Evaluation of X-Y coupling / Requirements on correction

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The 14th KEKB Accelerator Review Committee
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Outline

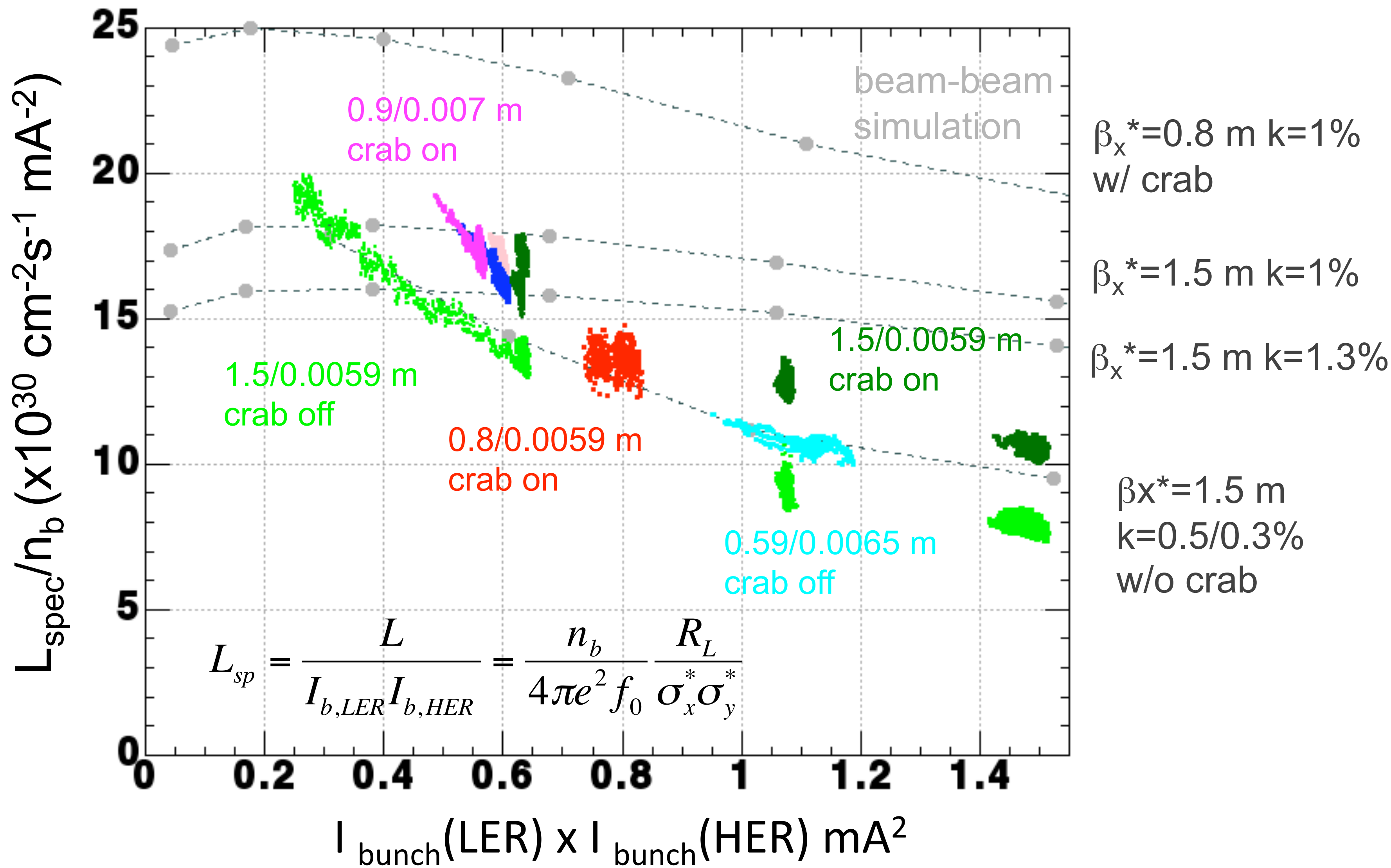
- Introduction
  - Estimation of vertical beam size from luminosity
  - Simulation with machine error (see Apendix)
- Chromaticity - Turn-by-turn BPM analysis -
- Twiss-chromaticity at IP
- R-chromaticity at IP
- Method of a X-Y coupling measurement
- Summary of X-Y coupling
- Miscellaneous

Specific luminosity

Estimation of vertical beam size

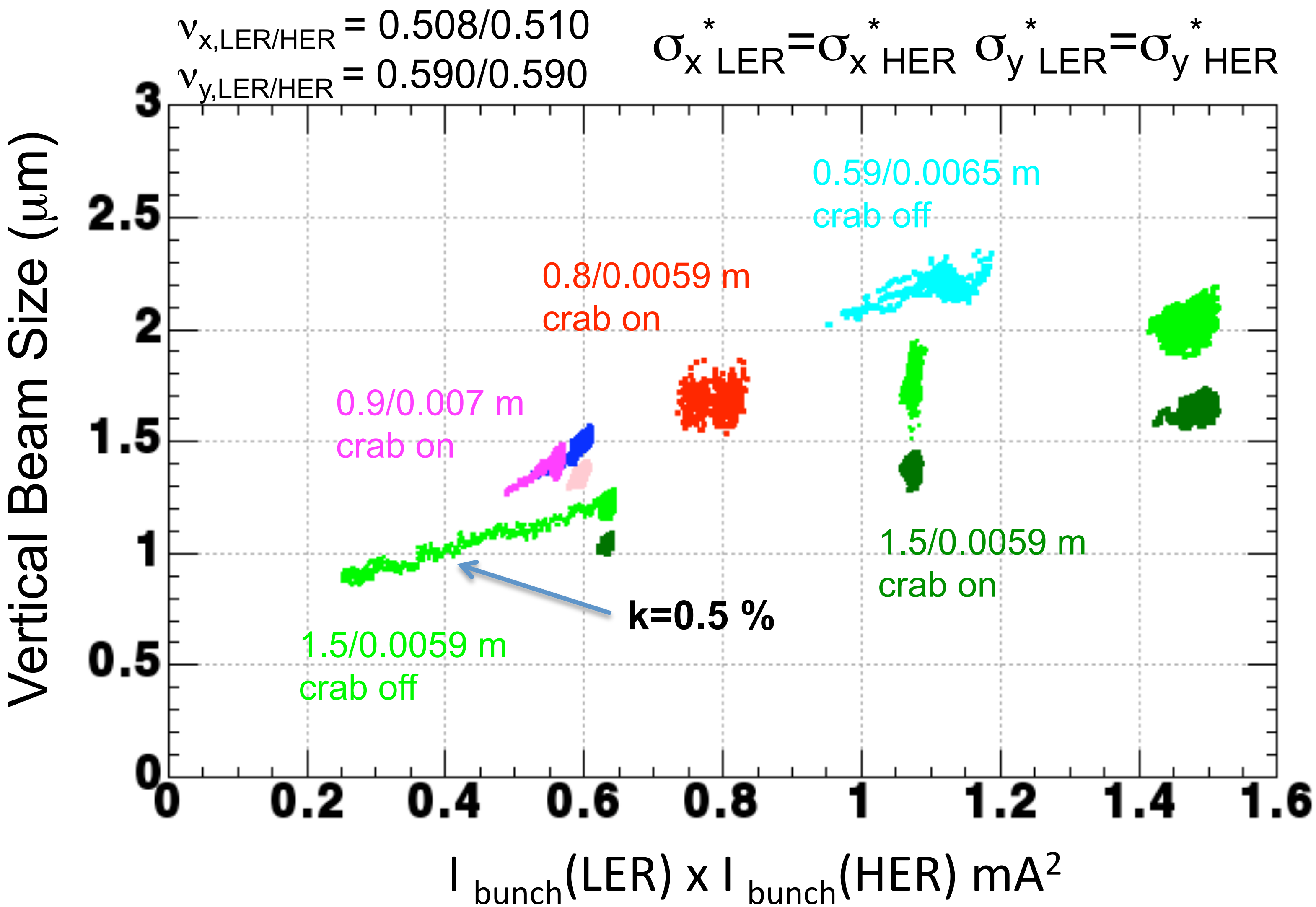
INTRODUCTION

Specific luminosity



$$L_{sp} = \frac{L}{I_{b,LER} I_{b,HER}} = \frac{n_b}{4\pi e^2 f_0} \frac{R_L}{\sigma_x^* \sigma_y^*}$$

Estimation of vertical beam size
with dynamic effect



Comparison of vertical beam size

- Estimated value from luminosity
  - 1.1 μm at 0.6 mA^2 -> 1.6 μm at 1.5 mA^2
  - increased by ~50 %
  - $k=\varepsilon_y/\varepsilon_x$= 0.5 % at the low bunch current products.
- SRM measurement
  - 1.8/1.8 μm at 0.6 mA^2 -> 2.8/2.4 μm at 1.5 mA^2
  - increased by ~50 %
- Blowup rate is similar each other except for absolute values.
- The coupling parameter, k is small at the small bunch current ?.

Chromaticity

Twiss-chromaticity

R-chromaticity

Method of X-Y coupling measurement

Summary of X-Y coupling

TURN-BY-TURN BPM ANALYSIS AT KEKB

EXPERIMENTS

Chromaticity

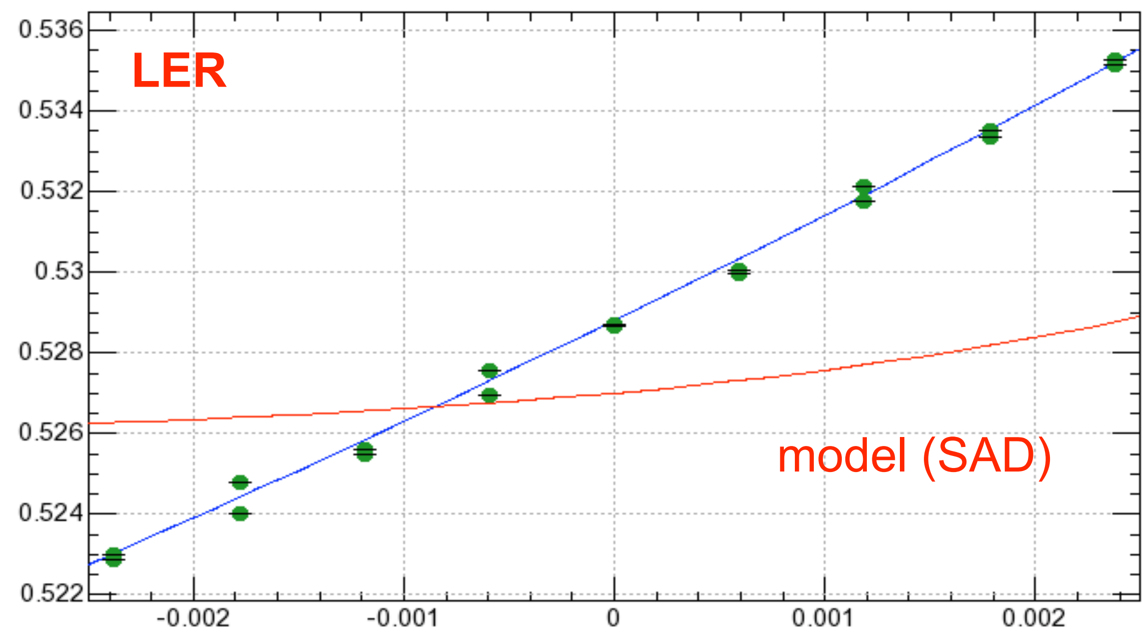
Measured by single-pass BPMs using a horizontal kicker

Date: 2008/Dec/14

ChiSquare = 508709. Goodness = .00000

p2 = 53.8055 +/- .15305 p1 = 2.55889 +/- 2.06E-4 p0 = .52881 +/- 4.69E-7

Horizontal Chromaticity : $\xi_x = 2.56$ $\alpha_p = 3.31\times10^{-4}$



ν_x

Function = (p0+(p1 x)+(p2 x x))

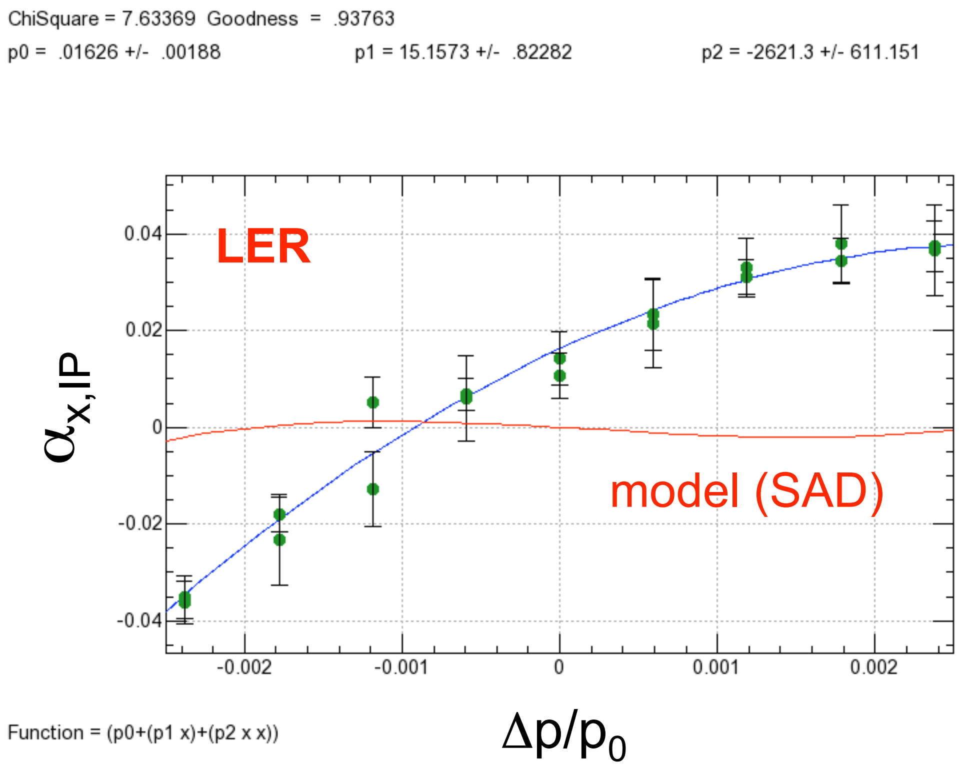
$\Delta p/p_0$

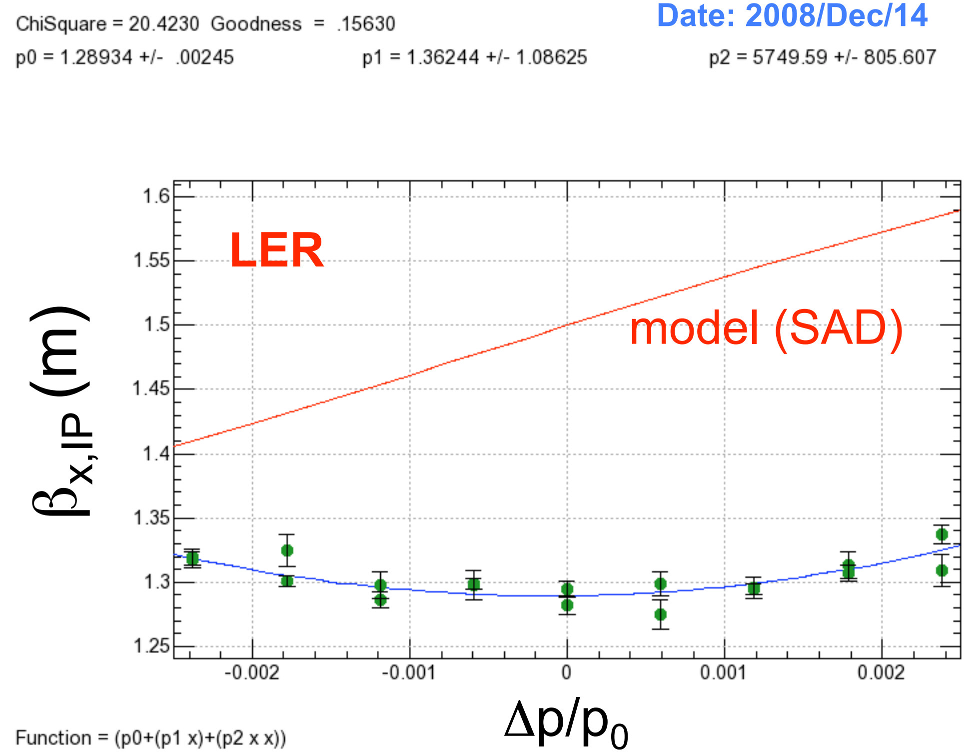
$$\frac{\Delta f}{f_0} = -\alpha_p \frac{\Delta p}{p_0}$$

Δf = -400 ~ +400 Hz, 100 Hz step

Twiss-chromaticity : Measurements

Date: 2008/Dec/14





$$\alpha_{x,IP} = -0.016$$

$$\frac{\partial \alpha_{x,IP}}{\partial \delta} = 15.2$$

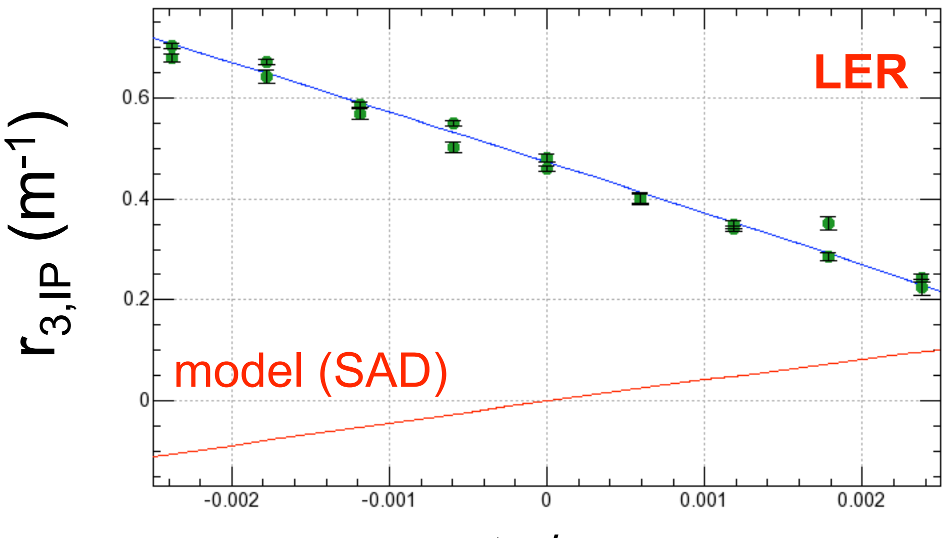
$$\beta_{x,IP} = 1.29\ m$$

$$\frac{\partial \beta_{x,IP}}{\partial \delta} = 1.4$$

R-chromaticity : Measurements

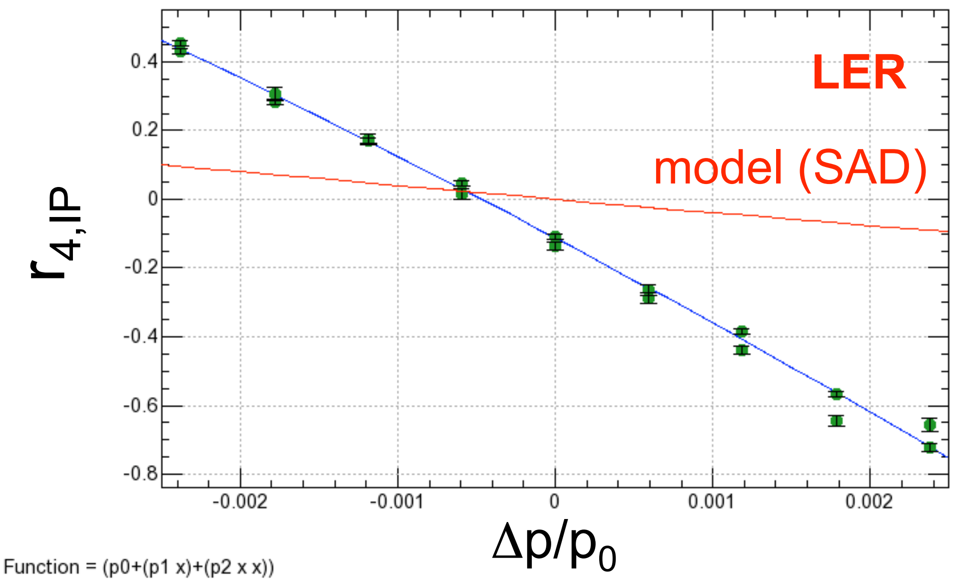
Date: 2008/Dec/14

ChiSquare = 110.220 Goodness = 1.5E-16
p0 = .47291 +/- .00248 p1 = -99.833 +/- 1.18460 p2 = -878.74 +/- 862.683



Function = (p0+(p1 x)+(p2 x x))

ChiSquare = 90.7673 Goodness = 7.1E-13
p0 = -.11212 +/- .00353 p1 = -243.24 +/- 1.57720 p2 = -5273.5 +/- 1176.09



Function = (p0+(p1 x)+(p2 x x))

$$r_{3,IP} = 0.473\ m^{-1}$$

+10 units

$$\frac{\partial r_3}{\partial \delta} = -100\ m^{-1}$$

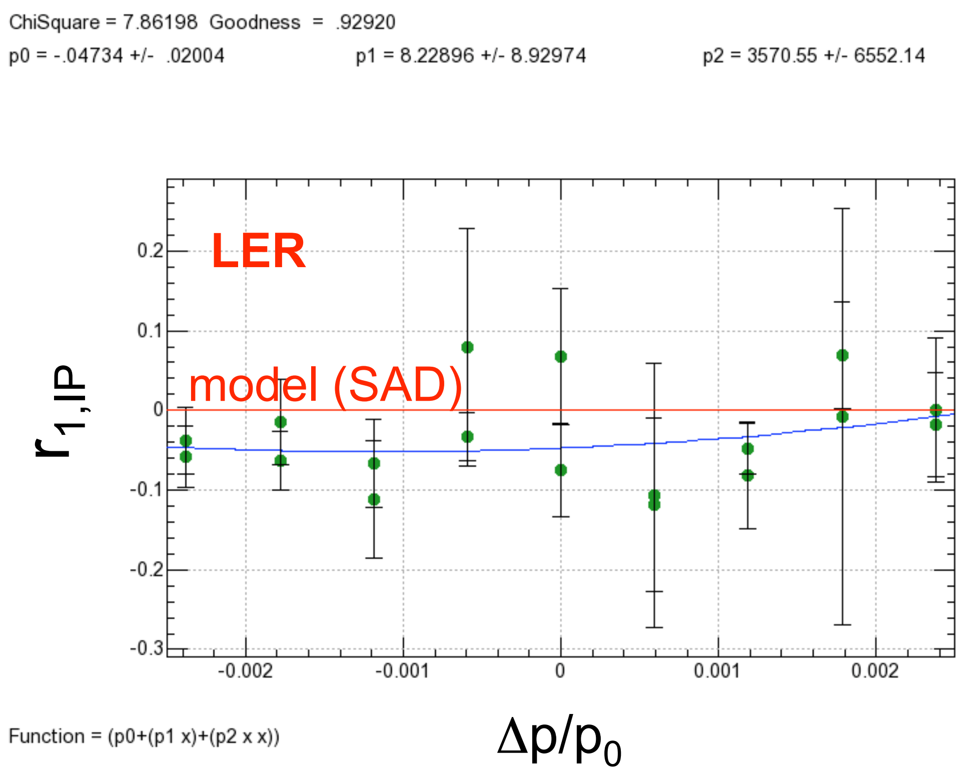
coupling (BPM coordina
R1 @IP (1 = 3.17 mrad)
R2 @IP (1 = 0.43 mm)
R3 @IP (1 = 47.69 km^{-1})
R4 @IP (1 = 57.60 mrad)

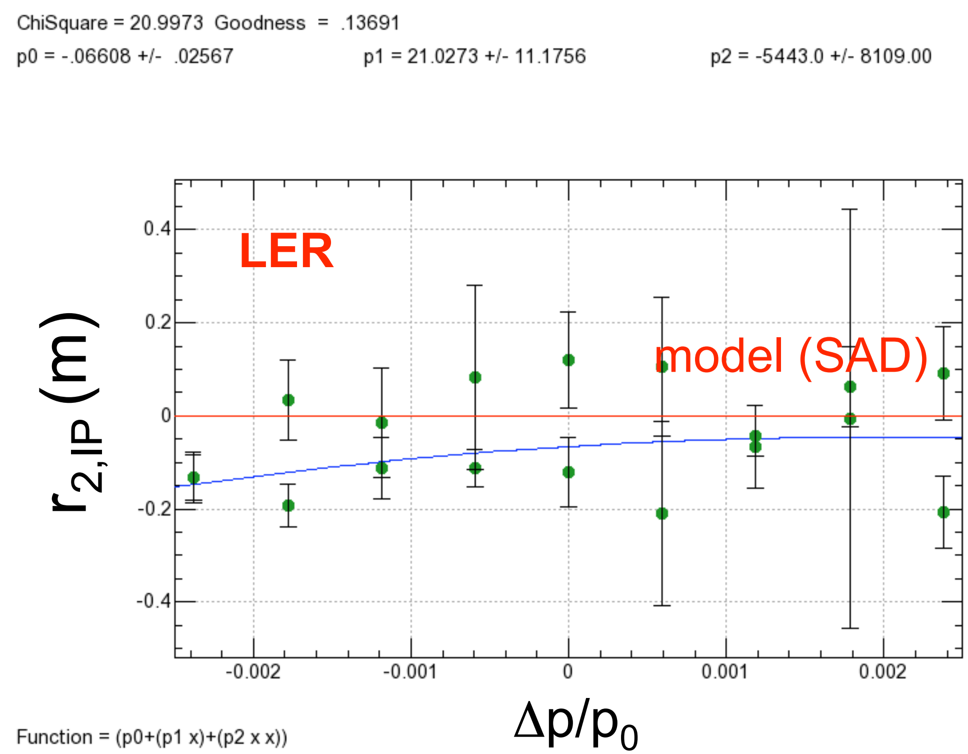
$$r_{4,IP} = -0.112$$

-2 units

$$\frac{\partial r_4}{\partial \delta} = -243$$

R-chromaticity : Measurements (cont'd)





$$r_{1,IP} = -0.047$$

-15 units

$$\frac{\partial r_1}{\partial \delta} = 8.2$$

coupling (BPM coordina

R1 @IP (1 = 3.17 mrad)

R2 @IP (1 = 0.43 mm)

R3 @IP (1 = 47.69 km^{-1})

R4 @IP (1 = 57.60 mrad)

$$r_{2,IP} = -0.066\ m$$

-150 units

$$\frac{\partial r_2}{\partial \delta} = -21\ m$$

Accuracy of BPM is not enough for r_1 and r_2. (V-mode is necessary.)

X-Y coupling (1)

Transformation from a decoupled coordinate to a physical coordinate:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} \mu & 0 & r_4 & -r_2 \\ 0 & \mu & -r_3 & r_1 \\ -r_1 & -r_2 & \mu & 0 \\ -r_3 & -r_4 & 0 & \mu \end{pmatrix} \begin{pmatrix} X \\ X' \\ Y \\ Y' \end{pmatrix} \qquad \mu^2 + \left(r_1 r_4 - r_2 r_3\right) = 1$$

When Y=0 and Y'=0 (H-mode):

$$x = \mu X$$
$$x' = \mu X'$$
$$y = -r_1 X - r_2 X'$$
$$y' = -r_3 X - r_4 X'$$

$$X = \sqrt{2J_x\beta_x}\cos\psi_x$$
$$X' = -\sqrt{\frac{2J_x}{\beta_x}}\left(\alpha_x\cos\psi_x + \sin\psi_x\right)$$

⇒ This induces a vertical betatron oscillation.

X-Y coupling (2)

[1]
$$
\begin{aligned}
y &= -r_1 X - r_2 X' \\
&= \left(-r_1\sqrt{2J_x\beta_x} + r_2\sqrt{\frac{2J_x}{\beta_x}}\alpha_x\right)\cos\psi_x + r_2\sqrt{\frac{2J_x}{\beta_x}}\sin\psi_x \\
&= A\cos\psi_x + B\sin\psi_x \\
&= \sqrt{A^2+B^2}\sin(\psi_x+\phi)
\end{aligned}
$$

Phase is important.

$$
\begin{cases}
\sin\phi = \dfrac{A}{\sqrt{A^2+B^2}} \\
\cos\phi = \dfrac{B}{\sqrt{A^2+B^2}}
\end{cases}
$$

$$A = -r_1\frac{x_{amp}}{\mu} + r_2\frac{x'_{amp}}{\mu}\sin\theta = y_{amp}\sin\phi$$

$$B = r_2\frac{x'_{amp}}{\mu}\cos\theta = y_{amp}\cos\phi$$

$$r_1 = -\mu\frac{y_{amp}}{x_{amp}}\sin\phi + r_2\frac{x'_{amp}}{x_{amp}}\sin\theta$$

$$r_2 = \mu\frac{y_{amp}}{x'_{amp}}\frac{\cos\phi}{\cos\theta}$$

$$X' = -\sqrt{\frac{2J_x}{\beta_x}\left(1+\alpha_x^2\right)}\sin(\psi_x+\theta)$$

$$\sin\theta = \frac{\alpha_x}{\sqrt{1+\alpha_x^2}} \qquad \cos\theta = \frac{1}{\sqrt{1+\alpha_x^2}}$$

x_{amp}, x'_{amp}, y_{amp}, ϕ, θ can be obtained from single-pass BPMs.

The ratio of amplitudes and phase provides an information of X-Y coupling.

When α_x=0 at IP, sinθ=0 and cosθ=1.

X-Y coupling (3)

[2]
$$y' = -r_3 X - r_4 X'$$
$$= \left(-r_3\sqrt{2J_x\beta_x} + r_4\sqrt{\frac{2J_x}{\beta_x}}\alpha_x\right)\cos\psi_x + r_4\sqrt{\frac{2J_x}{\beta_x}}\sin\psi_x$$
$$= C\cos\psi_x + D\sin\psi_x$$
$$= \sqrt{C^2 + D^2}\sin(\psi_x + \varphi)$$

Phase is important.

$$\begin{cases} \sin\varphi = \dfrac{C}{\sqrt{C^2 + D^2}} \\ \cos\varphi = \dfrac{D}{\sqrt{C^2 + D^2}} \end{cases}$$

$$C = -r_3\frac{x_{amp}}{\mu} + r_4\frac{x'_{amp}}{\mu}\sin\theta = y'_{amp}\sin\varphi$$

$$D = r_4\frac{x'_{amp}}{\mu}\cos\theta = y'_{amp}\cos\varphi$$

$$r_3 = -\mu\frac{y'_{amp}}{x_{amp}}\sin\varphi + r_4\frac{x'_{amp}}{x_{amp}}\sin\theta$$

$$r_4 = \mu\frac{y'_{amp}}{x'_{amp}}\frac{\cos\varphi}{\cos\theta}$$

$$X' = -\sqrt{\frac{2J_x}{\beta_x}(1+\alpha_x^2)}\sin(\psi_x + \theta)$$

$$\sin\theta = \frac{\alpha_x}{\sqrt{1+\alpha_x^2}} \qquad \cos\theta = \frac{1}{\sqrt{1+\alpha_x^2}}$$

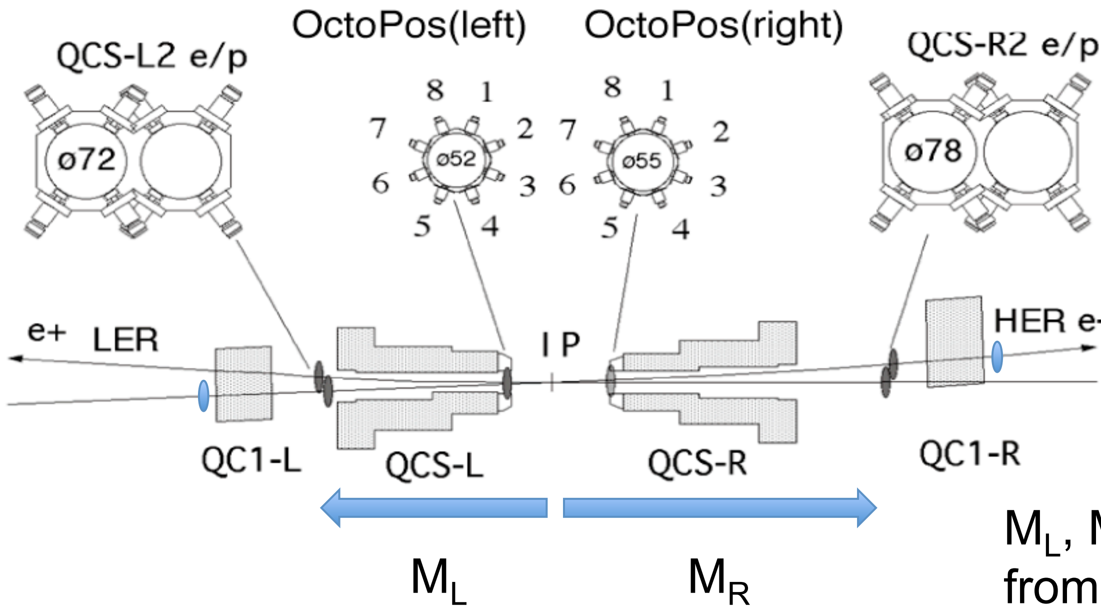
x_{amp}, x'_{amp}, y'_{amp}, φ, θ can be obtained from single-pass BPMs.

The ratio of amplitudes and phase provides an information of X-Y coupling.

When α_x=0 at IP, sinθ=0 and cosθ=1.

Reconstruction of phase space

Phase space (x,x',y,y') at IP is reconstructed by using neighbor single-pass BPMs (location: QCS-L and QCS-R)

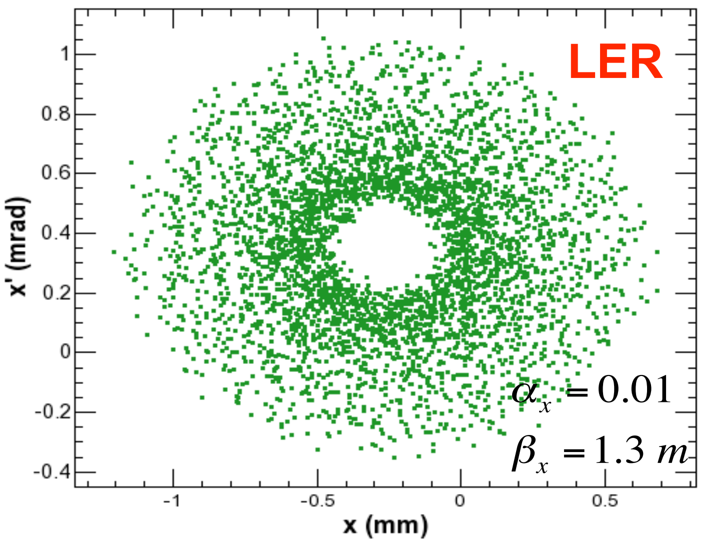


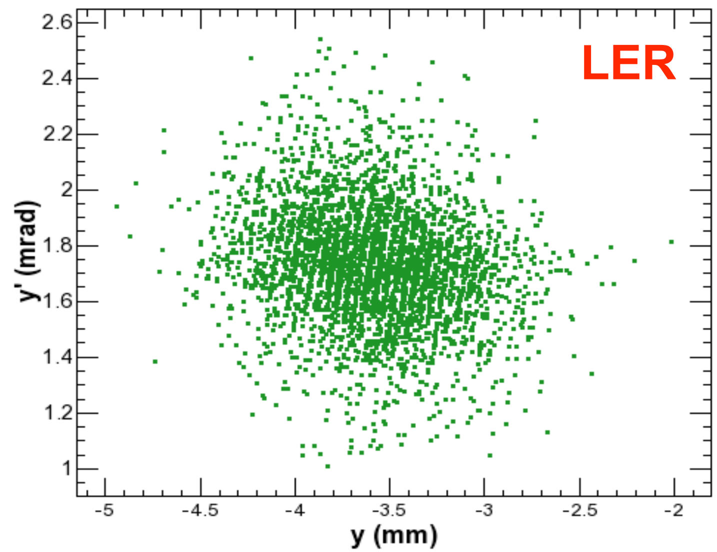
M_L, M_R is a transfer matrix from IP to the single-pass BPM. The model is used.

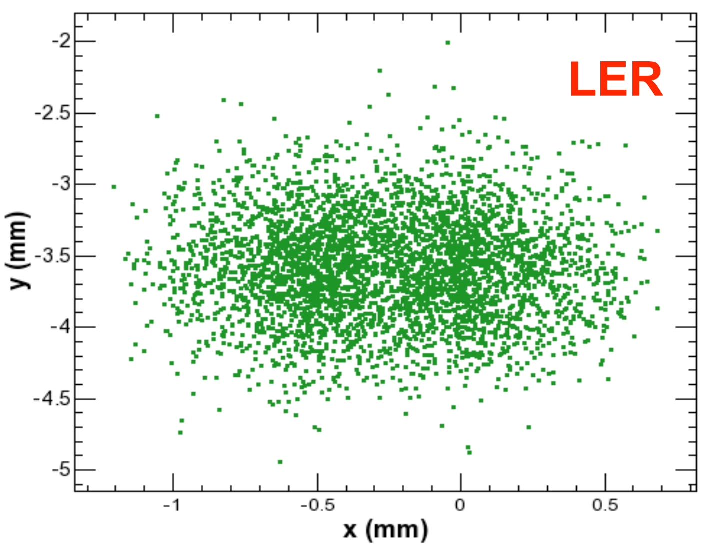
$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{IP} = \begin{pmatrix} m_{L,11} & m_{L,12} & m_{L,13} & m_{L,14} \\ m_{L,31} & m_{L,32} & m_{L,33} & m_{L,34} \\ m_{R,11} & m_{R,12} & m_{R,13} & m_{R,14} \\ m_{R,31} & m_{R,32} & m_{R,33} & m_{R,34} \end{pmatrix}^{-1} \begin{pmatrix} x_L \\ y_L \\ x_R \\ y_R \end{pmatrix}_{QCS}$$

Experiments

Date: 2008/Dec/14







Beam oscillation is induced by a horizontal kicker. Phase space plots at IP reconstructed by two BPMs(QCS-L and QCS-R)

$$\alpha_x = \tan\theta$$

$$\beta_x = \frac{x_{amp}}{x'_{amp}} \frac{1}{\cos\theta}$$

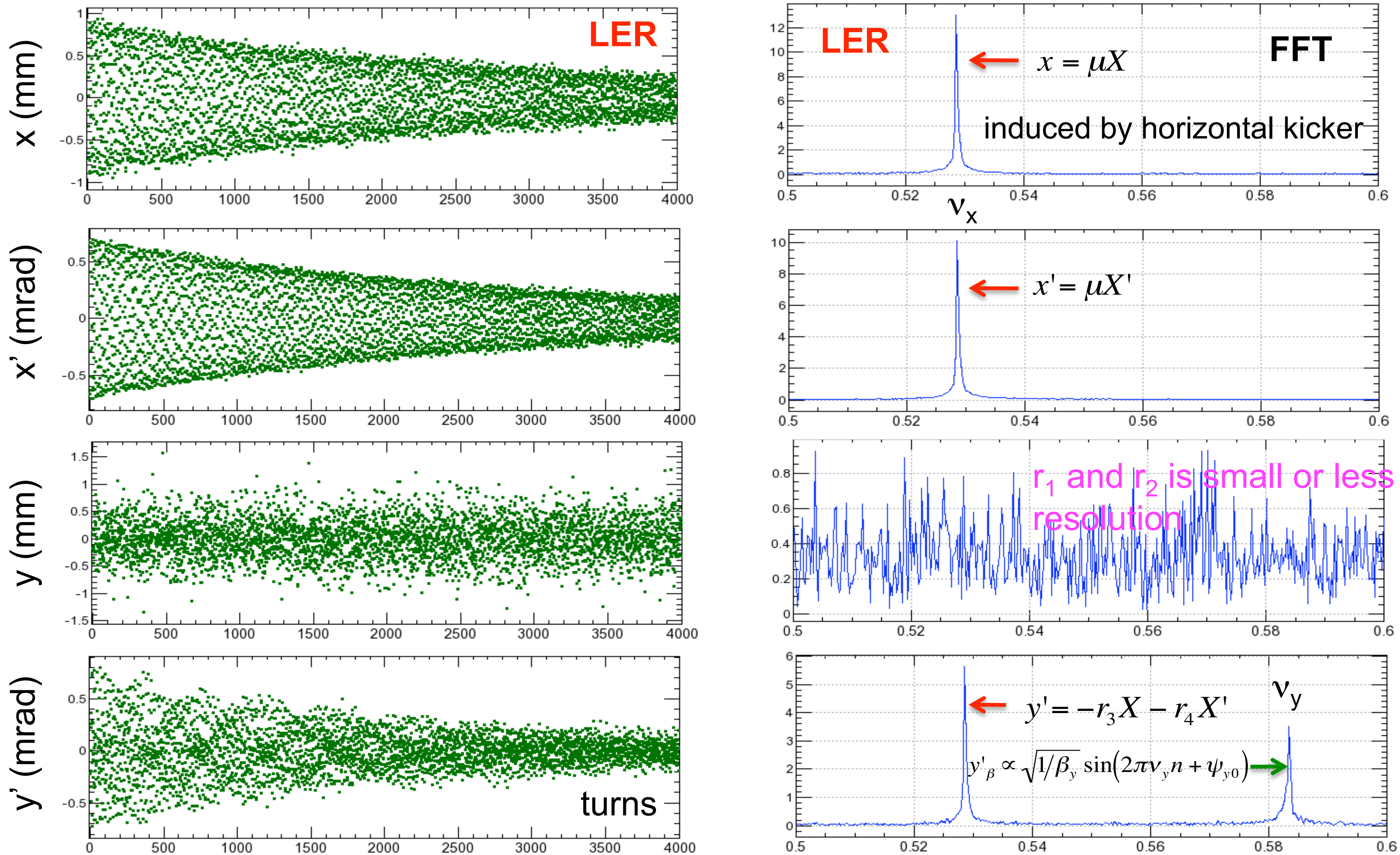
Ratio of amplitudes and phase

← Horizontal tune is near half integer.

Turn-by-turn data at IP

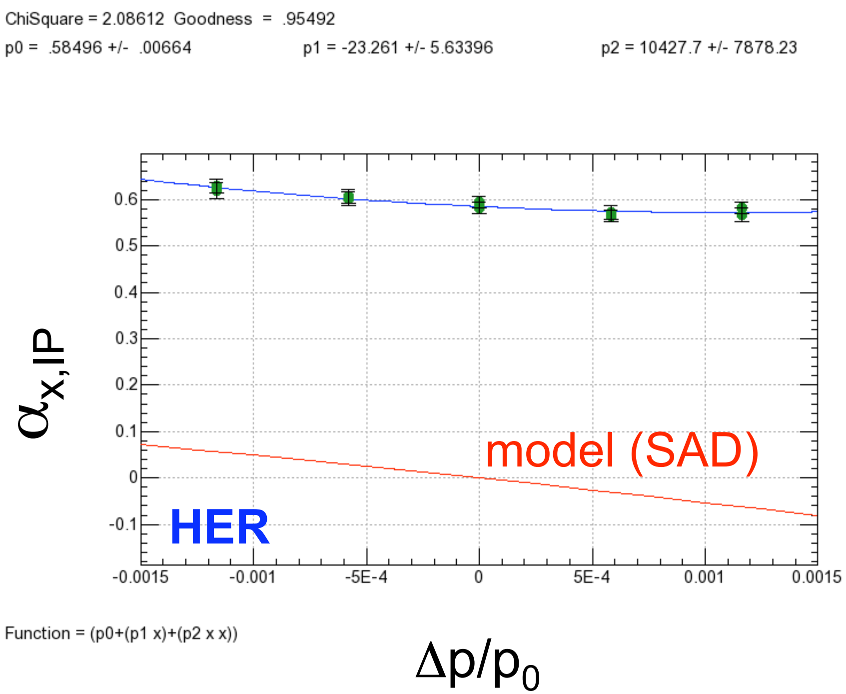
H-mode

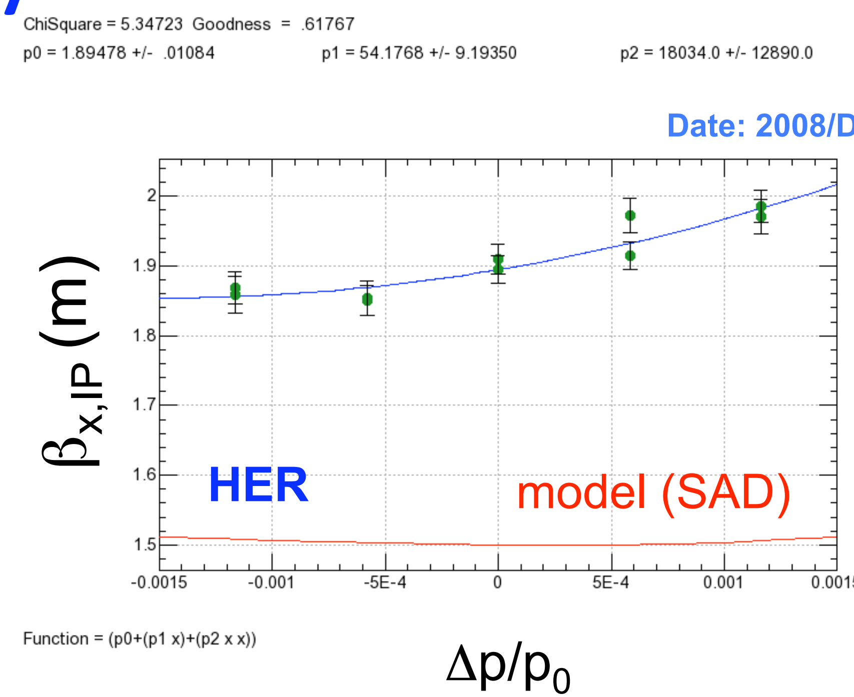
Date: 2008/Dec/14

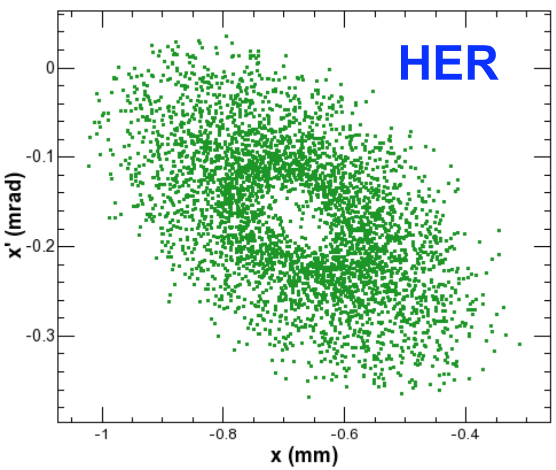


Twiss-chromaticity : Measurements

Date: 2008/Dec/12







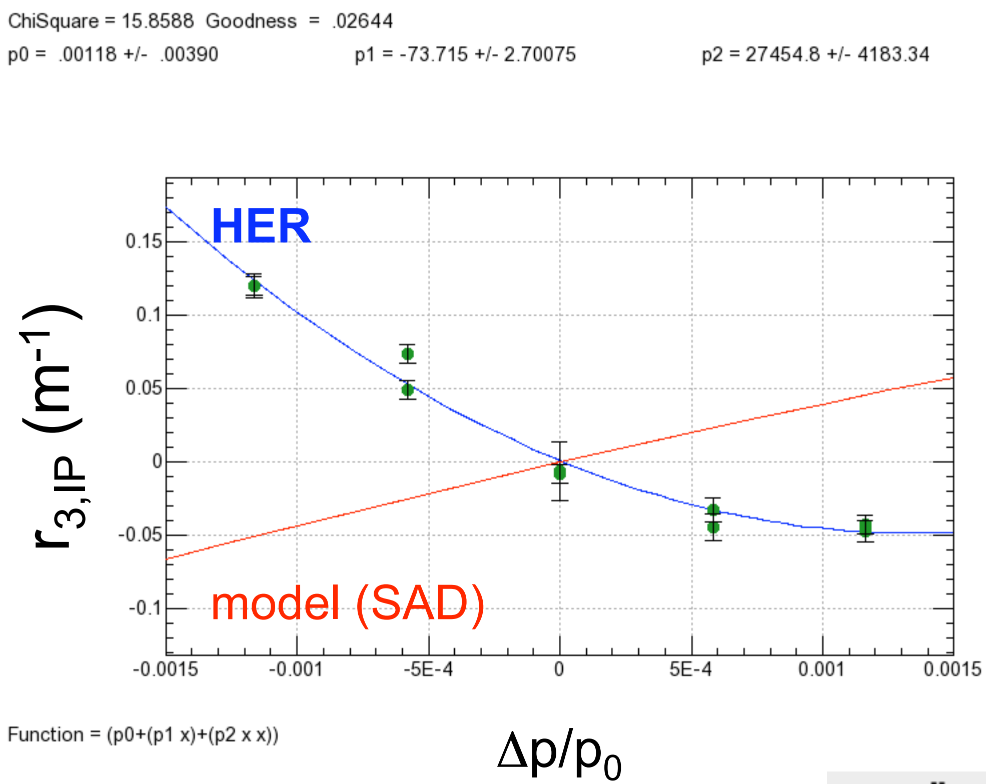
$$\alpha_{x,IP} = 0.59 \qquad \beta_{x,IP} = 1.90\ m$$

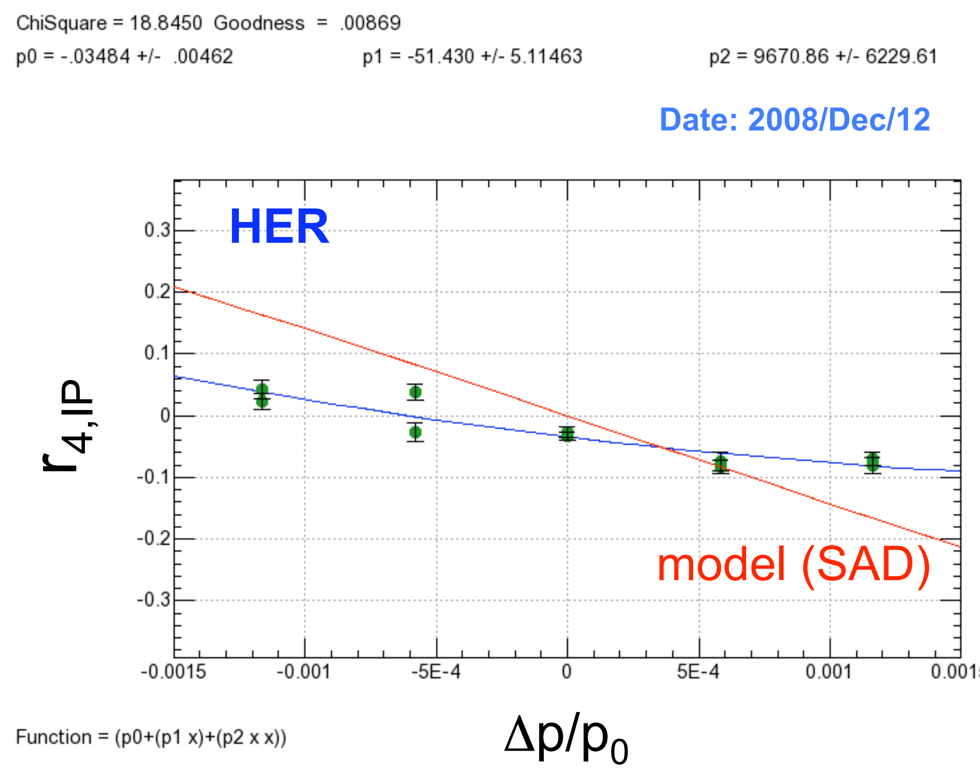
$$\frac{\partial \alpha_{x,IP}}{\partial \delta} = -23 \qquad \frac{\partial \beta_{x,IP}}{\partial \delta} = 54\ m$$

Large alpha, something wrong !.

(BPMs at QC1LE and QC1RE are used.)

R-chromaticity : Measurements





$$r_{3,IP} = 0.0012\ m^{-1}$$

$$\frac{\partial r_3}{\partial \delta} = -74\ m^{-1}$$

coupling (BPM coordinate)
R1 @IP (1 = 0.30 mrad)
R2 @IP (1 = 0.54 mm)
R3 @IP (1 = 24.69 km⁻¹)
R4 @IP (1 = 38.63 mrad)

$$r_{4,IP} = -0.035$$

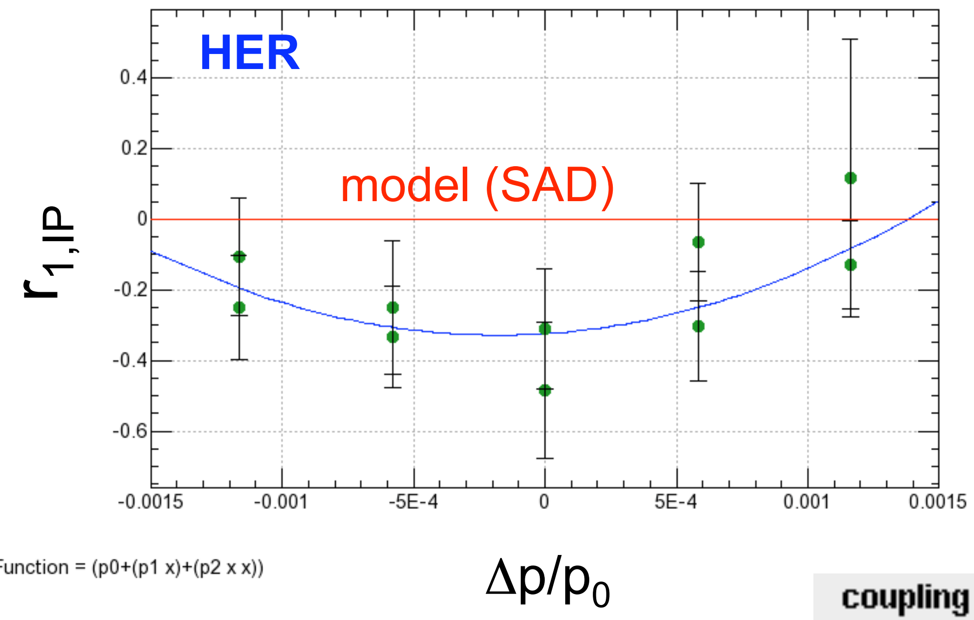
$$\frac{\partial r_4}{\partial \delta} = -51$$

R-chromaticity : Measurements

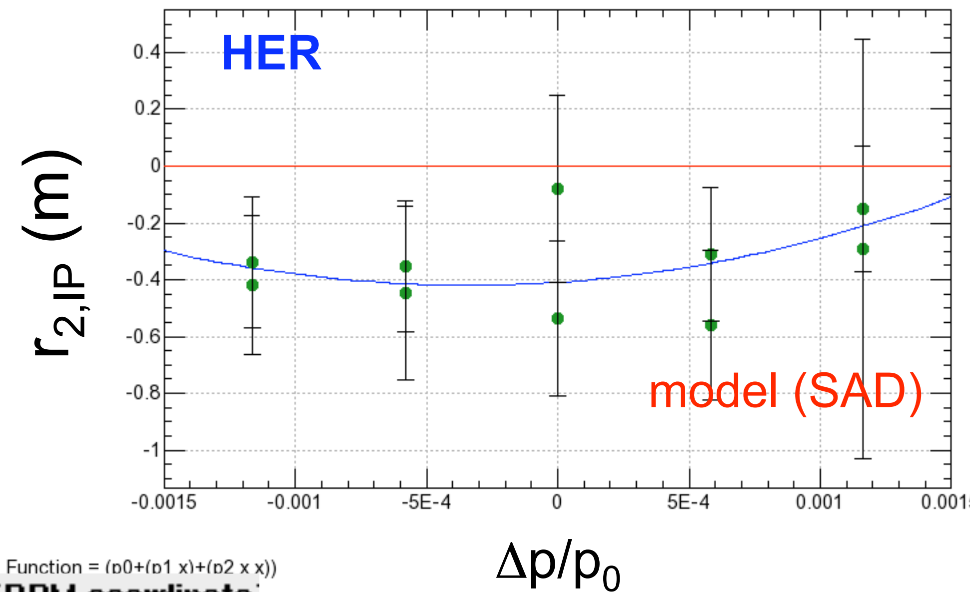
ChiSquare = 3.00474 Goodness = .88456
p0 = -.32387 +/- .08366 p1 = 47.6748 +/- 62.2849 p2 = 136100. +/- 93220.3

ChiSquare = 2.20045 Goodness = .94792
p0 = -.40835 +/- .13471 p1 = 63.0786 +/- 102.388 p2 = 90882.6 +/- 153160.

Date: 2008/Dec/12



Function = (p0+(p1 x)+(p2 x x))



Function = (p0+(p1 x)+(p2 x x))

$$r_{1,IP} = -0.324$$

$$\frac{\partial r_1}{\partial \delta} = 48$$

coupling (BPM coordinate)
R1 @IP (1 = 0.30 mrad)
R2 @IP (1 = 0.54 mm)
R3 @IP (1 = 24.69 km^{-1})
R4 @IP (1 = 38.63 mrad)

$$r_{2,IP} = -0.408\ m$$

$$\frac{\partial r_2}{\partial \delta} = 63\ m$$

Accuracy of BPM is not enough for r_1 and r_2. (V-mode is necessary.)

Summary of X-Y coupling

- *x(n), x'(n), y(n), y'(n)* are fitted by a linear combination of *cosine* and *sine* with an exponential damping. Resol. at each BPM is ~100 μm.
  - Model transfer matrix is used in this analysis. 1000 turns are used or analysis.
- Twiss parameters, α_x and β_x can be obtained from *x(n)* and *x'(n)*:

$$X'(n) = -\sqrt{\frac{2J_x}{\beta_x}\left(1+\alpha_x^2\right)}\sin\left(2\pi\nu_x n + \psi_{x0} + \theta\right)\exp\left(-\Gamma n\right)$$

$$\sin\theta = \frac{\alpha_x}{\sqrt{1+\alpha_x^2}} \qquad \cos\theta = \frac{1}{\sqrt{1+\alpha_x^2}}$$

$$\Rightarrow \quad \alpha_x = \tan\theta \qquad \beta_x = \frac{x_{amp}}{x'_{amp}}\frac{1}{\cos\theta}$$

  - If the location is IP, α_x is zero in the model.
- Uncertainty about μ. If r_1 and r_2 is small, μ is 1 approximately.

$$\mu^2 + \left(r_1 r_4 - r_2 r_3\right) = 1$$

$$X_{amp} = x_{amp} \qquad X'_{amp} = x'_{amp} \qquad \left(\alpha_x = 0\right)$$

$$\Rightarrow \quad r_3 = -\frac{y'_{amp}}{x_{amp}}\sin\varphi \qquad r_4 = \frac{y'_{amp}}{x'_{amp}}\cos\varphi$$

The ratio of amplitudes and phase provides an information of X-Y coupling.

Summary of X-Y coupling (cont'd)

- The global X-Y coupling measurement is sensitive to r_1 and r_2, but less sensitive to r_3 and r_4.

$$x = \mu X$$
$$x' = \mu X'$$
$$y = -r_1 X - r_2 X'$$
$$y' = -r_3 X - r_4 X'$$

The correction makes y small as much as possible which is induced by a horizontal kick.

  - The X-Y coupling, r_3 and r_4 at IP are large in LER(14/Dec/2008), which are estimated by single-pass BPMs. Accuracy is not enough for r_1 and r_2. If r_1 and r_2 are well corrected, σ_y/σ_x should be small. However σ'_y/σ'_x might be large.
- R-chromaticity can be measured by changing rf frequency which is similar to chromaticity measurements.
- IP tilt knob can correct r_3 and r_4.
  - Skew sextupoles are needed to correct the R-chromaticity.

References

- D. Sagan and D. Rubin, PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **2, 074001 (1999)** .
- Y. Ohnishi, Y. Funakoshi, K. Mori, E. Perevedentsev, M. Tanaka, M. Tejima, M. Tobiyama, Measurement of xy coupling using turn-by-turn BPM at KEKB, EPAC2000.
- D. Rubin, “AC” dispersion measurement, ILCDR08.

LER damping rate

Head-Tail damping

Smearing effect due to nonlinearity

MISCELLANEOUS

LER Damping Rate

Measured by single-pass BPMs

ChiSquare = 1.45790 Goodness = .48241

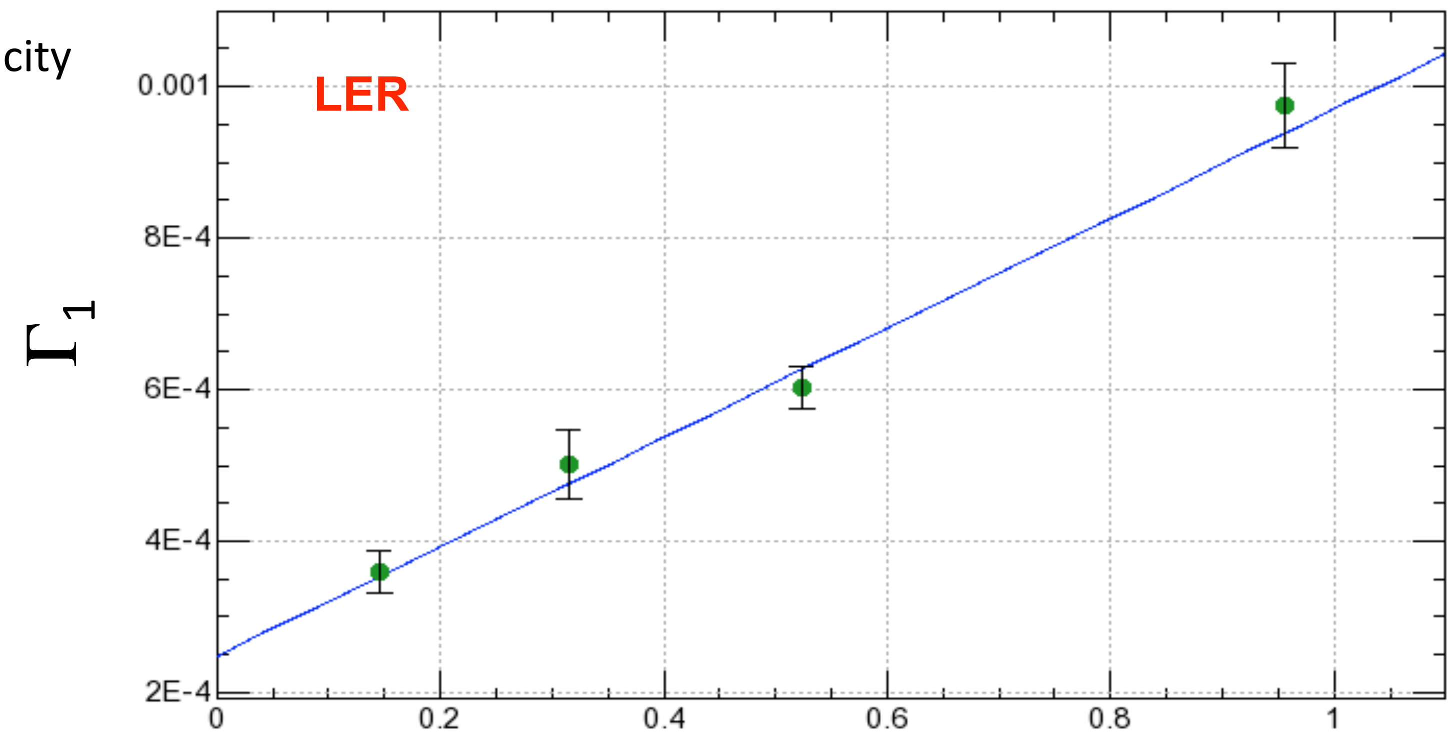
p1 = 7.23E-4 +/- 7.10E-5

p2 = 2.48E-4 +/- 3.32E-5

Radiation damping rate (1/turn) : Γ = (2.48±0.33)x10^{-4} (meas)/ 2.47x10^{-4} (model)

Chromaticity

ξ_x = 2.56



Function = (p2+(p1 x))

*DCCT

Head-Tail Damping (LER)

$$\Gamma_{HT} = \frac{\sqrt{\pi}\langle\beta\rangle I_b Z_\perp \xi}{(E_b/e)\alpha_p}$$

(1/turn)

$$\langle\beta\rangle = \frac{R}{\nu_\beta} = \frac{C}{2\pi\nu_\beta}$$

LER transverse impedance:

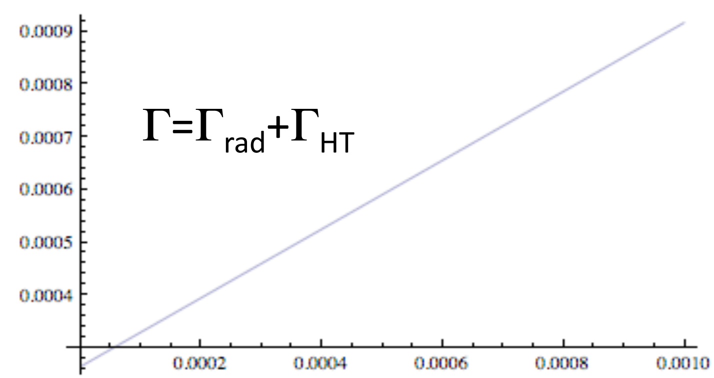
$$Z_\perp = 17 \pm 2 \text{ k}\Omega\text{m}$$

Head – Tail Damping

```
In[24]:= G := √π β Ib Z ξ / (Eb α);
(*HER 2008/12/14 *)
β = 3016 / 2 / π / 45.527;
Ib = 0.2 * 10^-3;
Z = 16.50 * 10^3;
ξ = 2.56;
Eb = 3.3 * 10^9;
α = 3.31 * 10^-4;
G
Out[31]= 0.000144534
```

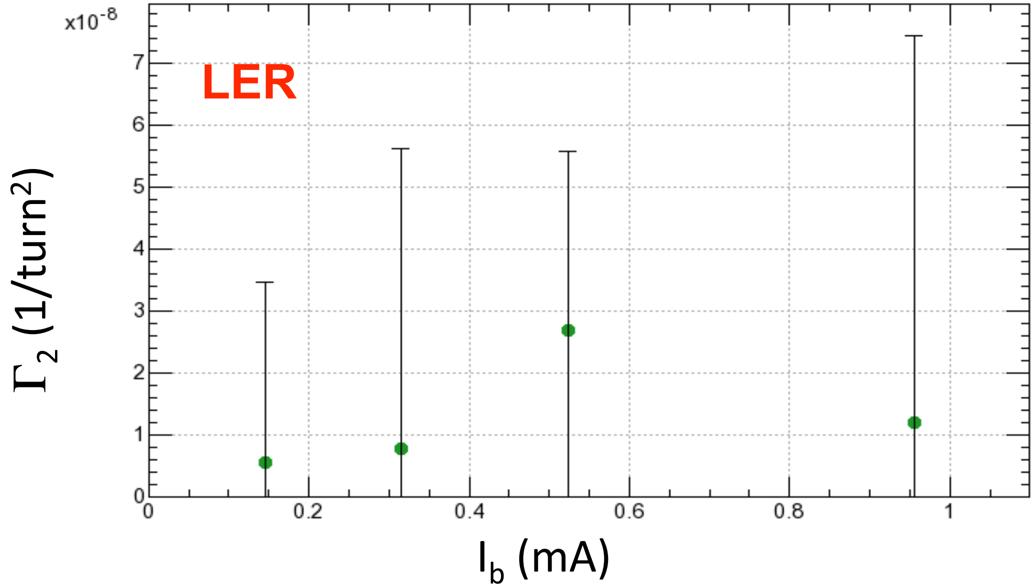
Υ(2S)

```
Plot[G + 2.64 * 10^-4, {Ib, 0, 0.001}]
```




```
In[21]:= ZZ := Eb α GG / (√π β ξ);
(* ZZ [kΩ m] *)
GG = 7.23 * 10^-4;
ZZ
16.5076
```

Smearing effect due to nonlinearlity

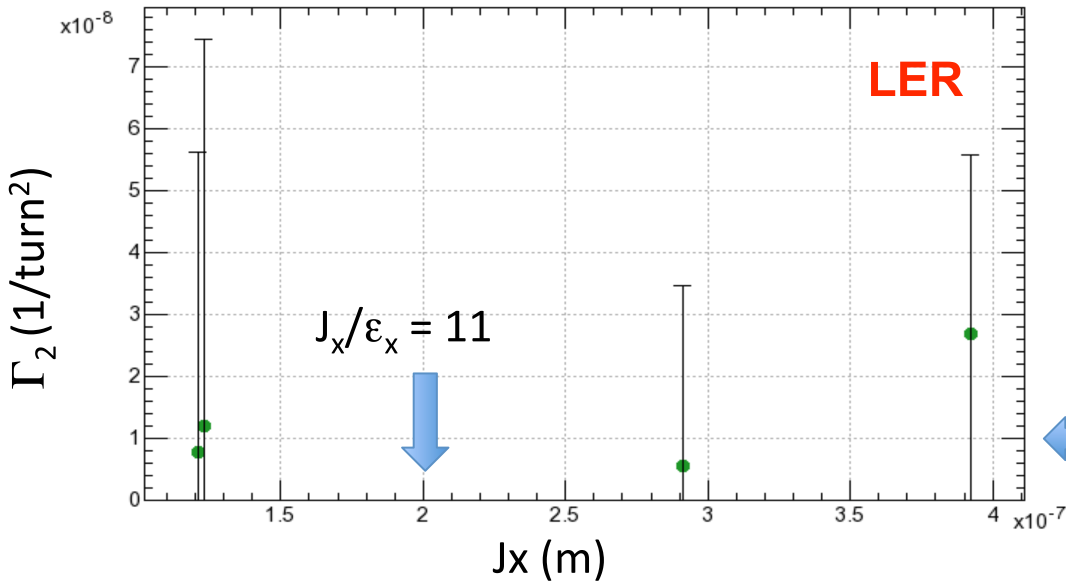


Measured by single-pass BPMs

$$\frac{x(n)}{x(0)} = \exp\left\{-\Gamma_1 n - \Gamma_2 n^2\right\}$$

$$\Gamma_2 = \frac{(2\pi\sigma_{\nu x})^2}{2} \qquad \text{(1/turn}^2\text{)}$$

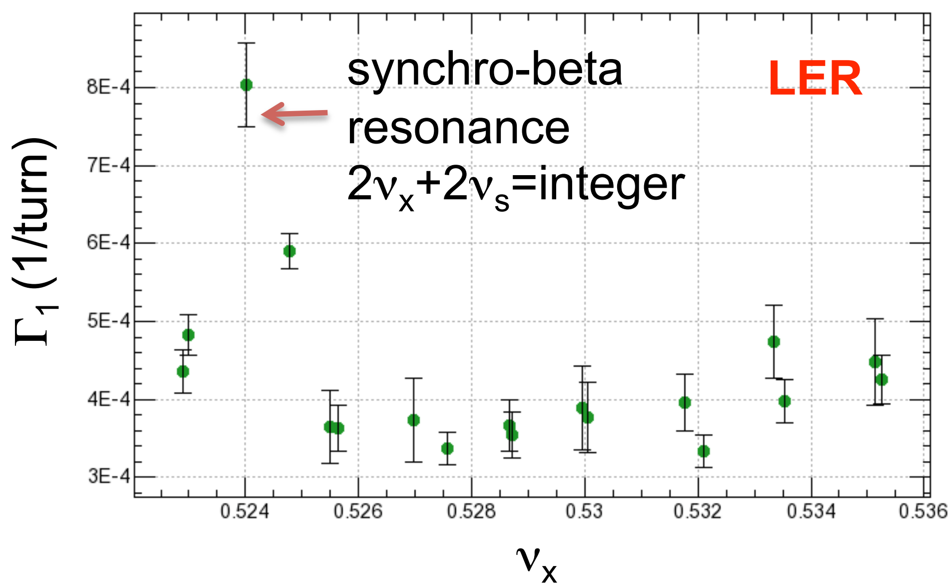
less dependence on bunch current

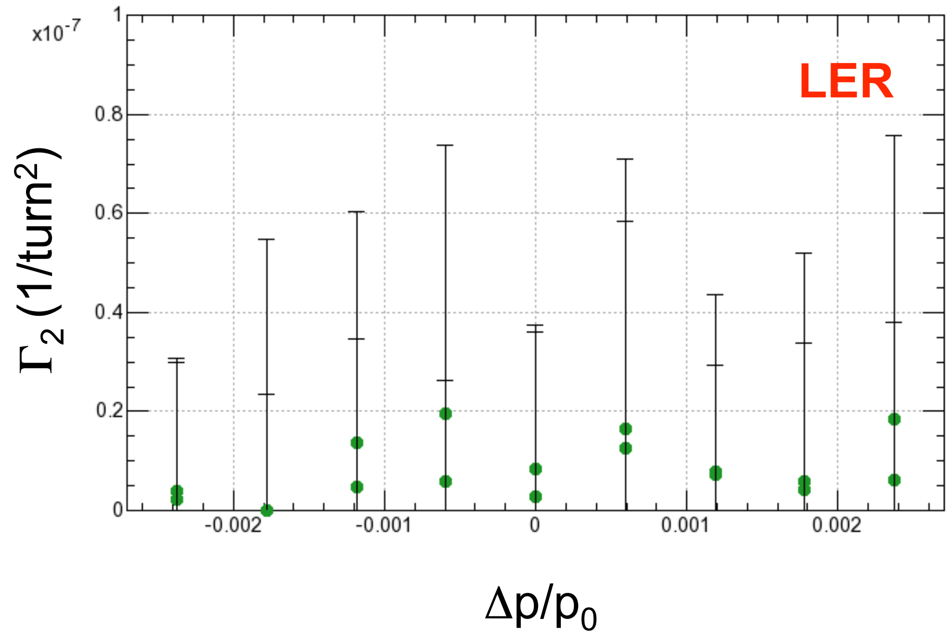


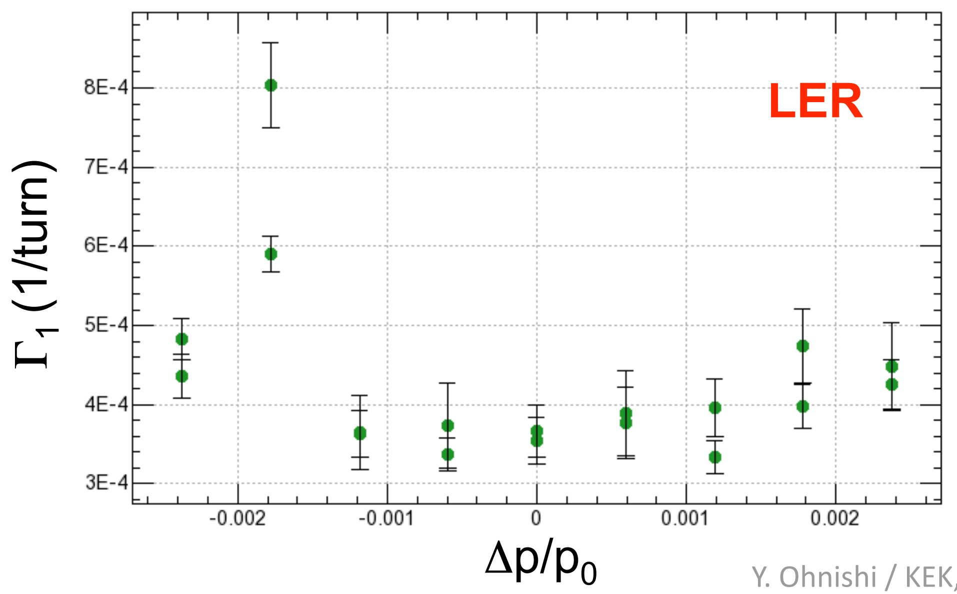
$\sigma_{\nu x} = 2.25\text{x}10^{-5} \ll \Gamma_1 = 2.47\text{x}10^{-4}$

10 times smaller than radiation damping

Nonlinearity at off-momentum







$$\frac{x(n)}{x(0)} = \exp\left\{-\Gamma_1 n - \Gamma_2 n^2\right\}$$

$$\Gamma_2 = \frac{(2\pi\sigma_{\nu x})^2}{2} \qquad (1/\mathrm{turn}^2)$$

APPENDIX

KEKB HER optics: Beta05_14_2008_22:07:34i

ε_x = 24 nm

β_x^* = 0.9 m

β_y^* = 5.9 mm

ν_x = 44.5138

ν_y = 41.590

SIMULATION WITH MACHINE ERROR

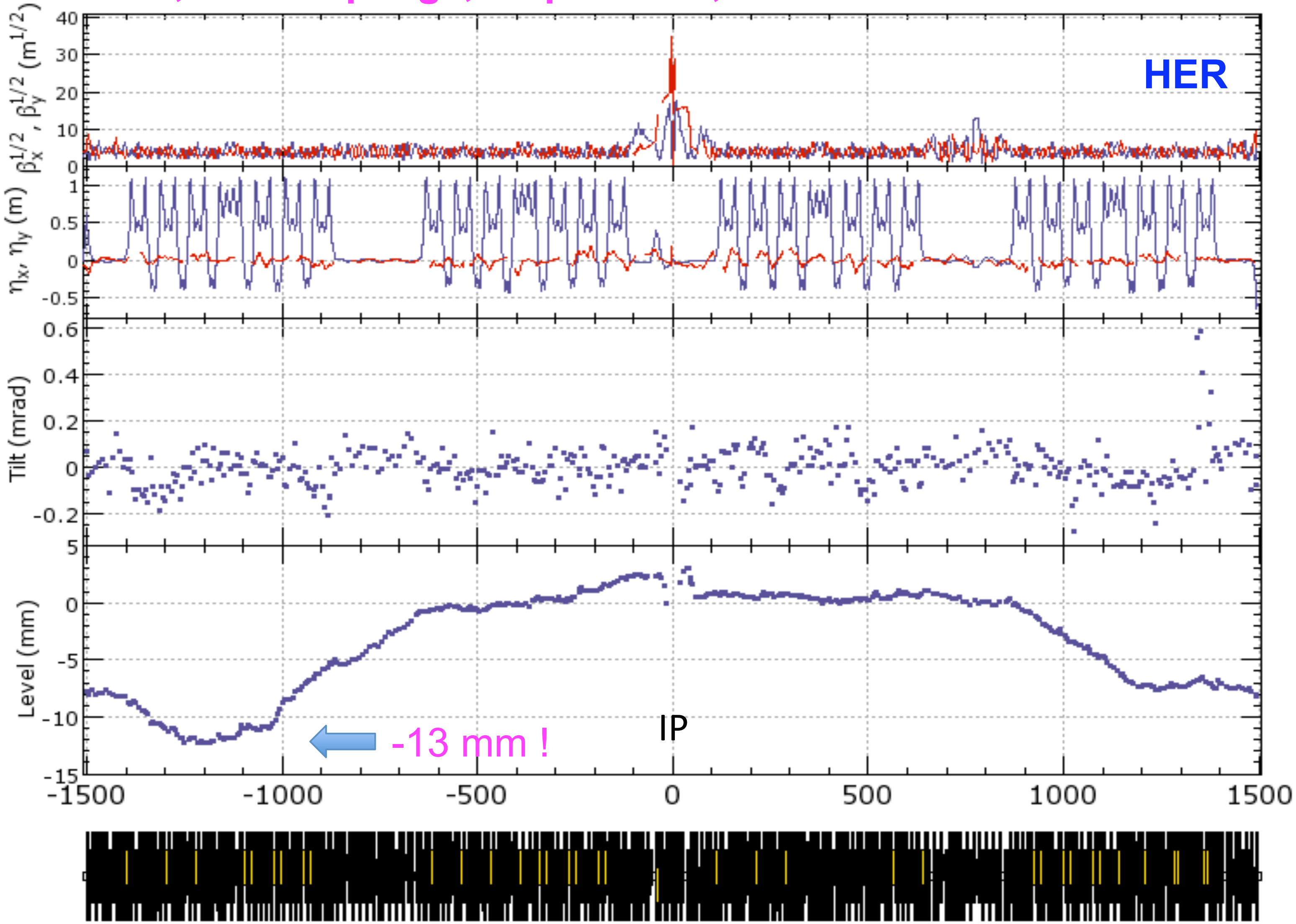
Machine Errors

	$\sigma_{\Delta x,rms}$ (μm)	$\sigma_{\Delta y,rms}$ (μm)	$\sigma_{\Delta\theta,rms}$ (mrad)	$\sigma_{\Delta K/K,rms}$
Quad	100	meas.*2	meas.*2	1×10^{-4}
Skew Quad	100	meas.*2	meas.*2	1×10^{-4}
Sextupole	100	extrapolation+100	0.1	1×10^{-4}
BPM*1	100	extrapolation+100	-	-
Steering	100	extrapolation+100	0.1	-

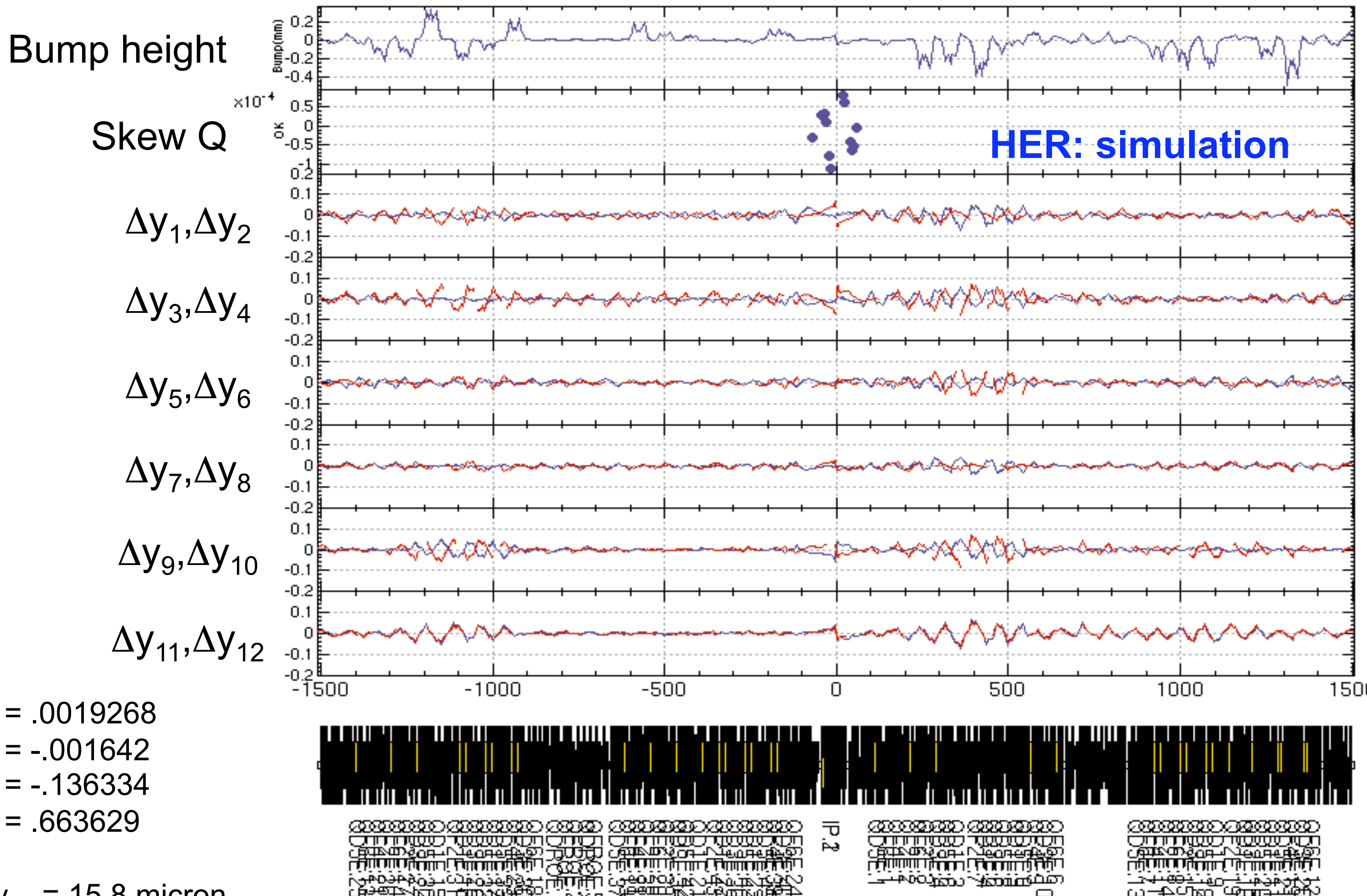
*1) BPM jitter error : $\sigma_{\Delta x,\ \Delta y,rms}$ = 2 μm
*2) Measured by MG group

Machine Error and Optics Correction : Simulations

Orbits, X-Y couplings, dispersions, beta functions are corrected.



Sample of X-Y coupling correction

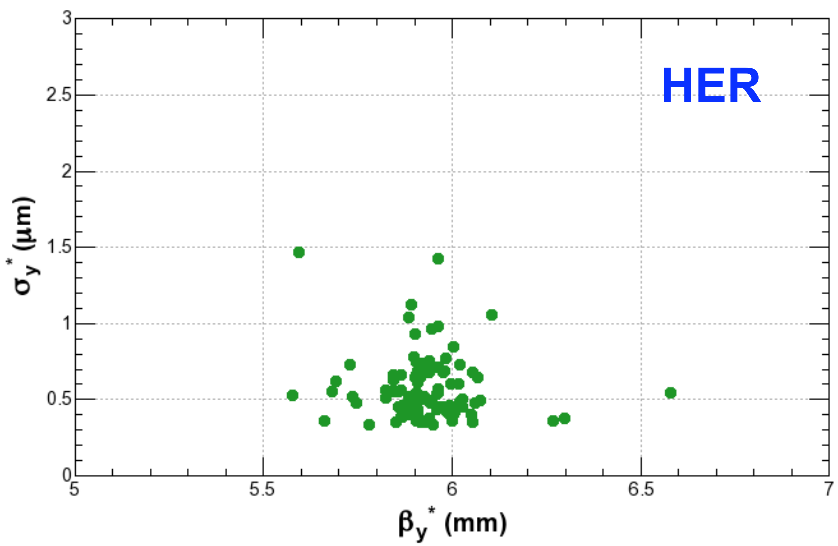


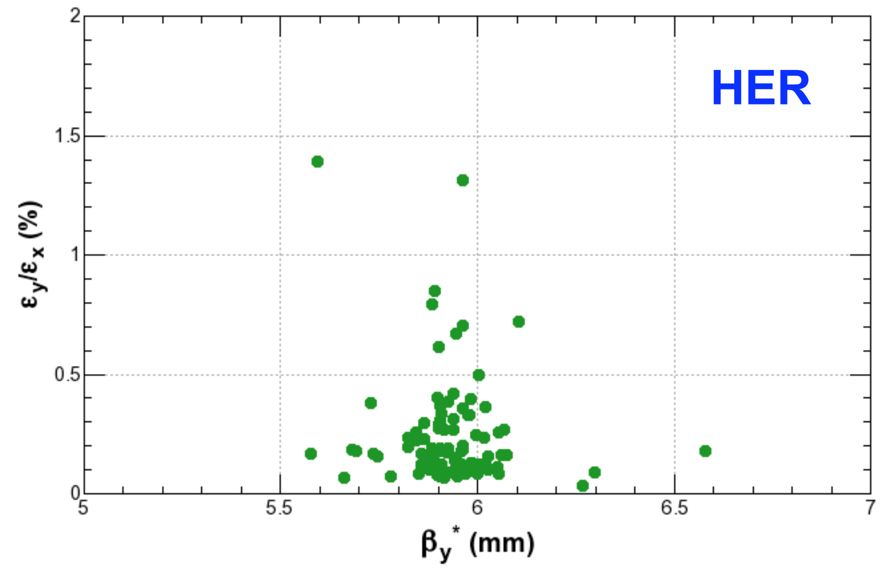
r_1 = .0019268
r_2 = -.001642
r_3 = -.136334
r_4 = .663629

Δy_{rms}= 15.8 micron

Emittance ratio : Simulations

100 samples: Each sample has a different random seed.





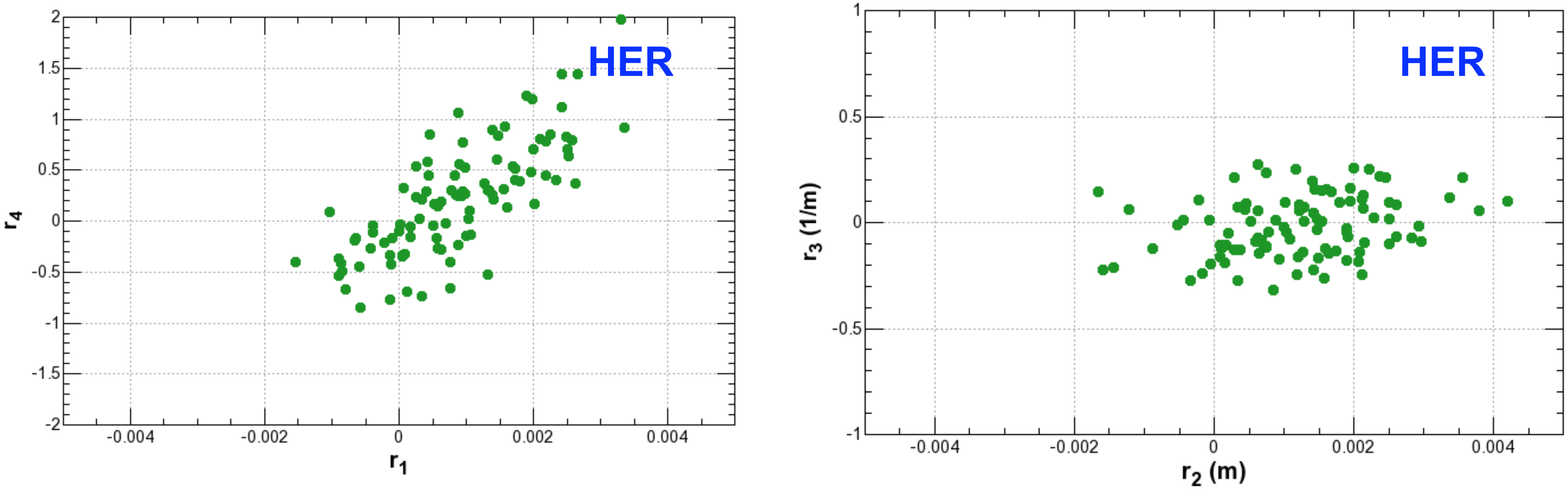
Beam size is less than 1 μm at IP

Emittance ratio is less than 0.5 %.

After the optics correction, the optics seems to be better condition.

X-Y coupling at IP : Simulations

100 samples: Each sample has a different random seed.



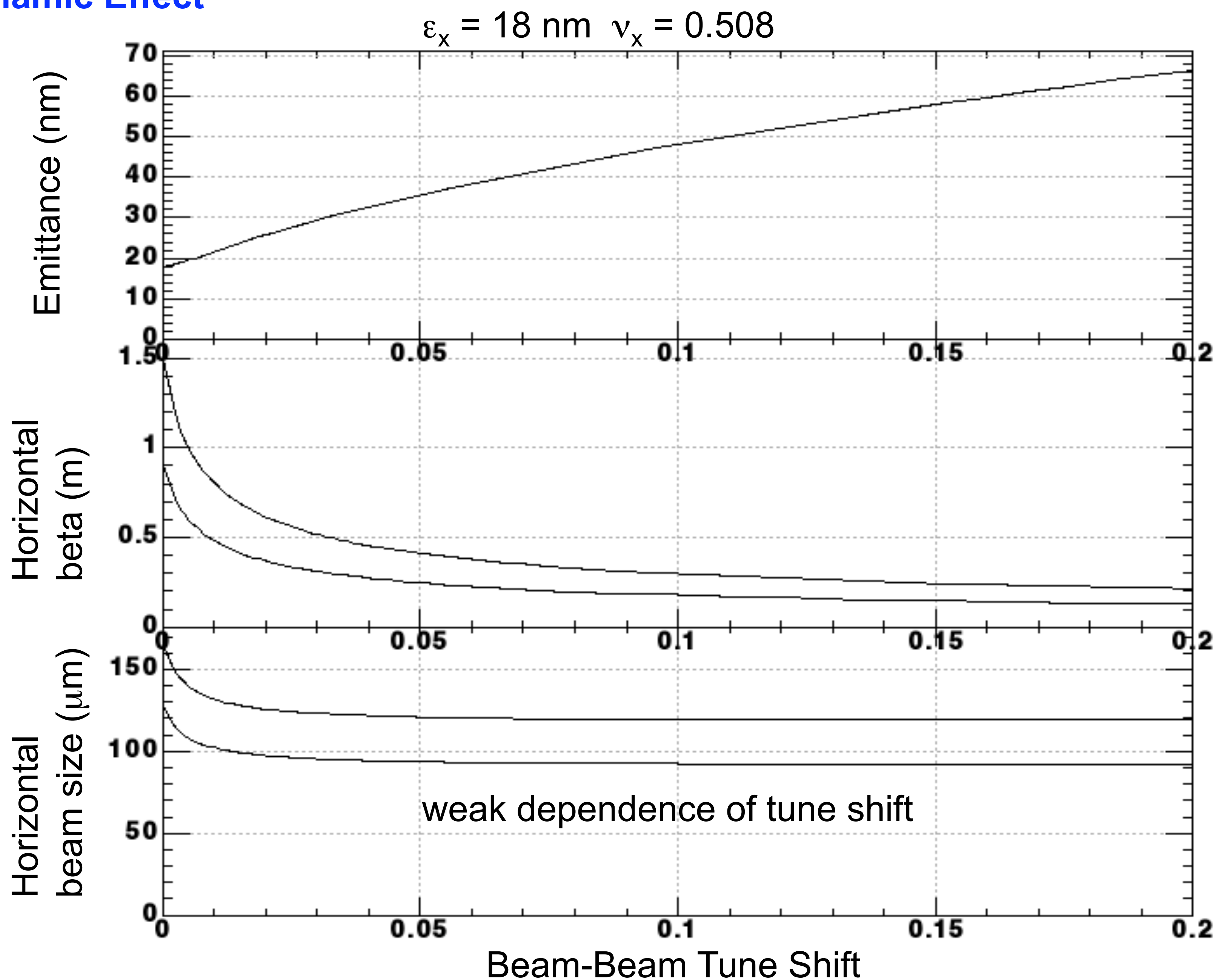
	average	standard deviation	
r_1*	0.00086	0.00105	3 units*
r_2* (m)	0.00125	0.00122	3 units
r_3* (1/m)	-0.01217	0.15403	6 units
r_4*	0.21266	0.54331	23 units

*IP tilt knob

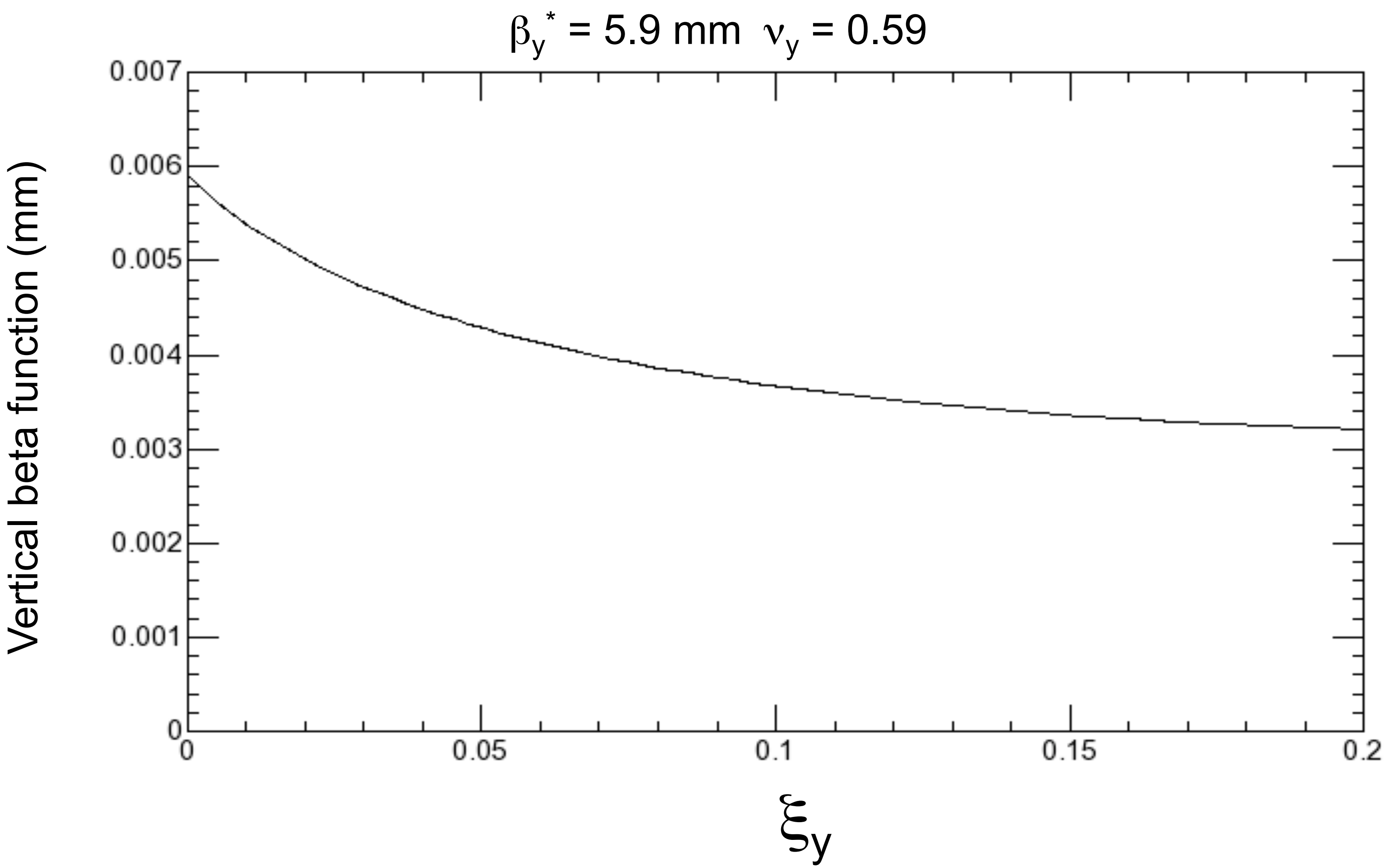
Large residuals

The r_1 and r_2 are well corrected, however r_3 and r_4 are scattered.

Dynamic Effect

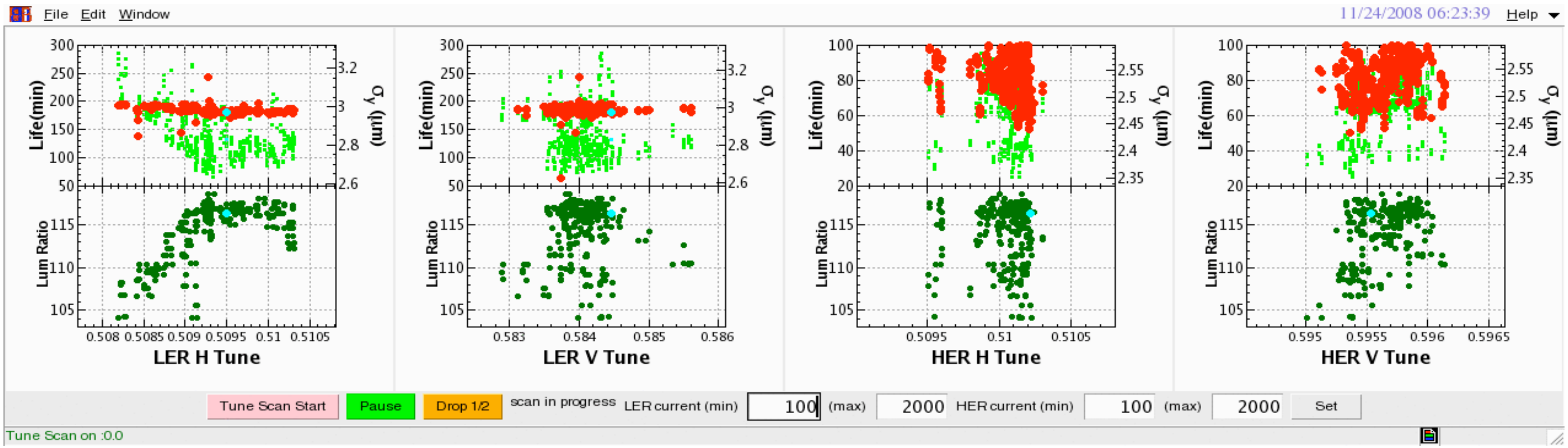


Dynamic Beta



Beam size measured by SRM

green: vertical beam size / red: lifetime



Vertical beam size ~2.8/2.4 μm for LER/HER (Crab ON, 1.5 mA^2).

c.f. Estimated value is 1.6 μm