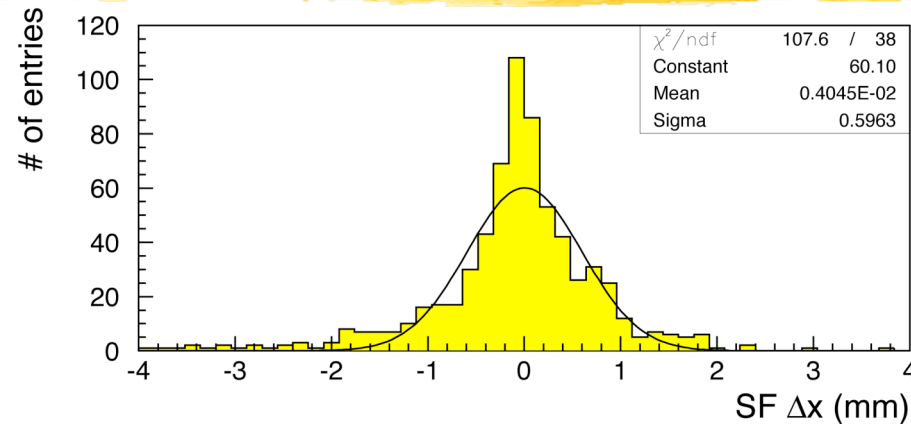
sextupole(SF)  
mover  
(horizontal)

sextupole(SD)  
mover  
(vertical)

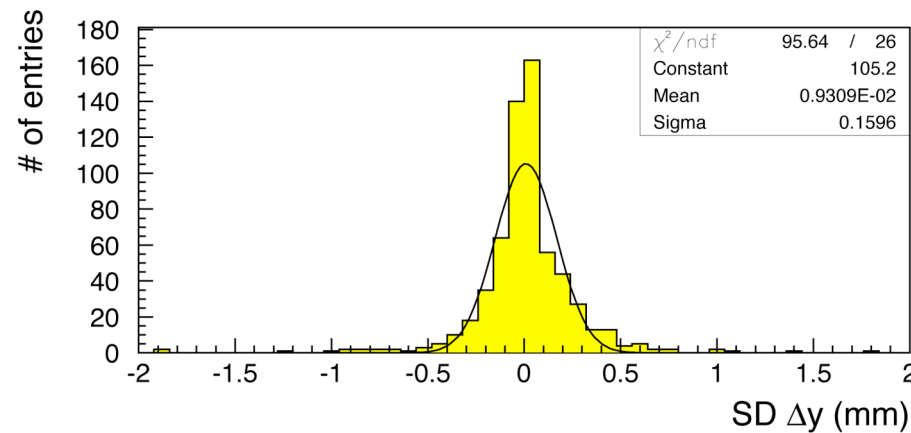


IP

# Sextupole Mover



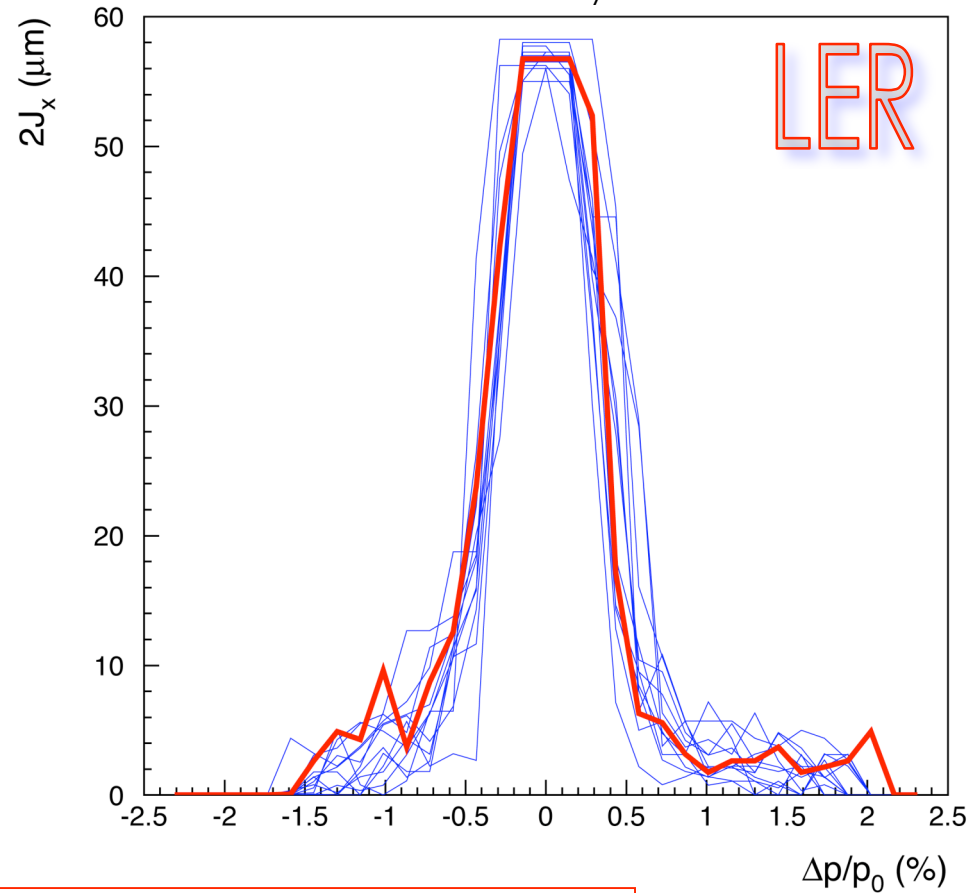
Mover displacement  
used for optics correction



- Maximum stroke of  $\pm 3$  mm is necessary for sextupole movers to correct optics.
- Sextupole local bumps are not available for SuperKEKB to keep synchrotron light path in the ante-chamber.

# Dynamic Aperture in LER

Stored beam  $J_y/J_x = 2\%$



red: no machine error  
blue: machine error + optics correction  
(12 lines indicate different seed numbers.)

Touschek lifetime : 75 min

# Required Acceptance for Injected Beam

	LER 3.5 GeV			HER 8 GeV		
	e <sup>+</sup>	e <sup>+</sup>	e <sup>-</sup>	e <sup>+</sup>	e <sup>+</sup>	e <sup>-</sup>
Damping Ring	NO	YES	-	NO	YES	-
A <sub>x</sub> (μm)	7.5	1.8	2.6	4.5	1.5	1.9
A <sub>y</sub> (μm)	1.2	0.056	0.18	0.52	0.025	0.08
A <sub>y</sub> /A <sub>x</sub> (%)	16	3	7	12	2	4

Energy Switch

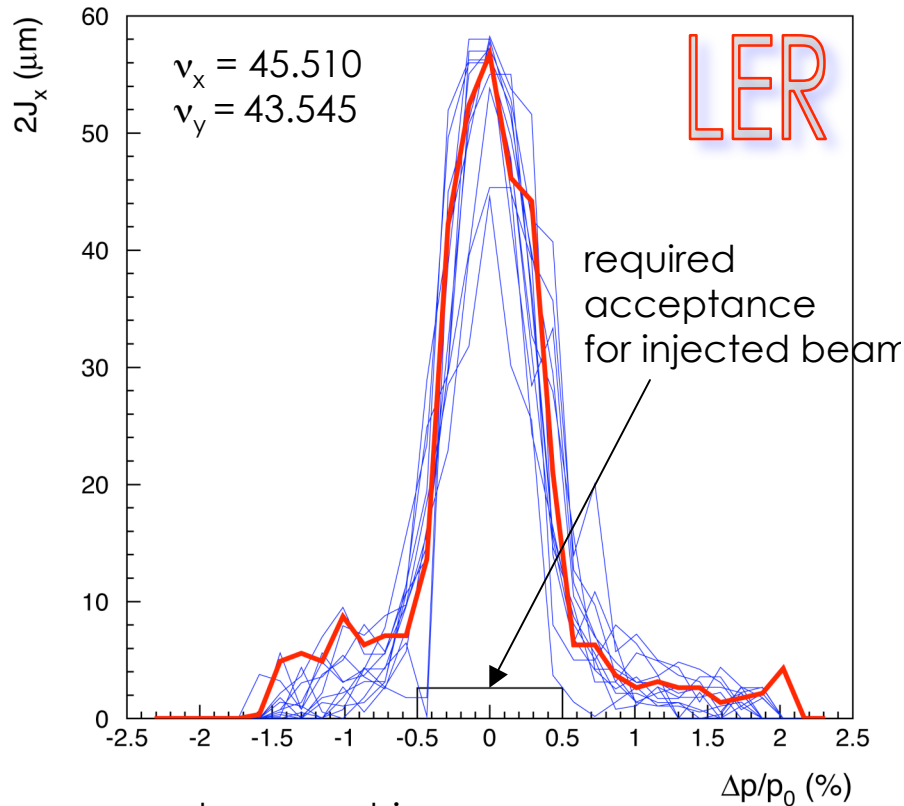
Energy Switch

Energy Switch

e<sup>+</sup> damping ring is assumed in SuperKEKB.

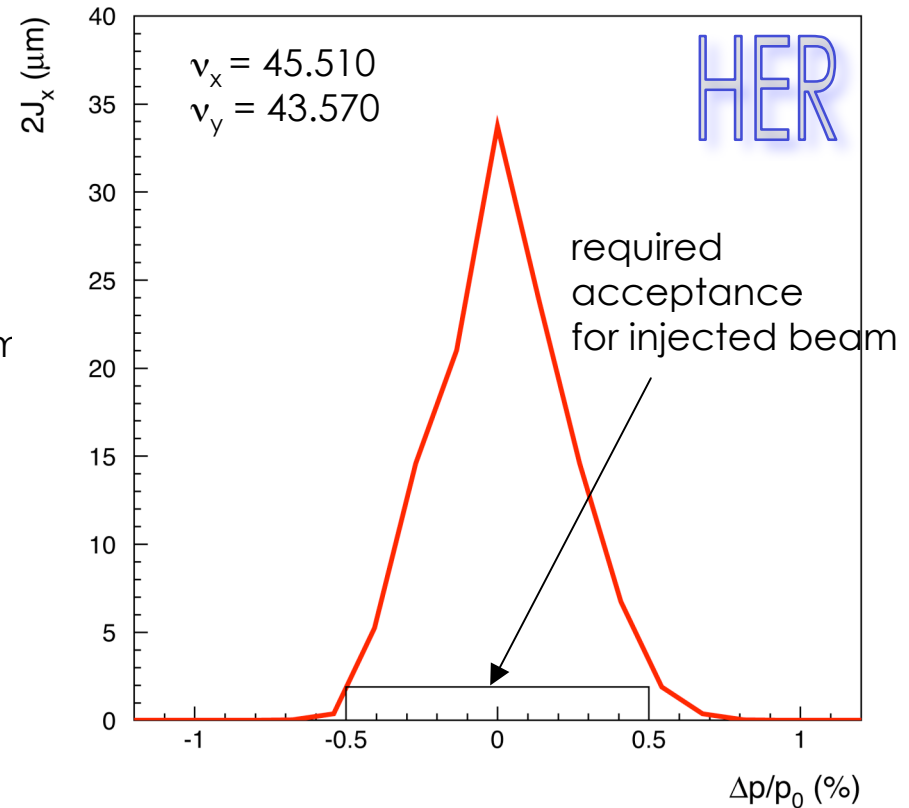
# Dynamic Aperture for Injected Beam

Injected beam  $J_y/J_x = 7\%$



red: no machine error  
blue: machine error + optics correction  
(12 lines indicate different seed numbers.)

Injected beam  $J_y/J_x = 4\%$



Dynamic aperture satisfies injected beam.



# Tracking Simulation with Beam-Beam Effect

- Beam-beam kick is given by Bassetti-Erskine formula.

- $x' \rightarrow x' + \Delta x'$ ,  $y' \rightarrow y' + \Delta y'$  at I.P

$$\Delta x' = -\frac{N_- r_e}{\gamma_+} F_x(x, y, \sigma_x^*, \sigma_y^*) \quad \text{LER}$$

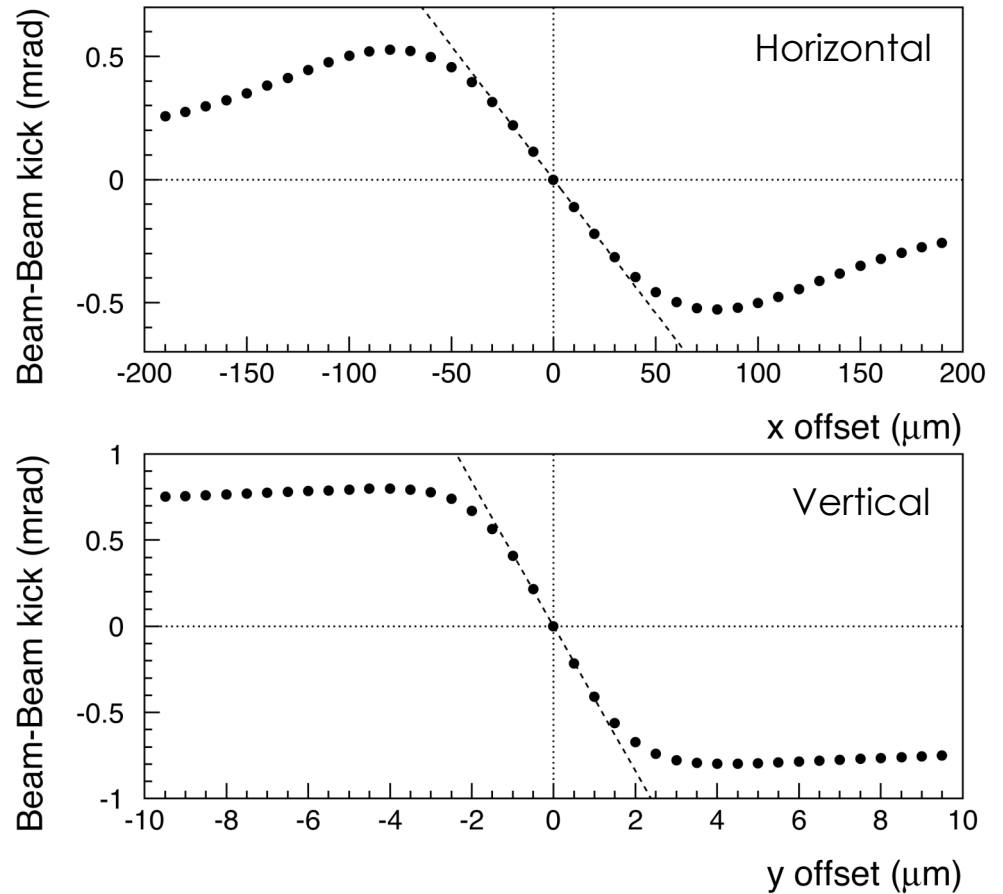
$$\Delta y' = -\frac{N_- r_e}{\gamma_+} F_y(x, y, \sigma_x^*, \sigma_y^*) \quad (\text{e}^+)$$

$$F_x(x, y, \sigma_x^*, \sigma_y^*) - iF_y(x, y, \sigma_x^*, \sigma_y^*) = -i \sqrt{\frac{2\pi}{\sigma_x^{*2} - \sigma_y^{*2}}}$$

$$\times \left\{ w \left( \frac{x + iy}{\sqrt{2(\sigma_x^{*2} - \sigma_y^{*2})}} \right) - \exp \left( -\frac{x^2}{\sigma_x^{*2}} - \frac{y^2}{\sigma_y^{*2}} \right) w \left( \frac{\frac{\sigma_y^{*2}}{\sigma_x^{*2}} x + i \frac{\sigma_x^{*2}}{\sigma_y^{*2}} y}{\sqrt{2(\sigma_x^{*2} - \sigma_y^{*2})}} \right) \right\}$$

- Tracking simulation is "Weak-Strong".

Demonstration of beam-beam kick with tracking



# Dynamic Aperture with Beam-Beam Effect

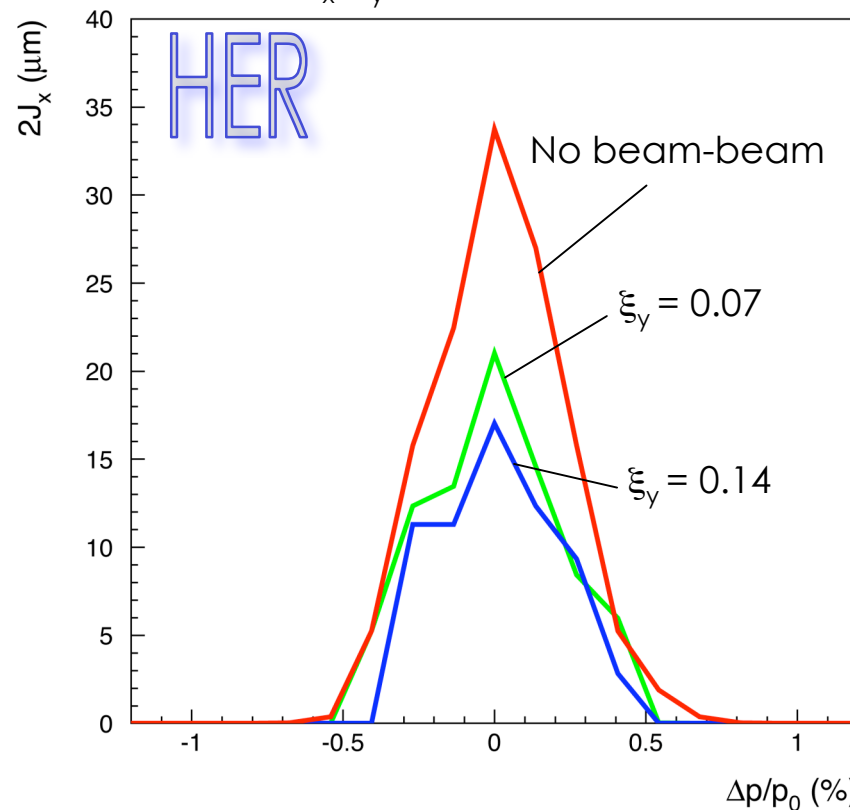
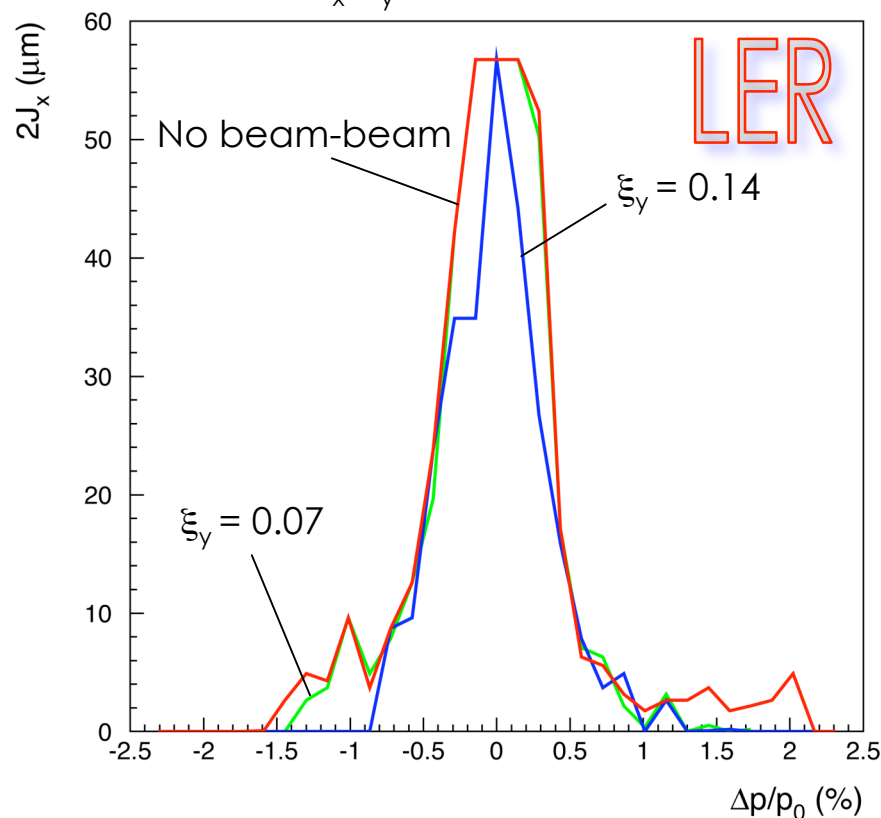
Stored beam  $J_y/J_x = 2\%$

$\nu_x/\nu_y = 45.510/43.545$

\* no machine error

Stored beam  $J_y/J_x = 2\%$

$\nu_x/\nu_y = 45.510/43.570$



- Case  $\xi_y = 0.14$ , dynamic aperture shrinks in large momentum deviation for LER.
- Transverse aperture decreases in HER due to beam-beam effect.
- Touschek lifetime with beam-beam ( $\xi_y = 0.14$ ): 50 min in LER / 180 min in HER

# Lifetime

Item	LER	HER
Quantum lifetime	very long	
Vacuum lifetime	~10 hours	
Luminosity lifetime	170 min	70 min
Touschek lifetime for single beam ( $J_y/J_x=2\%$ )	75 min	320 min
Touschek lifetime with beam-beam ( $J_y/J_x=2\%$ )	50 min	180 min
Total (collision lifetime)	40 min	50 min

To keep CIM, 40 nC · Hz/pulse injection for LER, 14 nC · Hz/pulse for HER are needed at least.

# Summary



- Non-interleaved sextupole scheme(NISS) is applied to SuperKEKB lattice.
- Tolerance of machine errors and optics correction
  - Sextupole mover is important to make optics correction.
- Dynamic aperture satisfies the requirement of injected beam in both rings.
- Dynamic aperture with beam-beam effect
  - Momentum aperture decreases in LER, transverse aperture decreases in HER.
  - Lifetime(beam-beam) : 50 min (LER) / 180 min (HER)
- Sextupole optimization still needs improvement.
  - Local chromaticity correction in HER

# Appendix

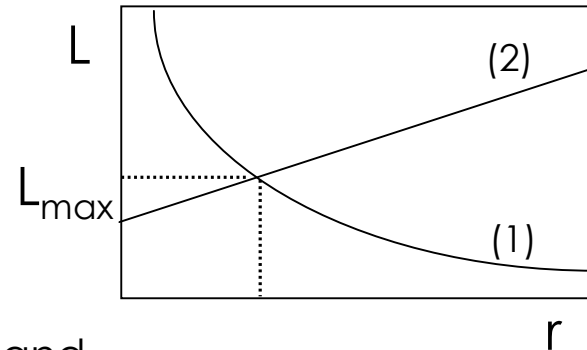


# Luminosity Formula

- Luminosity expressed by Beam-Beam parameters ( $\gamma_{e^+} N_{e^+} = \gamma_{e^-} N_{e^-}$ )

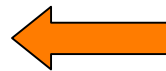
$$\begin{cases} L \leq \frac{\gamma_{e^\pm}}{2er_e} (1+r) \left( \frac{I_{e^\pm} \xi_x}{r\beta_x^*} \right) & (1) \\ L \leq \frac{\gamma_{e^\pm}}{2er_e} (1+r) \left( \frac{I_{e^\pm} \xi_y}{\beta_y^*} \right) & (2) \end{cases}$$

$$r \equiv \frac{\sigma_y^*}{\sigma_x^*}$$



- When  $\xi_x/r\beta_x^* = \xi_y/\beta_y^*$ , luminosity is maximum and

$$L = \frac{\gamma_{e^\pm}}{2er_e} (1+r) \left( \frac{I_{e^\pm} \xi_y}{\beta_y^*} \right)$$



Luminosity expression used in machine design

- Flat beam:  $r \sim O(1\%)$

$$L \propto \frac{I_{e^\pm} \xi_y}{\beta_y^*}$$

# Geometrical Reductions to Luminosity

- Geometrical reduction to luminosity:  $R_L$

$$L \rightarrow L = \frac{N_{e^+} N_{e^-} f}{4\pi\sigma_x^* \sigma_y^*} R_L \quad R_L = \frac{a}{\sqrt{\pi}} e^b K_0(b)$$

$\uparrow$   
 modified Bessel

$$a = \frac{\beta_y^*}{\sigma_z}$$

$$b = \frac{a^2}{2} \left[ 1 + \left( \frac{\sigma_z}{\sigma_x^*} \tan \frac{\theta_x}{2} \right)^2 \right]$$

- Geometrical reduction to beam-beam parameter:  $R_{\xi y}$

$$\xi_{y,e\pm} \rightarrow \xi_{y,e\pm} = \frac{r_e N_{e^+} \beta_y^*}{2\pi \gamma_{e\pm} \sigma_y^* (\sigma_x^* + \sigma_y^*)} R_{\xi y} \quad R_{\xi y} = \int dz' \rho(z') \sqrt{1 + \left( \frac{S}{\beta_y^*} \right)^2} f_y \left( z' \tan \frac{\theta}{2}, \sigma_x^*, \sigma_y^* \right)$$

Montague's function

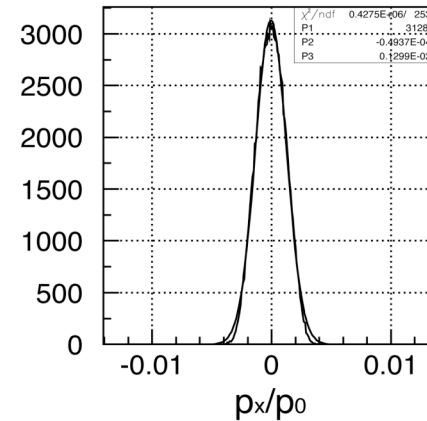
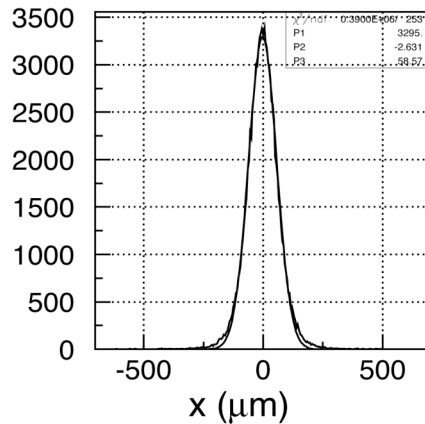
- Luminosity expression used in machine design:  $R_L/R_{\xi y}$

$$L \rightarrow L = \frac{\gamma_{e\pm}}{2er_e} (1+r) \left( \frac{I_{e\pm} \xi_y}{\beta_y^*} \right) \left( \frac{R_L}{R_{\xi y}} \right)$$

# Beam-Beam simulation using Supercomputer

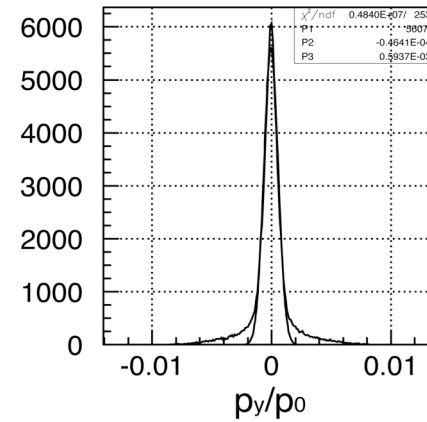
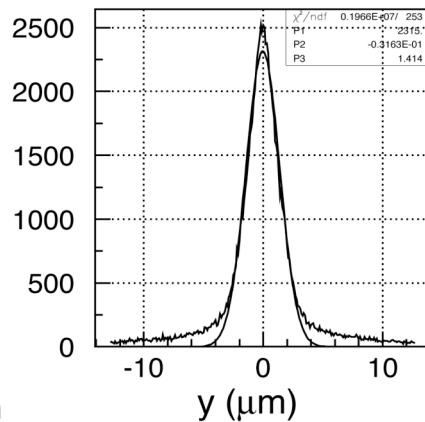
## Positron beam

$$\sigma_x = 59 \text{ mm}$$



$$\sigma_{px/p0} = 1.3 \times 10^{-3}$$

$$\sigma_y = 1.4 \text{ mm}$$



$$\sigma_{py/p0} = 0.59 \times 10^{-3}$$

initial condition

Beam-Beam effect

$$\beta_x^* = 20 \text{ cm} / \beta_y^* = 3 \text{ mm}$$

$$\varepsilon_x = 24 \text{ nm} / \varepsilon_y = 0.18 \text{ nm}$$

**Dynamic effects**

$$\beta_x^* = 4.5 \text{ cm} / \beta_y^* = 2.3 \text{ mm}$$

$$\varepsilon_x = 77 \text{ nm} / \varepsilon_y = 0.83 \text{ nm}$$