Systematic study of nonlinear dynamics and dynamic aperture optimization

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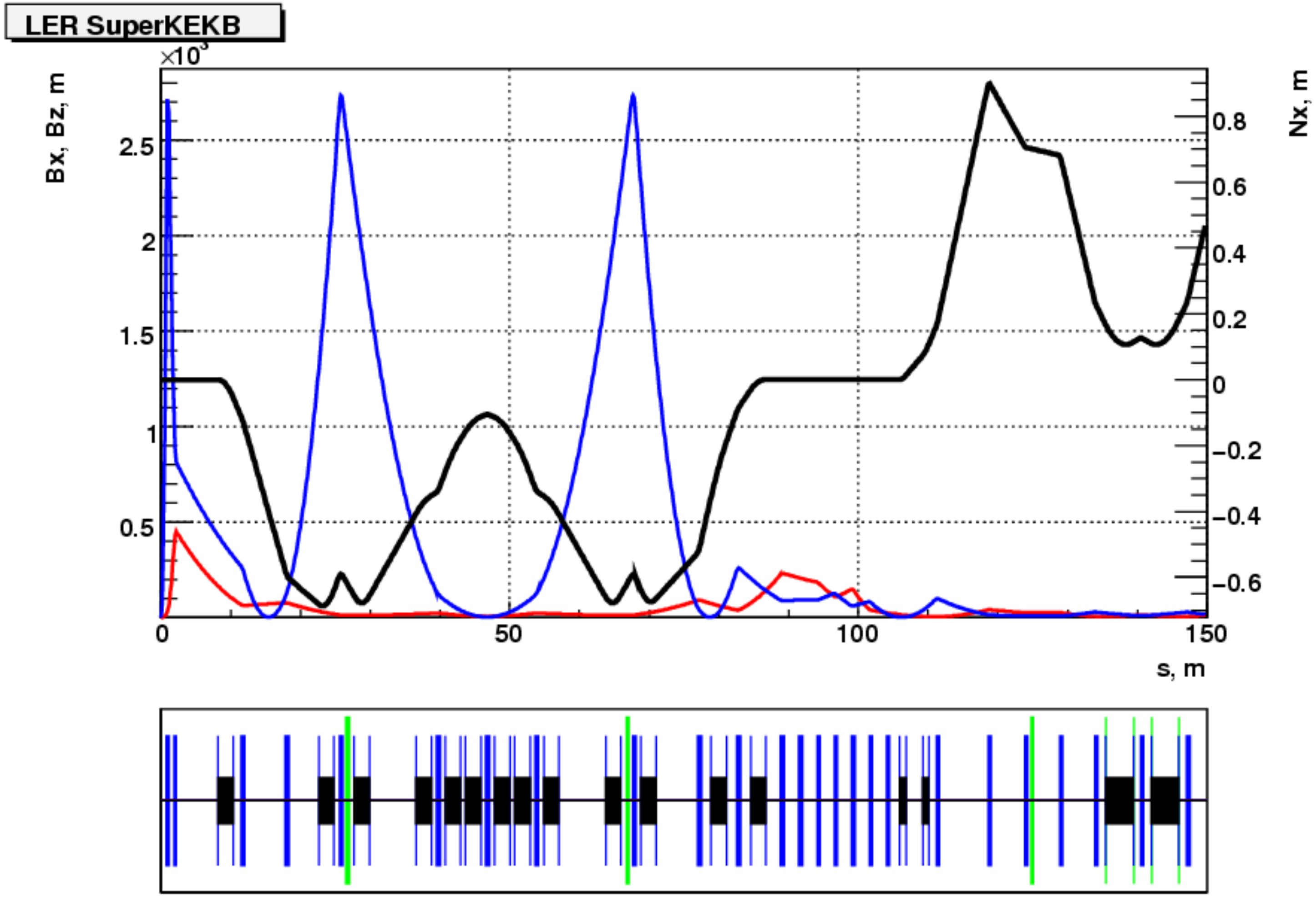
Outline

1. Relevant SuperKEKB lattice parameters
2. Nonlinearities
4. Phase space plots
5. Nonlinear detuning
6. Dynamic aperture study
7. DA optimization
8. Conclusion

SuperKEKB LER Lattice



IR with final focus and vertical chromatic section. Bymax = 2.7 km

SuperKEKB LER relevant parameters

Betatron tunes	Q_x/Q_y	45.527/45.570
Compaction factor	α	$2.75 \cdot 10^{-4}$
Damping times (ms)	τ_x/τ_s	31.8/16.8
Horizontal emittance (nm-rad)	ε_x	5.78
Energy spread	σ_E/E	$8.2 \cdot 10^{-4}$
Natural chromaticity	ξ_{x0}/ξ_{y0}	–107.4/–807.5
Corrected chromaticity	ξ_x/ξ_y	0.7/3.5
Betatron coupling	κ	0.4%
IP betas (mm)	β^*_x/β^*_y	31.1/0.270
Beam size at IP (µm)	σ^*_x/σ^*_y	13.5/0.079

All plot data below are given for IP

Nonlinearities

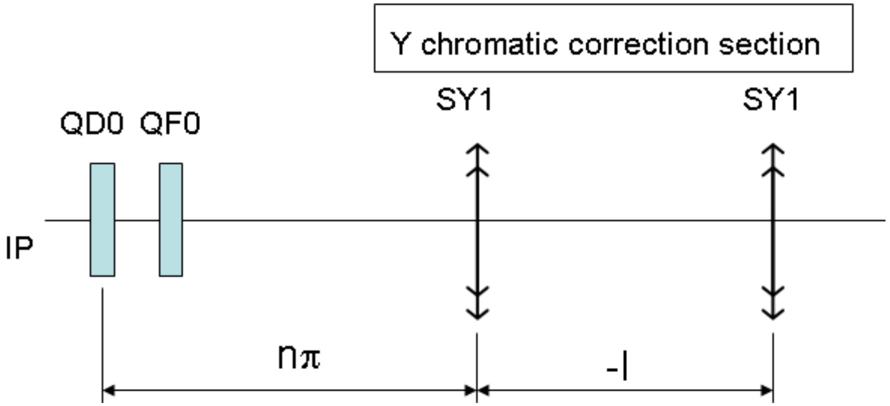
Strong chromatic sextupoles (FF horizontal and vertical correction sections) + arc chromatic sextupoles. All sextupoles are arranged in pairs separated by –I

Fringe fields in quadrupoles (third order)

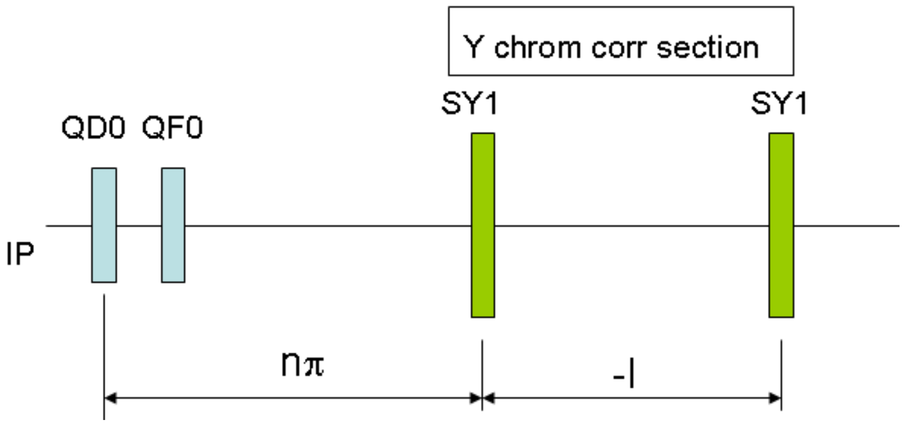
Fringe fields in bends (second order)

Kinematic terms (third order)

Nonlinearities - sextupoles



For kick sextupoles all high order aberrations are cancelled exactly



For thick sextupoles only the second order terms are cancelled but third and higher appear

$$x_3 = -x_0 - p_{x0}L - \frac{(K_2L)^2}{12}\left(x_0^3 + x_0 y_0^2\right)\cdot L^2 + \ldots$$

$$p_{x3} = -p_{x0} - \frac{(K_2L)^2}{6}\left(x_0^3 + x_0 y_0^2\right)\cdot L + \ldots$$

$$y_3 = -y_0 - p_{y0}L - \frac{(K_2L)^2}{12}\left(x_0^2 y_0 + y_0^3\right)\cdot L^2 + \ldots$$

$$p_{y3} = -p_{y0} - \frac{(K_2L)^2}{6}\left(x_0^2 y_0 + y_0^3\right)\cdot L + \ldots$$

Nonlinearities – quad fringe fields

Particle transport through the quadrupole fringe (leading term)

$$x = x_0 + \frac{k_{10}}{12(1+\delta)}\left(x_0^3 + 3x_0y_0^2\right), \qquad p_x = p_{x0} - \frac{k_{10}}{4(1+\delta)}\left[p_{x0}\left(x_0^2 + y_0^2\right) - 2p_{y0}x_0y_0\right],$$

$$y = y_0 - \frac{k_{10}}{12(1+\delta)}\left(y_0^3 + 3x_0^2y_0\right), \qquad p_y = p_{y0} + \frac{k_{10}}{4(1+\delta)}\left[p_{y0}\left(x_0^2 + y_0^2\right) - 2p_{x0}x_0y_0\right],$$

For simulation we use more complex but symplectic expressions from SAD

Rough comparison of two effects:

$$\Delta x_{qe} = -\frac{k_1}{12}\cdot x_0^3 = -4\cdot 10^{-5}(cm^{-2})\cdot x_0^3$$

FF QC1LPH

$$\Delta x_{ts} = -\frac{\left(k_2L^2\right)^2}{12}\cdot x_0^3 = -8\cdot 10^{-6}(cm^{-2})\cdot x_0^3$$

Thick sextupole pair in vertical IR chromatic section

Quadrupole fringe field effect is stronger than the tick sextupole effect

Nonlinearities – kinematic term

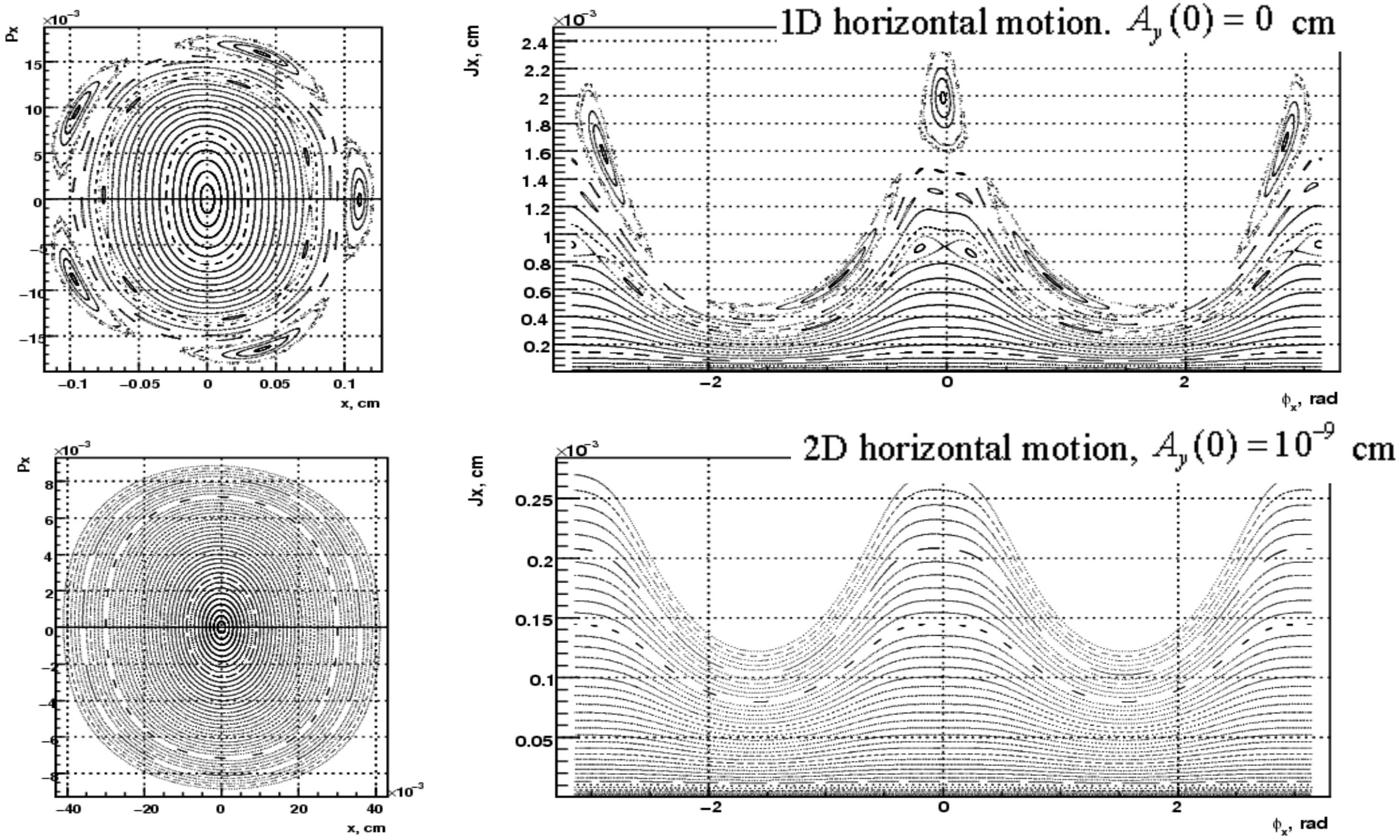
As we expect, a bulk of the kinematic effects

$$H_k(s) = \frac{1}{8}\left(p_x^2 + p_y^2\right)^2$$

should come from IP with Beta_Y = 270 um.

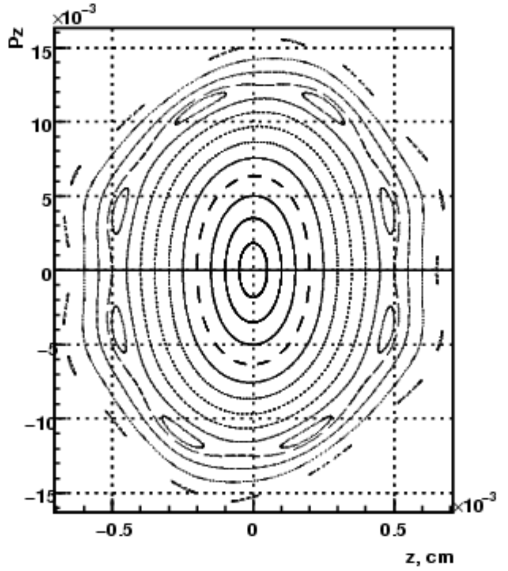
The kinematic effects are included in the simulation.

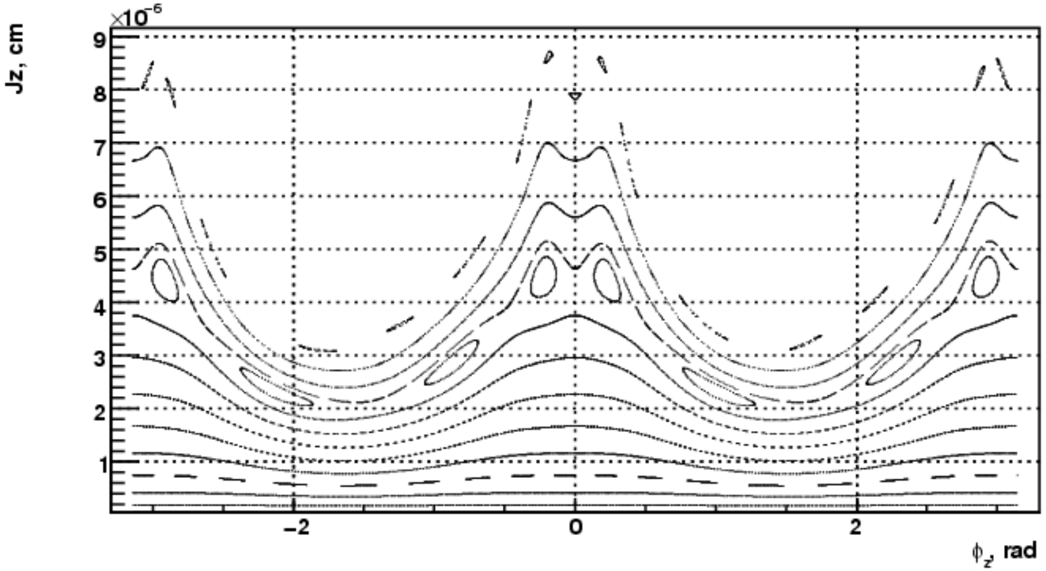
Phase space plot (horizontal)



Extremely small seed of the vertical motion reduces the horizontal aperture by factor 2 through the nonlinear coupling term

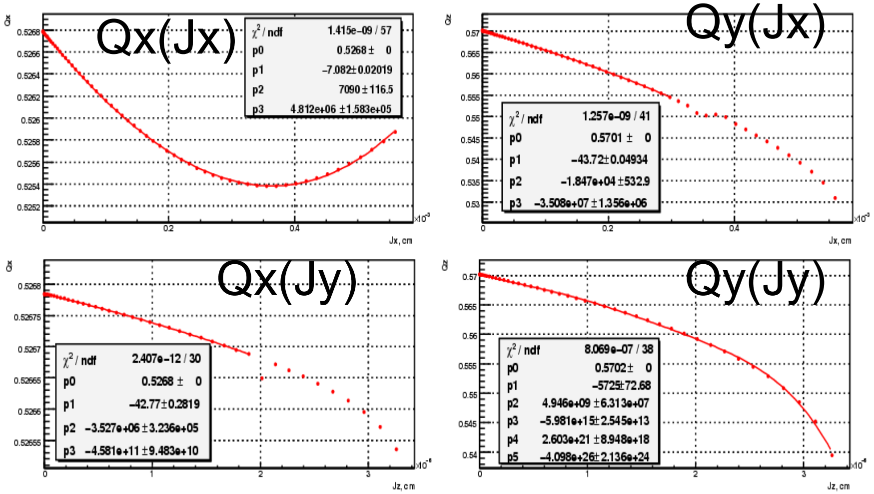
Phase space plot (vertical)



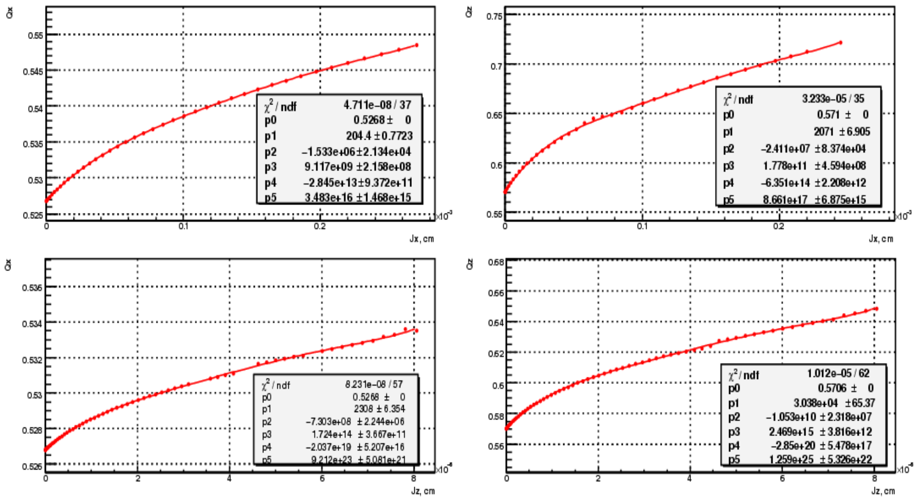


1D and 2D (with small seed of horizontal motion) vertical apertures have the same size. Vertical DA is defined by the vertical nonlinear terms

Tune amplitude dependence



Sextupoles only

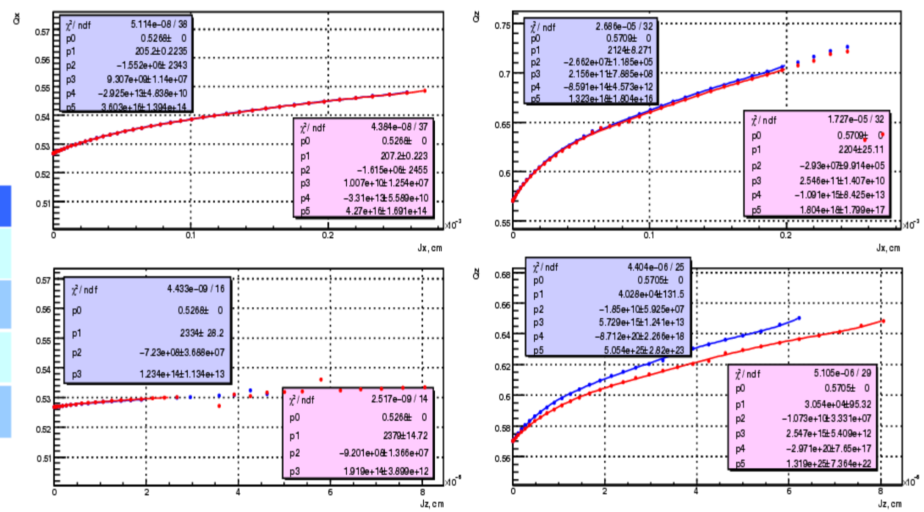


S + fringe fields

$$\Delta \nu_y = C_{xy} J_x + C_{yy} J_y$$

$$\Delta \nu_x = C_{xx} J_x + C_{xy} J_y$$

	Sext	QEdge	Kin	All
C_{xx}, cm^{-1}	–7.1	226	0.93	205
C_{xy}, cm^{-1}	–42.8	2370	62	2370
C_{yx}, cm^{-1}	–43.7	2237	69	2130
C_{yy}, cm^{-1}	–5700	35400	10020	40490

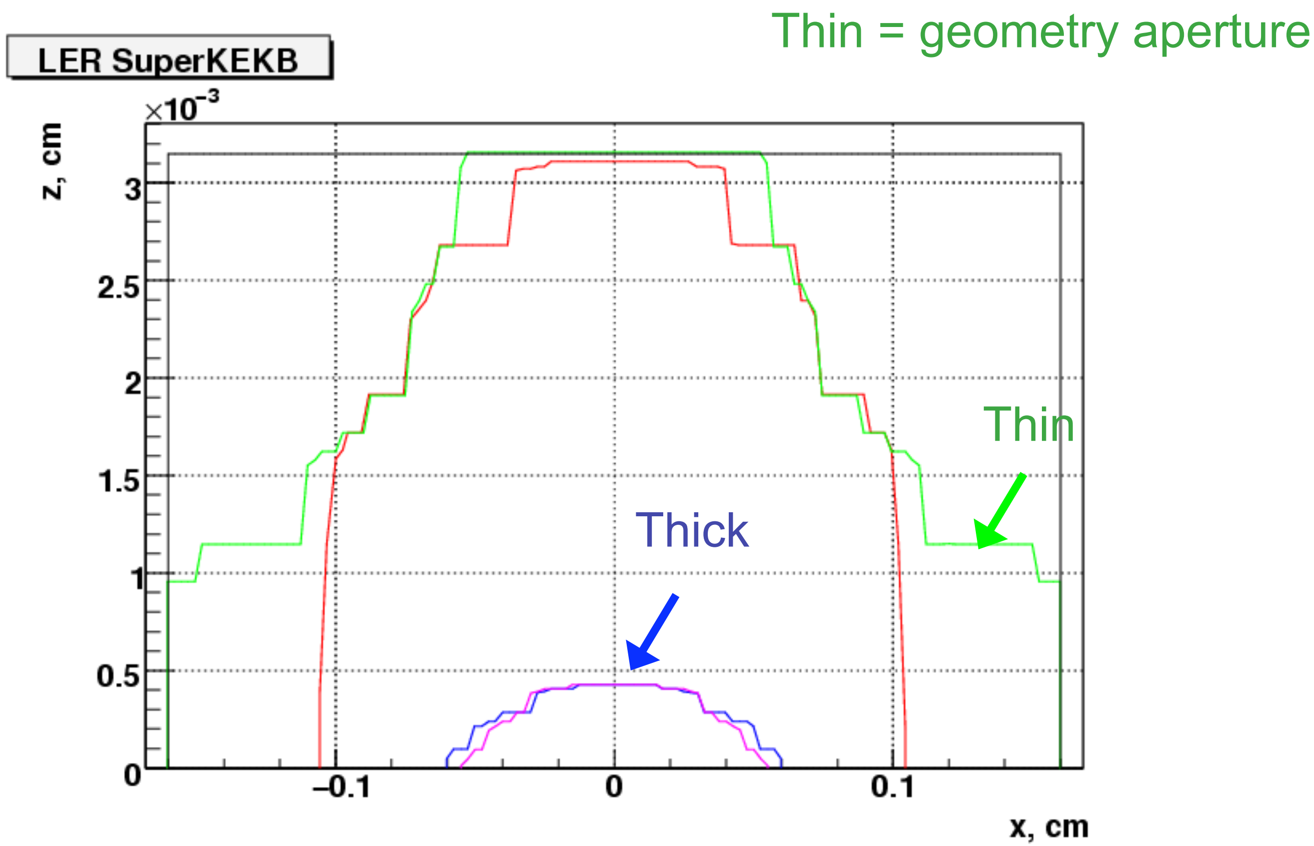


S + fringe fields+ kinematic terms

Main effect is due to the quadrupole fringe fields!

Dynamic aperture - I

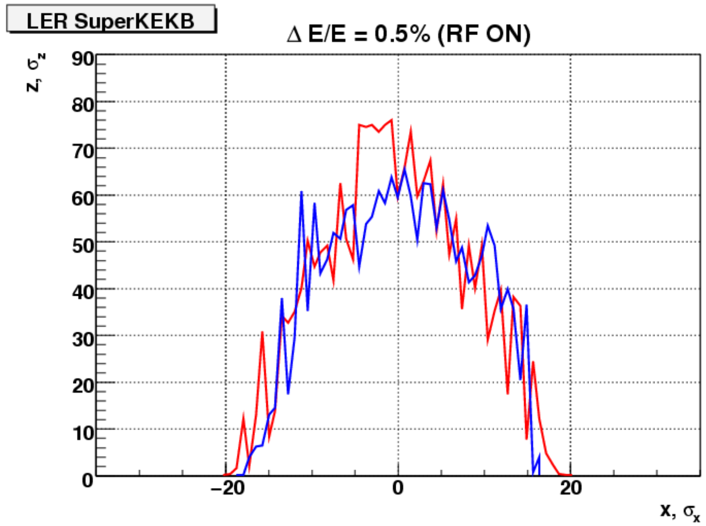
Illustration: dynamic aperture for thin and thick sextupoles arranged in pairs separated by -I map. Quadrupole fringe fields are switched off.



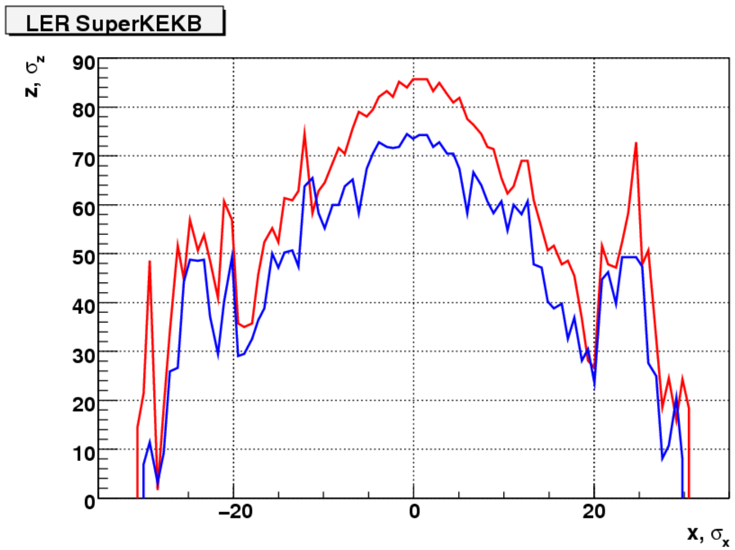
Dynamic aperture - II

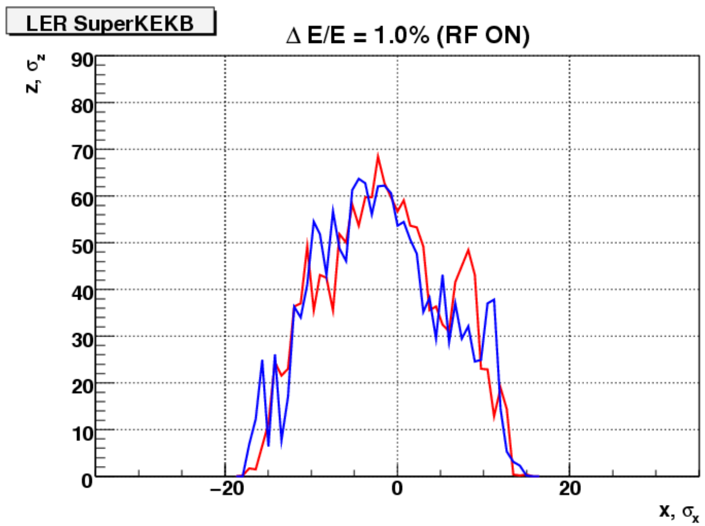
Dynamic aperture decrease due to the kinematic effects.

Off-energy



On-energy





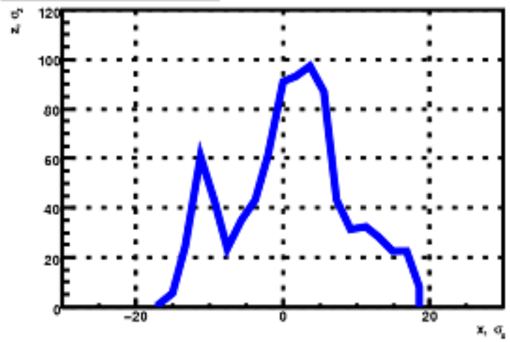
---- Without kinematic terms

---- With kinematic terms

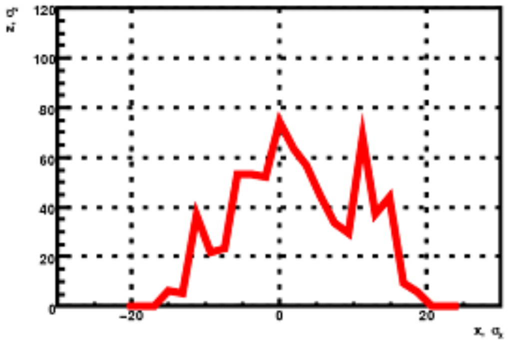
Dynamic aperture - III

W/o synch.osc. | With synch.osc.

All nonlinearities are included

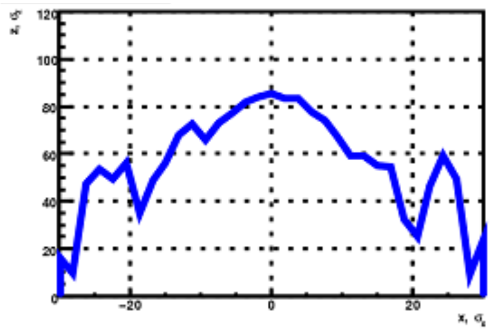


−1%, RF off

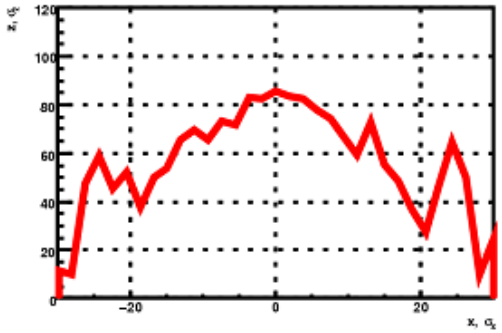


−1%, RF on

60 Sigma_y, ±15 Sigma_x

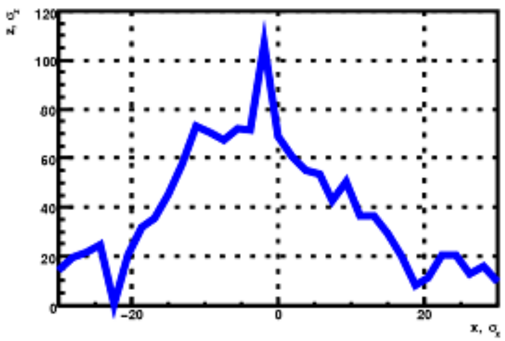


0%, RF off

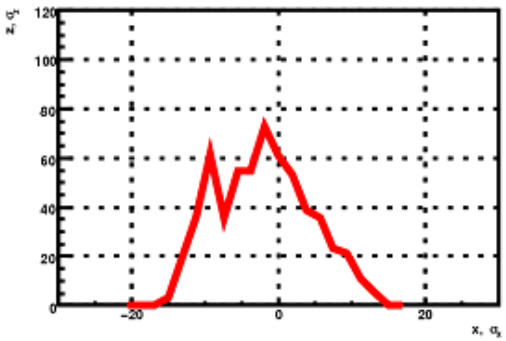


0%, RF on

80 Sigma_y, ±30 Sigma_x



+1%, RF off



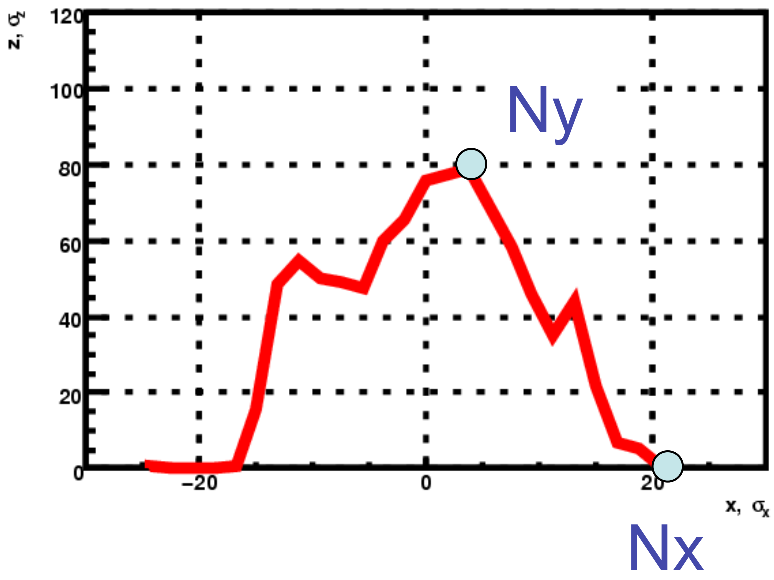
+1%, RF on

60 Sigma_y, ±15 Sigma_x

Conclusion before optimization

DA in rms beam size

Source	Nx	Ny
Fringe QC1	25	80
Fringe QC2	50	380
Sextupoles only	40	60
All	30	80



(1) DA due to the quadrupole fringe field and thick sextupoles have the same order of magnitude
(2) Among all the quadrupoles influence of QC1 is most crucial

Dynamic aperture optimization - I

(1) Correction of the thick sextupole effect (by weak sextupole correctors)
(2) Correction of the quadrupole fringe fields (how?)

One can try to cope with the quadrupole edges (third order aberration) by octupole correctors compensating relevant detuning terms

To improve vertical aperture we have to correct vertical detuning term

To improve horizontal aperture we have to correct coupling detuning term

Dynamic aperture optimization - II

Separate octupole correctors, detuning coefficients

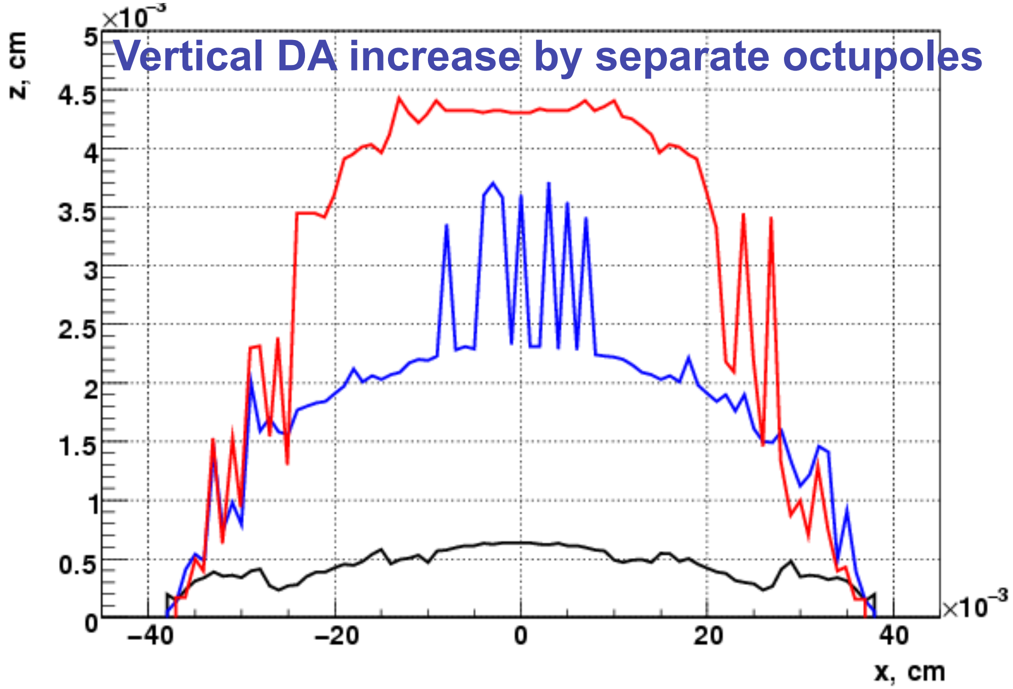
$$\Delta\nu_y = C_{xy}J_x + C_{yy}J_y$$

$$\Delta\nu_x = C_{xx}J_x + C_{xy}J_y$$

$$C^o_{xx} = \frac{1}{16\pi}\oint k_3(s)\beta_x^2(s)ds,$$

$$C^o_{xy} = -\frac{1}{8\pi}\oint k_3(s)\beta_x(s)\beta_y(s)ds,$$

$$C^o_{yy} = \frac{1}{16\pi}\oint k_3(s)\beta_y^2(s)ds.$$



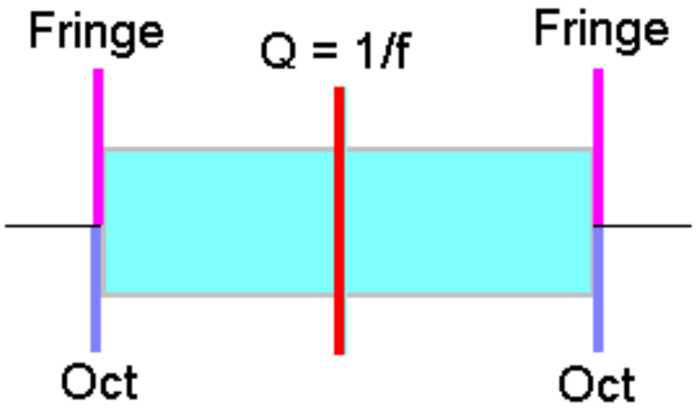
(1) In IR, where the bulk of the perturbation sources are concentrated, By>>Bx

(2) In any case $\beta_x \cdot \beta_y \le \max(\beta_x^2, \beta_y^2)$ so we can not control effectively the coupling term

(3) But with the vertical one everything works well

Dynamic aperture optimization - III

Octupole correctors incorporated in the quadrupole



$$x = x_0\,, \qquad y = y_0\,,$$

$$\Delta p_x = -\frac{1}{3}A\cdot x_0^3 + B\cdot x_0 y_0^2\,,$$

$$\Delta p_y = -\frac{1}{3}A\cdot y_0^3 + B\cdot x_0^2 y_0\,,$$

$$A = k_3 L_o + k_1^2 L_q\,,\quad B = k_3 L_o - k_1^2 L_q\,.$$

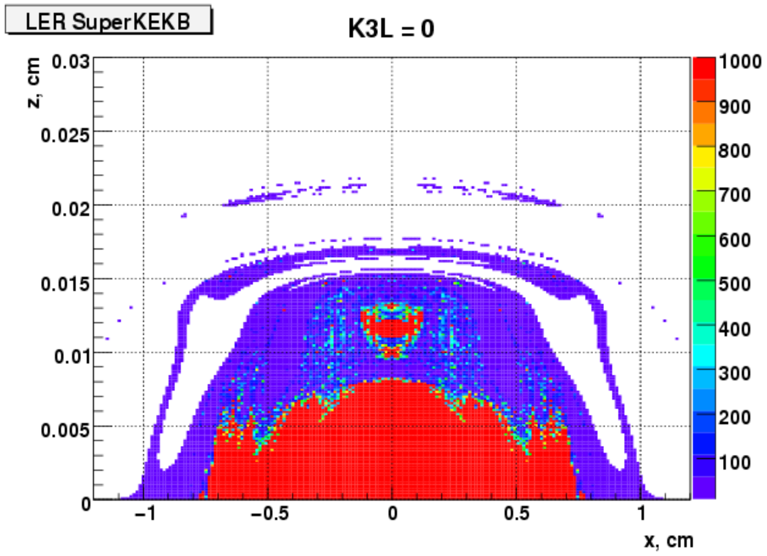
Transformation through this simple model shows that we can effectively compensate either direct or coupling terms for both planes. The octupole corrector strength for each octupole kick is

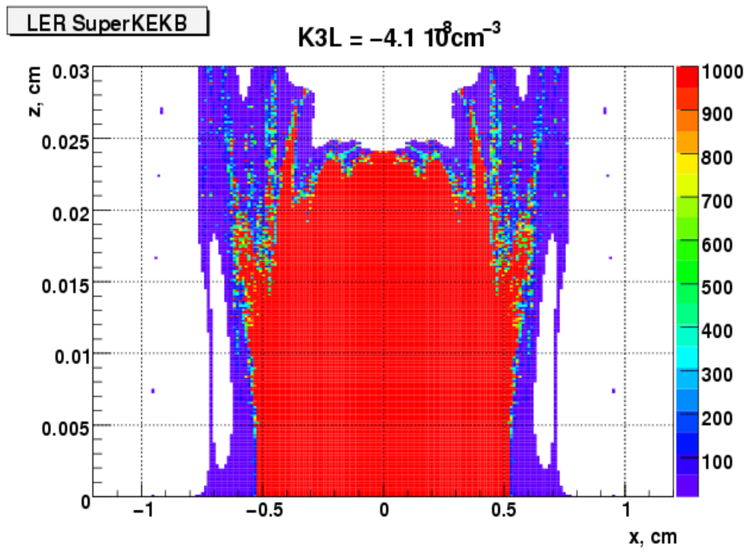
$$k_3 L_o = \pm k_1^2 L_q$$

Later we found that a single kick or the octupole corrector distributed along the quadrupole length work as well.

Dynamic aperture optimization - IV

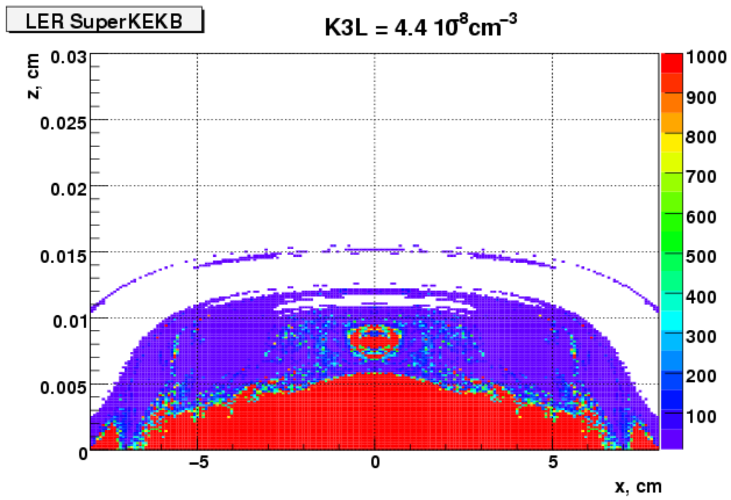
Example of the local correction scheme





DA reduced by the edges of strong quadrupole

Octupole corrector open it either horizontally (~10 times) (coupling term correction) or vertically (~3 times)



Dynamic aperture optimization - V

As we have two knobs to control the horizontal (by local correction) and vertical (by global correction) aperture, the following procedure was applied

(1) Optimization of the horizontal aperture in main quadrupoles

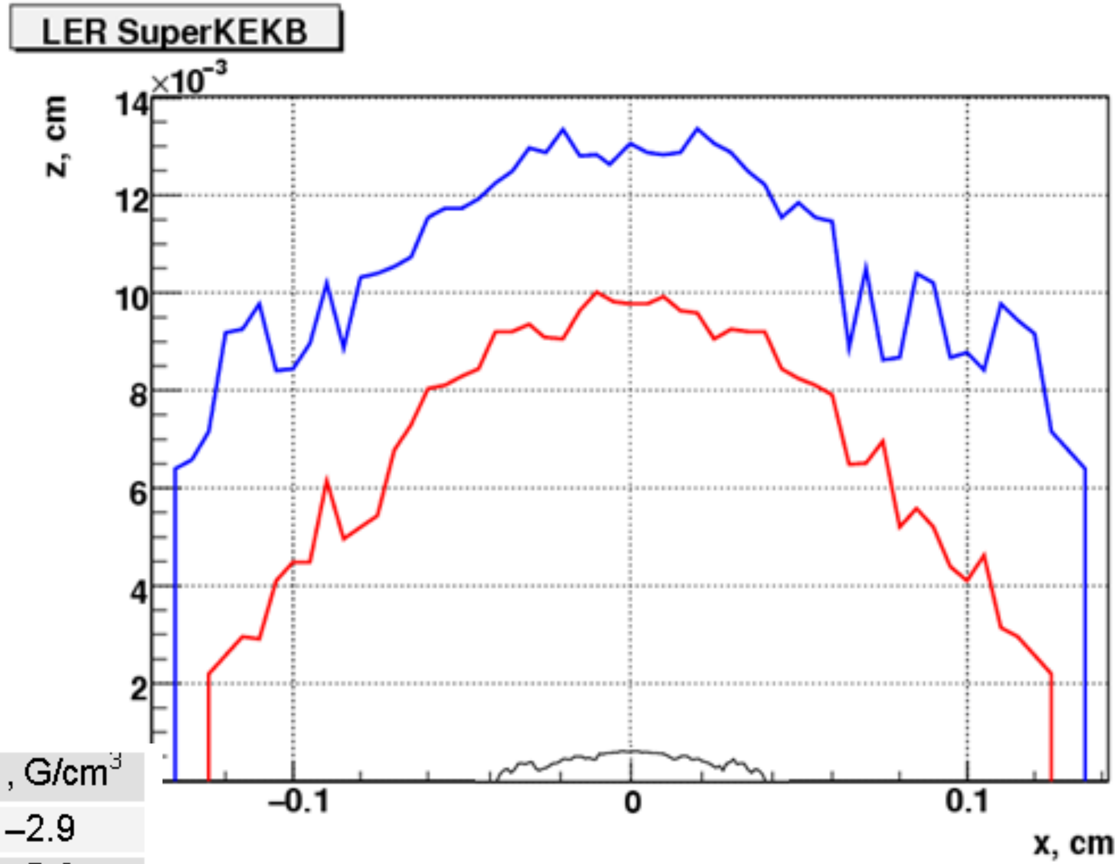
(2) Optimization of the vertical aperture by two octupoles in the vertical section of the FF chromaticity correction

(3) Numerical optimization of all octupole correctors because in real lattice every corrector not only suppresses a required term but also affects others (though slightly)

Dynamic aperture optimization - VI

Optimization of the quadrupole fringe fields by octupole correctors

Source	Nx	Ny
Fringe QC1	25	80
Fringe QC2	50	380
Sextupoles only	40	60
All	30	80
Corrected fringes	100	1300



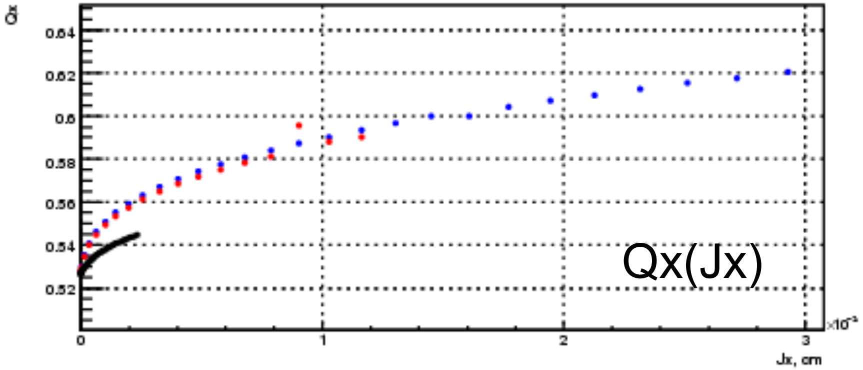
Octupole correctors

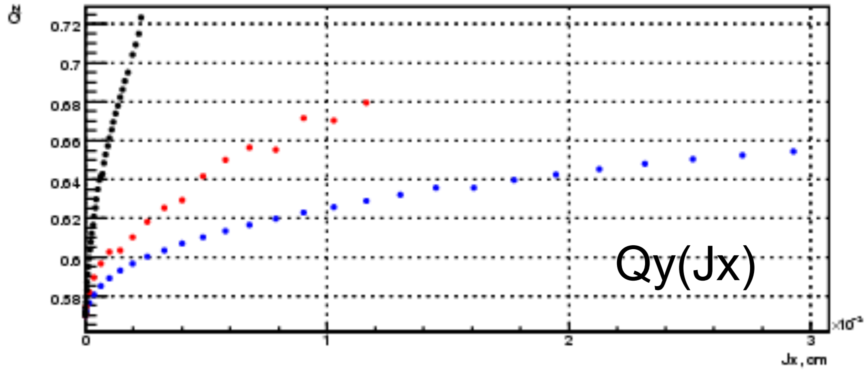
Name	L, cm	k_3L, cm^{-3}	B''', G/cm^3
OQC1LP	39	$-8.56\cdot10^{-6}$	–2.9
OQC1RP	28	$-1.18\cdot10^{-5}$	–5.6
OQC2LP	35	$1.11\cdot10^{-5}$	4.2
OQC2RP	35	$1.11\cdot10^{-5}$	4.2
OY1L=OY1R	10	$-9.95\cdot10^{-4}$	133
OY2L=OY2R	10	$3.28\cdot10^{-6}$	4.4

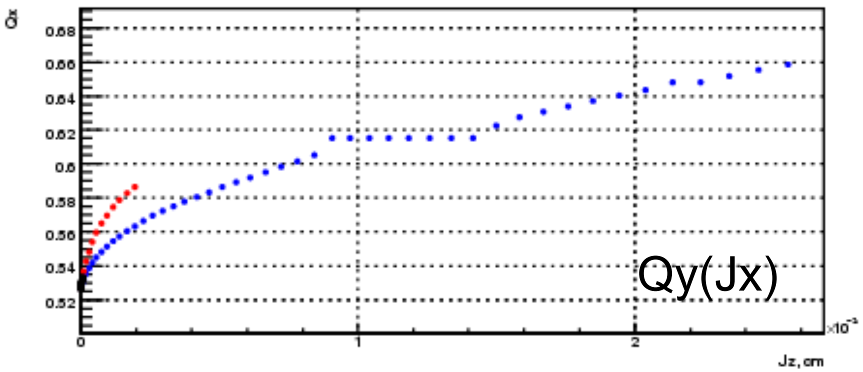
Black – initial aperture
Blue – FF quadrupoles corrected
Red – all quadrupoles corrected

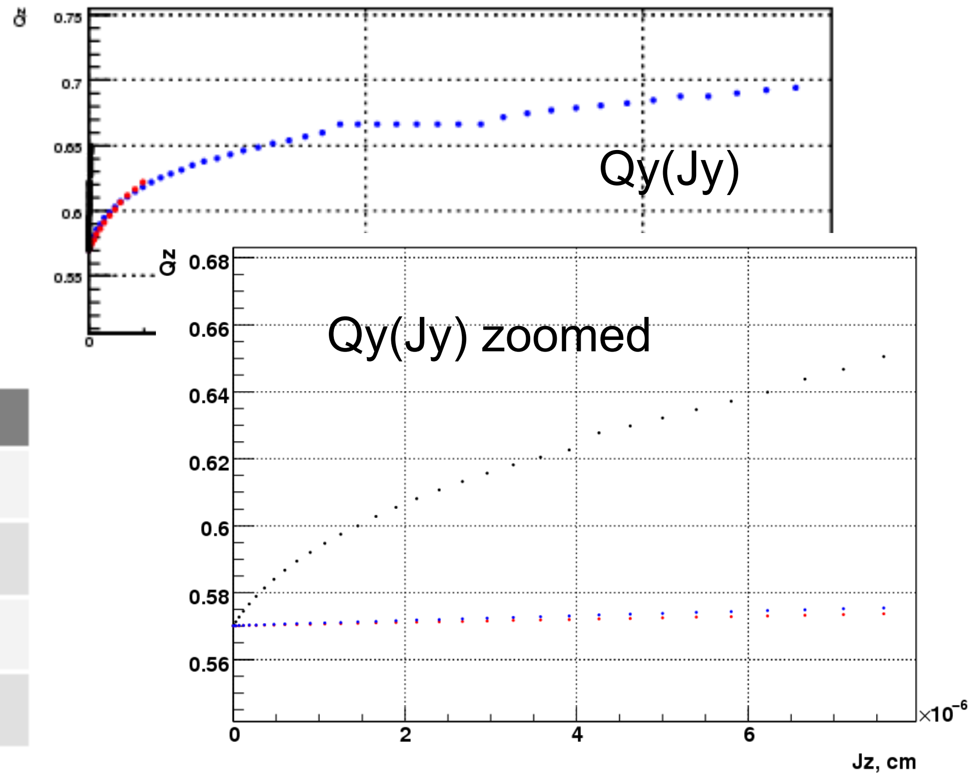
Dynamic aperture optimization - VII

Tune vs amplitude before and after optimization





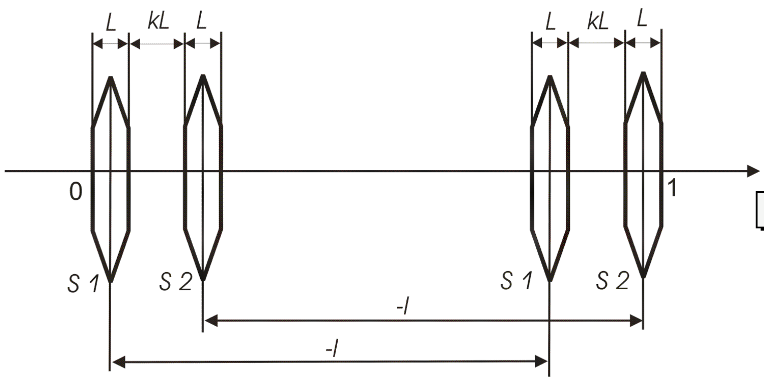




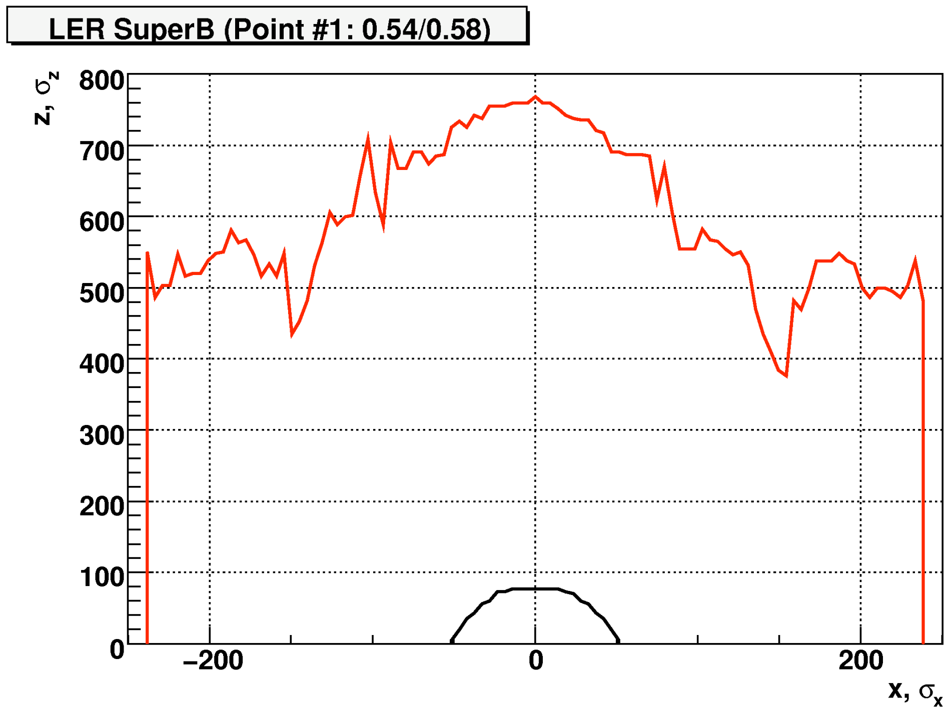
	Initial	Optimized
C_{xx}, cm^{-1}	204	450
C_{xy}, cm^{-1}	2310	420
C_{yx}, cm^{-1}	2070	430
C_{yy}, cm^{-1}	30400	470

Dynamic aperture optimization - VIII

Thick sextupole correction by two additional weak (strength = 3-7% of the main one) sextupole correctors (not applied yet to SuperKEKB)



Black – initial aperture
Red – improved by sextupole correctors



Conclusion

(1) In SuperKEKB LER the dynamic aperture is limited in the first place by the fringe field in the strong FF quadrupoles

(2) In the second place by the finite length strong chromatic sextupoles

(3) The quadrupole fringe fields can be corrected by octupole correctors, individual and incorporated in the FF quadrupoles

(4) The next steps are: (a) including and optimization of the sextupoles, (b) global optimization of all effects for on- and off-momentum particle, (c) optical errors (!)