

## ERROR TOLERANCE AND OPTICS CORRECTION

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# Five Big Issues (FBI)

**1. Injection and Touschek background** H. Nakayama et al. 2. SR from QC1/QC2 H. Nakayama et al. 3. Collimator Y. Suetsugu et al. 4. Machine error and optics correction Y. Ohnishi et al. 5. Beam-Beam interaction (inc. continuous injection, iBump FB) N. Iida, Y. Funakoshi et al.

# Five Big Issues (FBI)

- 1. Injection and Touschek background
- 2. SR from QC1/QC2
- 3. Collimator
- 4. Machine error and optics correction
- 5. Beam-Beam interaction (inc. continuous injection, iBump FB)

# Five Big Issues (FBI)

### 4. Machine error and optics correction

- 4-1 Measurement of x-y coupling
- 4-2 Measurement of dispersions
- 4-3 Measurement of betatron phase-advance
- 4-4 Optics correction scheme

## **4** INTRODUCTION

- Optics measurement based on closed orbit distortion(COD) can be used at low beam-current and non-collision condition. No fundamental difficulty is found at KEKB except for a timedependent of orbit fluctuation.
- 2. Alternatively, it is possible to measure optics at high-beam current by using a single-pass BPM when small betatron oscillation is induced by an exciter. If the measurement of a non-collision bunch among many bunches is possible, the optics measurement is feasible during a collision condition. Rotation error of exciter along the beam-axis does not affect x-y coupling measurement at the first order(advantage over the COD-based measurement).
- 3. The COD-based measurement will be used as well as single-pass BPMs, especially beginning of the commissioning or until singlepass BPMs are ready.

## **4** INTRODUCTION

- 4. Besides the optics correction, stability of the optics during correction to correction is necessary. The optics correction had been performed once every two weeks at KEKB.
- 5. Error tolerance without optics corrections is indicative for the requirement of short-term stability at least.

• drifting field strength of magnets, magnet misalignment, closedorbit-distortions, and so on.

6. The coupling parameter is one of the most important parameters in the nano-beam scheme.

### 4 ERROR TOLERANCE (1)

A. Morita

Rotation error of normal quadrupoles
 Orbit offset of sextupoles





## **4 ACCEPTABLE ERROR TOLERANCE**

#### Stability limit without optics corrections

	s.d.	LER	HER
Rotation error of normal quadrupoles	σΔθ	0.2 mrad	0.2 mrad
Misalignment of normal sextupoles	σ <sub>Δy</sub>	30 µm	10 µm
Misalignment of local chromaticity sextupoles	$\sigma_{\Delta y}$	10 µm	10 µm

The canonical momenta normalized by a design momentum are defined by:

$$p_x = (1+\delta)x' - \frac{B_z}{2B\rho_0}y$$
  $p_y = (1+\delta)y' + \frac{B_z}{2B\rho_0}x$   $\delta = \frac{\Delta p}{p_0}$ 

The canonical variables can be expressed as:

$$\begin{pmatrix} u \\ p_{u} \\ v \\ p_{v} \end{pmatrix} = \begin{pmatrix} \mu & 0 & -r_{4} & r_{2} \\ 0 & \mu & r_{3} & -r_{1} \\ r_{1} & r_{2} & \mu & 0 \\ r_{3} & r_{4} & 0 & \mu \end{pmatrix} \begin{cases} \begin{pmatrix} x \\ p_{x} \\ y \\ p_{y} \end{pmatrix} - \begin{pmatrix} \eta_{x} \\ \eta_{px} \\ \eta_{y} \\ \eta_{py} \end{pmatrix} \delta \\ \mu^{2} + (r_{1}r_{4} - r_{2}r_{3}) = 1 \end{cases}$$
decoupled coordinate

#### In the case of H-mode(v=0, p<sub>v</sub>=0):

$$x = \mu u$$

$$p_x = \mu p_u$$

$$y = -r_1 u - r_2 p_u + \sum a_m \cos \theta_m$$

$$p_y = -r_3 u - r_4 p_u + \sum b_n \cos \phi_m$$

Vertical betatron oscillation due to rotation error of exciter along the beam-axis or x-y coupling is insensitive to the measurement.

$$\langle xy \rangle = -\mu(r_1 < u^2 > +r_2 < up_u >) = -\frac{1}{\mu}(r_1 < x^2 > +r_2 < xp_x >)$$

$$\langle p_x y \rangle = -\mu(r_1 < up_u > +r_2 < p_u^2 >) = -\frac{1}{\mu}(r_1 < xp_x > +r_2 < p_x^2 >)$$

$$\langle xp_y \rangle = -\mu(r_3 < u^2 > +r_4 < up_u >) = -\frac{1}{\mu}(r_3 < x^2 > +r_3 < xp_x >)$$

$$\langle p_x p_y \rangle = -\mu(r_3 < up_u > +r_4 < p_u^2 >) = -\frac{1}{\mu}(r_4 < xp_x > +r_4 < p_x^2 >)$$

 $\begin{pmatrix} \langle xy \rangle \\ \langle p_xy \rangle \end{pmatrix} = -\frac{1}{\mu} \Sigma \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}$  $\begin{pmatrix} \langle xp_y \rangle \\ \langle p_xp_y \rangle \end{pmatrix} = -\frac{1}{\mu} \Sigma \begin{pmatrix} r_3 \\ r_4 \end{pmatrix}$ 

 $\Sigma = \begin{pmatrix} \langle x^2 \rangle & \langle xp_x \rangle \\ \langle xp_x \rangle & \langle p_x^2 \rangle \end{pmatrix}$ 

Correlation matrix method

where



 $\ll$  In the case of H-mode(v=0, p<sub>v</sub>=0):

$$x = \mu u$$

$$p_x = \mu p_u$$

$$y = -r_1 u - r_2 p_u$$

$$p_y = -r_3 u - r_4 p_u$$

The x-y coupling parameters are derived as:

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = -\mu \Sigma^{-1} \begin{pmatrix} \langle xy \rangle \\ \langle p_x y \rangle \end{pmatrix}$$
$$\begin{pmatrix} r_3 \\ r_4 \end{pmatrix} = -\mu \Sigma^{-1} \begin{pmatrix} \langle xp_y \rangle \\ \langle p_x p_y \rangle \end{pmatrix}$$

 $\Sigma = \begin{pmatrix} \langle x^2 \rangle & \langle xp_x \rangle \\ \langle xp_x \rangle & \langle p_x^2 \rangle \end{pmatrix}$ 

$$\beta_u = \frac{\langle x^2 \rangle}{\sqrt{\det \Sigma}}$$
$$\alpha_u = -\frac{\langle xp_x \rangle}{\sqrt{\det \Sigma}}$$

 $\ll$  In the case of V-mode(u=0, p<sub>u</sub>=0):

$$y = \mu v$$
  

$$p_y = \mu p_v$$
  

$$x = r_4 v - r_2 p_v$$
  

$$p_x = -r_3 v + r_1 p_v$$

The x-y coupling parameters are derived as:

$$\begin{pmatrix} r_4 \\ -r_2 \end{pmatrix} = \mu \Sigma^{-1} \begin{pmatrix} \langle yx \rangle \\ \langle p_y x \rangle \end{pmatrix}$$
$$\begin{pmatrix} -r_3 \\ r_1 \end{pmatrix} = \mu \Sigma^{-1} \begin{pmatrix} \langle yp_x \rangle \\ \langle p_y p_x \rangle \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \langle y^2 \rangle & \langle yp_y \rangle \\ \langle yp_y \rangle & \langle p_y^2 \rangle \end{pmatrix}$$

$$\beta_v = \frac{\langle y^2 \rangle}{\sqrt{\det \Sigma}}$$
$$\alpha_v = -\frac{\langle yp_y \rangle}{\sqrt{\det \Sigma}}$$

Ref. Phys. Rev. SP-AB, 12, 091002 (2009) K. Ohmi et al., Proc. of IPAC'10

\* px and py is calculated by using two BPMs:

$$p_{x1} = \frac{x_2}{\sqrt{\beta_{x1}\beta_{x2}}\sin\psi_{x,21}} - \left(\frac{\alpha_{x1}}{\beta_{x1}} + \frac{\cos\psi_{x,21}}{\beta_{x1}\sin\psi_{x,21}}\right)x_1$$
$$p_{y1} = \frac{y_2}{\sqrt{\beta_{y1}\beta_{y2}}\sin\psi_{y,21}} - \left(\frac{\alpha_{y1}}{\beta_{y1}} + \frac{\cos\psi_{y,21}}{\beta_{y1}\sin\psi_{y,21}}\right)y_1$$

- Here, assumed that transfer matrix between location 1 and 2 to calculate px and py. Design values of phase advance and Twiss parameters are used.
- Alternatively, in the general form:

$$\vec{x}_{2} = M\vec{x}_{1} \quad M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \quad \vec{x} = (x, p_{x}, y, p_{y})$$

$$\begin{pmatrix} p_{x} \\ p_{y} \end{pmatrix} = \begin{pmatrix} m_{12} & m_{14} \\ m_{32} & m_{34} \end{pmatrix}^{-1} \left\{ \begin{pmatrix} x_{2} \\ y_{2} \end{pmatrix} - \begin{pmatrix} m_{11} & m_{13} \\ m_{31} & m_{33} \end{pmatrix} \begin{pmatrix} x_{1} \\ y_{1} \end{pmatrix} \right\}$$

### **4-1 MEASUREMENT OF X-Y COUPLING** <u>Tracking simulation: HER lattice</u>

- Single particle / Free oscillation
- \* No radiation damping
- Synchrotron oscillation
- 2000 turns
- Excite oscillation/ H-mode
- \* Assumed that  $\mu=1$ , then solve iterative procedure. (not found  $\mu^2<0$  in this simulation)
- \* Number of BPMs is 160 for the measurement in a ring.
- \* Coupling source: vertical displacement of sextupoles and rotation of quadrupoles:  $\sigma_{\Delta y} = 100 \ \mu m$  for sxtupoles,  $\sigma_{\Delta \theta} = 0.1 \ mrad$  for quadrupoles

#### 4-1 MONITORS (HER ARC CELL)



#### **4-1 MEASUREMENT OF X-Y COUPLING** x-y coupling due to machine error r<sub>1</sub>=-0.1392, r<sub>2</sub>=-0.8139, r<sub>3</sub>=0.0097, r<sub>4</sub>=0.0383 @ QD3E.3

2000 turns





green dot: raw data

red: correlation method





QD3E.3

The slant of ellipse corresponds to xy-coupling.

#### 4-1 AMPLITUDE DEPENDENCE (MONITOR:QD3E.3) x: 50 μm BPM resolution (jitter error) different random seeds



\* Definition: one-turn transfer matrix (4x4), T:

℁ CESR:

$$T = V^{-1}UV = \begin{pmatrix} \gamma I & C \\ -C^+ & \gamma I \end{pmatrix} \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} \gamma I & -C \\ C^+ & \gamma I \end{pmatrix}$$

℁ KEKB

$$T = V^{-1}UV = \begin{pmatrix} \mu I & -SR^{T}S \\ -R & \mu I \end{pmatrix} \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} \mu I & SR^{T}S \\ R & \mu I \end{pmatrix}$$
$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad R = \begin{pmatrix} r_{1} & r_{2} \\ r_{3} & r_{4} \end{pmatrix}$$
$$A = I\cos\psi_{u} + J_{u}\sin\psi_{u} \qquad B = I\cos\psi_{v} + J_{v}\sin\psi_{v}$$
$$J_{u,v} = \begin{pmatrix} \alpha_{u,v} & \beta_{u,v} \\ -\gamma_{u,v} & -\alpha_{u,v} \end{pmatrix}$$

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} r_4 & -r_2 \\ -r_3 & r_1 \end{pmatrix} \qquad \gamma = \mu$$

\* In the case of H-mode excitation, we observe x and y position at a BPM:

$$\vec{w}_0 = V\vec{x}_0 = (u_0, 0, 0, 0)^T$$

 $\vec{x}(n) = (V^{-1}UV)^n \vec{x}_0 = V^{-1}U^n V \vec{x}_0 = V^{-1}U^n \vec{w}_0$ 

D. C. Sagan, D. L. Rubin et al. Ref. CBN 96-20 Proc. of PAC2003, p.2267

$$\begin{aligned} x(n) &= \hat{x} \cos(n\psi_u - \delta_1) \\ y(n) &= \hat{y} \cos(n\psi_u - \delta_2) \quad \text{where} \\ \hat{x} &= \mid \mu u_0 \mid \sqrt{1 + \alpha_u^2} \end{aligned}$$
(1)

$$\hat{y} = |u_0| \sqrt{r_1^2 + (r_1 \alpha_u - r_2 \gamma_u)^2}$$
<sup>(2)</sup>

$$\tan \Delta \phi_u \equiv \tan(\delta_1 - \delta_2) = \frac{r_2/r_1}{\beta_u - (r_2/r_1)\alpha_u} \tag{3}$$

From Eq. (1)~(3):  $(\hat{y})$ 

$$r_{1} = -\mu \left(\frac{y}{\hat{x}}\right)_{u} \left(\cos \Delta \phi_{u} + \alpha_{u} \sin \Delta \phi_{u}\right)$$
$$r_{2} = -\mu \left(\frac{\hat{y}}{\hat{x}}\right)_{u} \beta_{u} \sin \Delta \phi_{u}$$

 $r_2$  and  $r_4$  can be obtained by a similar way of V-mode.

\* Alternative explanation for the case of H-mode excitation,

$$\langle x^{2} \rangle = \mu^{2} \langle u^{2} \rangle = \mu^{2} J_{u} \beta_{u}$$

$$\langle xy \rangle = \mu(-r_{1} \langle u^{2} \rangle - r_{2} \langle up_{u} \rangle) = -\mu J_{u} \beta_{u} \left(r_{1} - \frac{\alpha_{u}}{\beta_{u}} r_{2}\right)$$

$$\langle y^{2} \rangle = r_{1} \langle u^{2} \rangle + 2r_{1} r_{2} \langle up_{u} \rangle + r_{2}^{2} \langle p_{u}^{2} \rangle = \frac{\langle x^{2} \rangle}{\mu^{2}} \left\{ \mu^{2} \frac{\langle xy \rangle^{2}}{\langle x^{2} \rangle^{2}} + \frac{r_{2}^{2}}{\beta_{u}} \right\}$$

$$(1)$$

$$(2)$$

$$(2)$$

$$(3)$$

From Eq. (1)~(3):

$$r_{1} = \frac{\alpha_{u}}{\beta_{u}}r_{2} - \mu \frac{\langle xy \rangle}{\langle x^{2} \rangle}$$

$$r_{2} = \mu \beta_{u} \sqrt{\frac{\langle x^{2} \rangle \langle y^{2} \rangle - \langle xy \rangle^{2}}{\langle x^{2} \rangle^{2}}} = r_{1} = -\mu \left(\frac{\hat{y}}{\hat{x}}\right)_{u} (\cos \Delta \phi_{u} + \alpha_{u} \sin \Delta \phi_{u})$$

$$r_{2} = -\mu \left(\frac{\hat{y}}{\hat{x}}\right)_{u} \beta_{u} \sin \Delta \phi_{u}$$

- \*  $r_3$  can not be measured by this method,  $p_x$  and  $p_y$  are necessary.
- Both CESR method and correlation matrix method are applicable in the SuperKEKB lattice except for r<sub>3</sub>.

#### Effect of BPM resolution

H. Fukuma

Put BPM error  $\sigma_x = \sigma_v = 0.0001 \text{ m}$ 

100 trials at each point. Error bar shows rms.

Extrapolate the result to 0 amplitude (blue line).





## 4-2 MEASUREMENT OF DISPERSIONS

#### **4-2 DISPERSION MEASUREMENT**

#### Dispersion

	$\vec{u} = (u, p_u, v, p_v)$	decoupled coordinate	
$\vec{u} + \vec{\eta}_u \delta = R\vec{x}$	$\vec{x} = (x, p_x, y, p_y)$	physical coordinate	
$\vec{\eta}_u = R\vec{\eta}_x$	$\vec{\eta}_u = (\eta_u, \eta_{pu}, \eta_v, \eta_{pv})$	decoupled dispersion	
$\vec{u} + R\vec{\eta}_x \delta_1 = R\vec{x}_1$	$\vec{\eta}_x = (\eta_x, \eta_{px}, \eta_y, \eta_{py})$	physical dispersion	
$\vec{u} + R\vec{\eta}_x\delta_2 = R\vec{x}_2$	<i>R</i> is a x-y coupling matrix		

 $\vec{\eta}_x = \frac{\vec{x}_2 - \vec{x}_1}{\delta_2 - \delta_1}$  A measured dispersion is a physical dispersion

COD based measurement uses a change of RF frequency

$$\delta = -\frac{1}{\left(\alpha_c - \frac{1}{\gamma^2}\right)} \frac{\Delta f}{f_0}$$

In the case of single-pass BPMs with RF kick,  $\eta_{px}$  and  $\eta_{py}$  can be estimated by using two locations

## 4-4 OPTICS CORRECTION

### X-Y COUPLING AND VERTICAL DISPERSION

## **4-4 OPTICS CORRECTION**

#### **Corrector:**

- 1. Vertical offset of sextupoles generates both x-y coupling and vertical dispersion(contamination of horizontal dispersion via x-y coupling).
- 2. Skew quadrupole windings of sextupoles(equivalent to sextupole offset, need to evaluate higher order multipole field).

#### Primary method:

x-y coupling and vertical dispersion are corrected simultaneously.

$$\Delta r_{n,i} = \sum_{j=1}^{\#sext} M_{ij}^{(n)} \Delta SK_{1j} \qquad n = 1,2$$

$$M_{ij} \text{ and}$$

$$\Delta \eta_{y,i} = \sum_{k=1}^{\#sext} N_{ij} \Delta SK_{1j} \qquad \text{by usin}$$

2, 3, 4

Nij is a response matrix calculated g a design lattice

SK<sub>1</sub> is a strength of skew quadrupole field

#### Partial correction:

1. x-y coupling correction( $r_1$ - $r_4$ ) only

j=1

2. one of x-y coupling parameter, r<sub>2</sub> and vertical dispersion  $r_2$  is insensitive to BPM rotation along the beam axis.

## **4-4 OPTICS CORRECTION**

K. Egawa

### Q or skew-Q field excited by sextupole correction coils



#### skew-Q configuration



### **4-4 MACHINE ERROR**

#### Assumption in this simulation

\* measurement at KEKB

misalignment	σ <sub>y</sub> (μm)	tilt (mrad)
Sextupoles (SD/SF)	100	0
Quadrupoles (normal magnet)	0	0.1 *
BPM	0	1, 10

**BPM** resolution

No gradient error of magnets

position resolution due to jitter error	σ <sub>x,y</sub> (μm)	
BPM (single-pass mode)	50, 75, 100	
BPM (average mode)	2, 5	

## 4-4 TILT ANGLE OF QUADRUPOLES

M. Masuzawa et al.

Measurement of tilt angle along the beam axis at KEKB



 $\sigma_{\theta} = 0.07 \sim 0.1 \text{ mrad for quadrupoles}$ 

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# **4-4 AFTER CORRECTION**No BPM error#BPMs = 160



#### **4-4 CORRECTION PERFORMANCE**



#### **4-4 BPM RESOLUTION AND TILT**

#samples = 25 for each point (error bar: standard deviation)
of common SD/SF/Q misalignment after 5 iterative procedures

Error source: SD and SF vertical offset + tilt of Q / Corrector: SD/SF skew Q windings



#### 4-4 CORRECTION PERFORMANCE How is correction of X-Y coupling only ?

No BPM error

Error source: SD and SF vertical offset + tilt of Q / Corrector: SD/SF skew Q windings



#### **4-4 DYNAMIC APERTURE**

100 samples: Sextupole  $\sigma_{\Delta y}$  = 100 micron / Q tilt  $\sigma_{\Delta \theta}$  = 0.1 mrad BPM jitter error  $\sigma_{x,y}$  = 50 micron / BPM tilt  $\sigma_{\Delta \theta}$  = 1 mrad

#### Before correction

#### After correction



Degradation of dynamic aperture is improved by the correction of x-y coupling and vertical dispersion.

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#### **4-4 DYNAMIC APERTURE**

100 samples: Sextupole  $\sigma_{\Delta y} = 100 \text{ micron } / \text{ Q tilt } \sigma_{\Delta \theta} = 0.1 \text{ mrad}$ BPM jitter error  $\sigma_{x,y} = 50 \text{ micron } / \text{ BPM tilt } \sigma_{\Delta \theta} = 1 \text{ mrad}$ 

Error source: SD and SF vertical offset + tilt of Q / Corrector: SD/SF skew Q windings



Larger coupling makes longer lifetime even if dynamic aperture is small

Distribution of Touschek lifetime is widely scattered after the correction of x-y coupling and vertical dispersion.

#### 4-4 REQUIRED FILED STRENGTH OF SKEW QUADRUPOLE WINDINGS

Requirement of field strength:  $SK_1 = \pm 5 \times 10^{-3} \text{ m}^{-1}$ 



104 samples x 100 sextupoles in HER

#### 4-4 EFFECT OF SKEW OCTUPOLE FROM SKEW QUADRUPOLE WINDINGS



Effect is about 10 % for Touschek lifetime.

Specification for single-pass BPM to achieve target emittance ratio:

	standard deviation	
BPM resolution (jitter error)	50 µm	Better
	75 µm	Good
	100 µm	Acceptable
BPM tilt (misalignment)	1 mrad	Insensitive to the
	10 mrad	correction for global coupling

- x-y coupling and vertical dispersion correction
  - 1. x-y coupling measurement
    - # multi-turn method (H-mode)
    - \* Position resolution smaller than ~100 micron for single-pass BPMs satisfies the requirement of tentative target of  $\varepsilon_y/\varepsilon_x = 0.15$  %. (with considering margin of 0.25% in HER, 0.27% in LER)
  - 2. Dispersion measurement
    - RF-frequency method (COD-based) is used tentatively.
    - Position resolution of BPMs (average mode) is assumed to be 2 micron. This resolution is enough.
    - The worse case of 5 micron is acceptable to correct the global x-y coupling and the vertical emittance.

- x-y coupling and vertical dispersion correction
  - 3. Simultaneous correction of x-y coupling and vertical dispersion
    - No fundamental difficulty
  - 4. Performance of x-y coupling correction only
    - \* Emittance ratio,  $\varepsilon_y/\varepsilon_x$  is reduced from 2 % to ~ 0.75 % (typical example).
  - 5. One of x-y coupling parameters, r2 and vertical dispersion
    - It seems to be similar to all x-y coupling parameters and vertical dispersion(need to check various machine error)
    - Both correlation matrix method and CESR method
    - # Advantage:

a. The r<sub>2</sub> parameter is insensitive to the BPM tilt.

b. No necessary of transfer matrix between two BPMs( $\beta$ -function is necessary)

 $r_1 = \frac{\alpha_u}{\beta_u} r_2 - \mu \frac{\langle xy \rangle}{\langle x^2 \rangle}$ 

 $r_2 = \mu \beta_u \sqrt{\frac{\langle x^2 \rangle \langle y^2 \rangle - \langle xy \rangle^2}{\langle x^2 \rangle^2}}$ 

Dynamic aperture (Touschek lifetime) after optics corrections

Higher-order multipole field due to skew quadrupoles winding of sextupoles

## APPENDIX

#### skew Q component excited by Sx's correction coils : Bx [x] for different current configurations



K. Egawa

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#### By [y]



#### 4-1 BPM TILT (MONITOR:QD3E.3)

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#### 4-4 CORRECTION PERFORMANCE OFFSET AND SKEW Q ELEMENT



## SEXTUPOLE OFFSET

 $\Delta y_{(\text{SD1})} = +0.1 \text{ mm}$  $\Delta y_{(\text{SD2})} = +0.1 \text{ mm}$ 

 $\Delta y_{(\text{SD1})} = +0.1 \text{ mm}$  $\Delta y_{(\text{SD2})} = -0.1 \text{ mm}$ 



SD1 -I' SD2

SD1 -I' SD2