Beam-beam interaction in SuperKEKB: simulations and experiments

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Acknowledgments

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Outline

- Scaling laws for luminosity and beam-beam tune shifts
- Comparison of simulations and experiments
- Beam-beam parameters
- Status of beam-beam simulations
- Limitations on the current performance of SuperKEKB
- Beam-beam perspective on achieving target luminosity in SuperKEKB
- Summary

Scaling laws for luminosity and beam-beam tune shifts

- "Nano-beam" + crab waist in SuperKEKB
  - Simple scaling laws are good enough to discuss luminosity and beam-beam parameters/tune shifts for the case of $\beta_y^* = 1$ mm [1].

$$L \approx \frac{N_b N_+ N_- f}{2\pi\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^2+\sigma_{z-}^2}\tan\frac{\theta_c}{2}} e^{-\frac{\Delta^2}{2(\sigma_{y+}^{*2}+\sigma_{y-}^{*2})}}$$

$$L_{sp} \approx \frac{1}{2\pi e^2 f\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^2+\sigma_{z-}^2}\tan\frac{\theta_c}{2}}$$

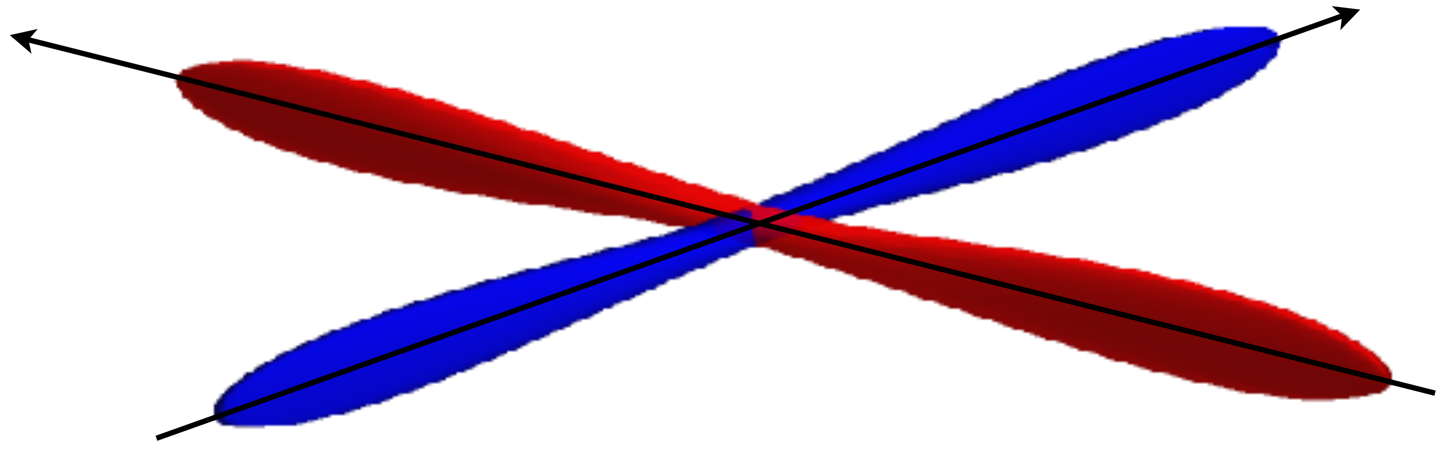
$$L = \frac{1}{2er_e}\frac{\gamma_\pm I_\pm}{\beta_{y\pm}^*}\boxed{\xi_{y\pm}^L} \longrightarrow \text{Beam-beam parameter [2]}$$

$$\text{Beam-beam tune shift [3]} \longleftarrow \boxed{\xi_{y+}^i} \approx \frac{r_e}{2\pi\gamma_+}\frac{N_-\beta_{y+}^*}{\sigma_{y-}^*\sqrt{\sigma_{z-}^2\tan^2\frac{\theta_c}{2}+\sigma_{x-}^{*2}}}$$

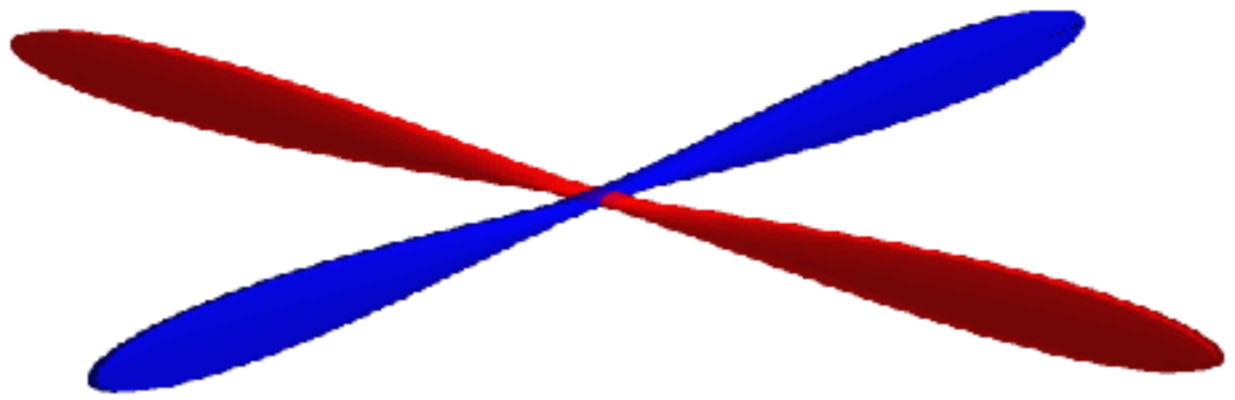
Piwinski angle: $\Phi_P = \frac{\sigma_z}{\sigma_x^*}\tan\frac{\theta_c}{2} \gg 1$

Hourglass condition: $\frac{\beta_y^*}{\sigma_x^*}\tan\frac{\theta_c}{2} \gtrsim 1$

Schematic view of collision schemes



SuperKEKB (2021c)

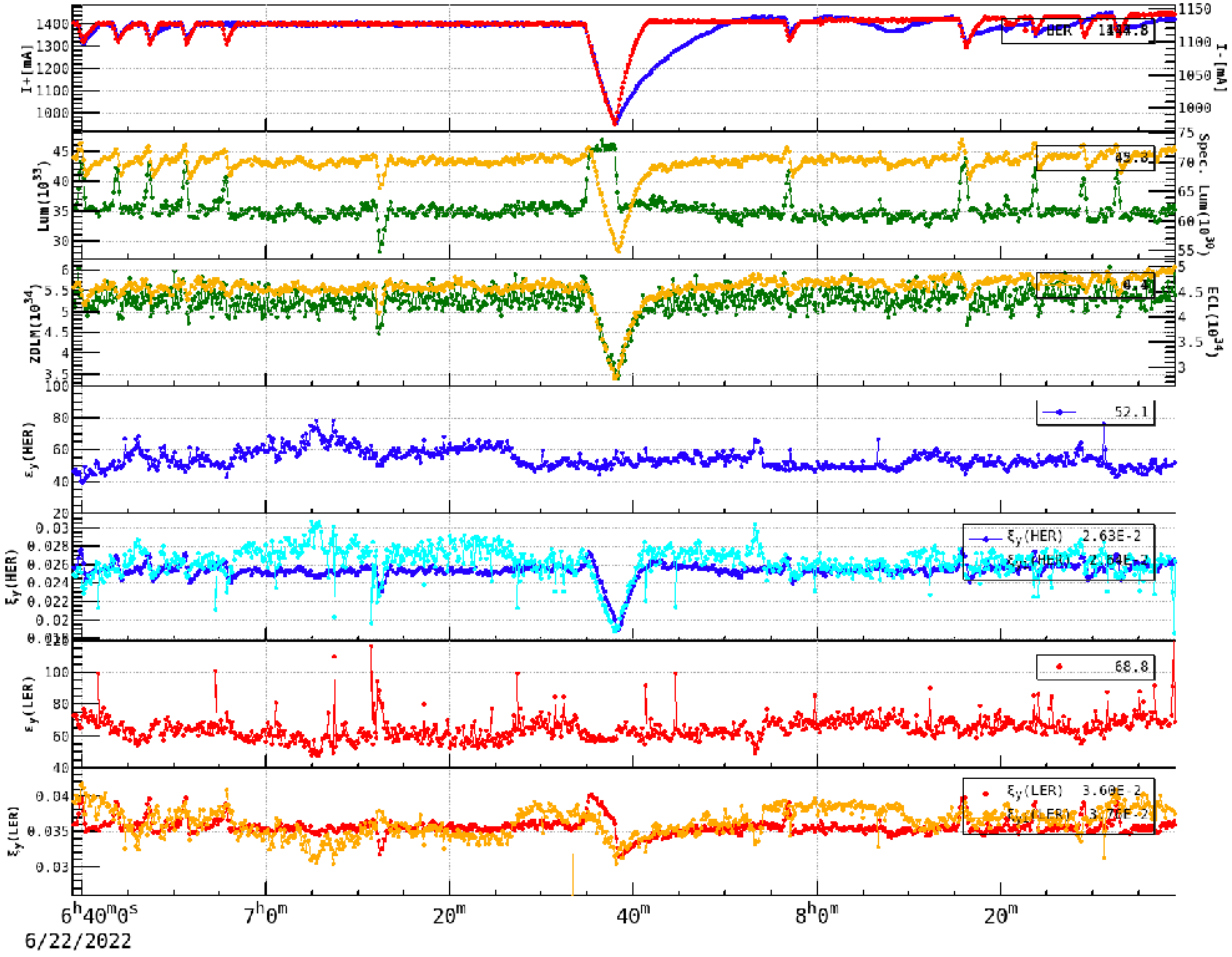


SuperKEKB (Final design)

[1] See backup slides for validations (p.30-42). [2] K. Ohmi et al., PRST-AB 7, 104401 (2004). [3] BB parameter and tune shift refer to different terms in this talk.

Achieved luminosity record

- The luminosity record 4.71×10^{34} cm^{-2}s^{-1} was achieved with Belle II HV OFF and injection stopped (Jun. 22, 2022).



Total currents: I_{+}/I_{-}=1.4/1.14 A

Total (Yellow) and specific luminosity (Green) by ECL

Instantaneous total luminosity by ZDLM (Yellow) and ECL (Green)

e- emittance by XRM: ~50 pm

e- beam BB parameter: ~0.028

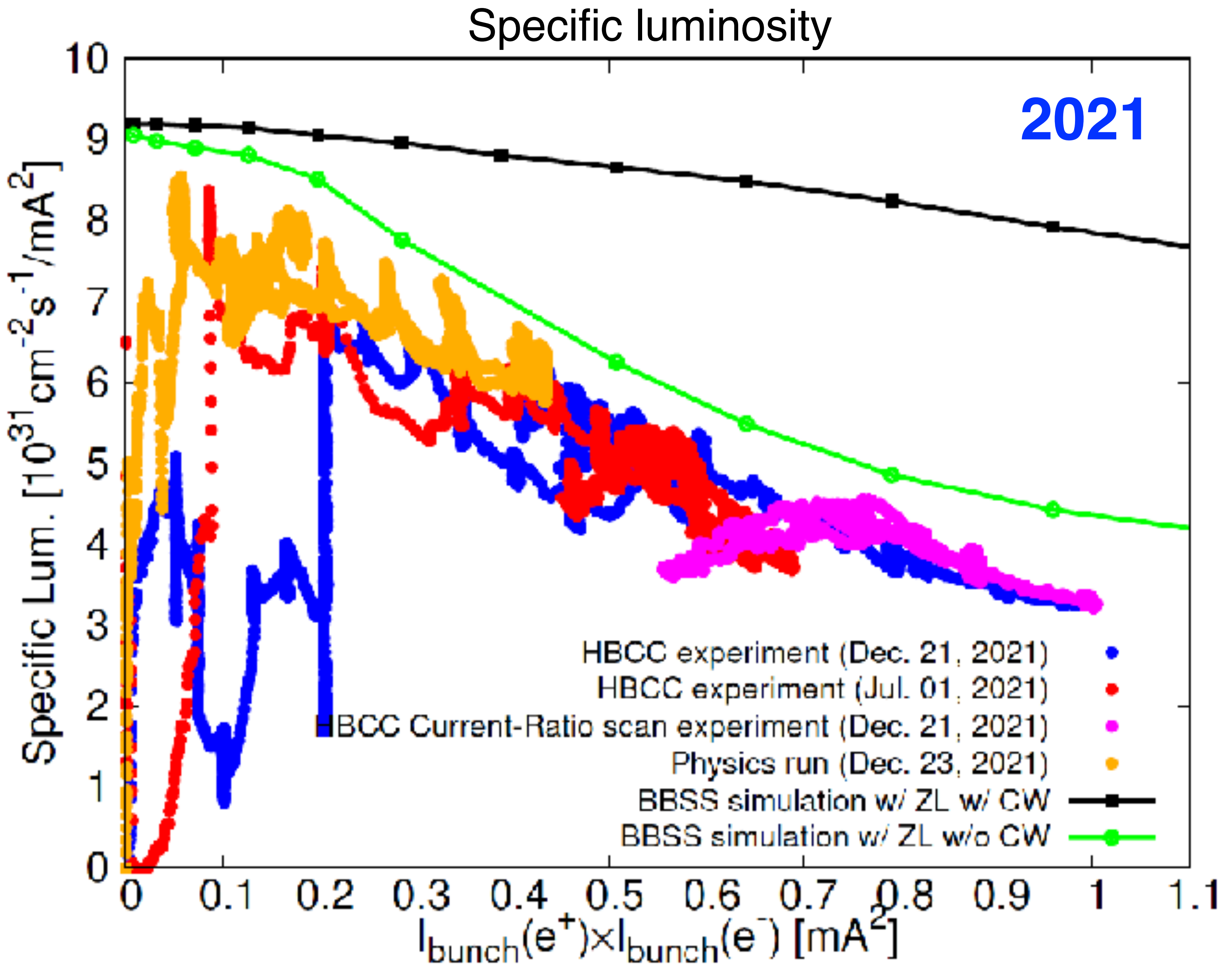
e+ emittance by XRM: ~60 pm

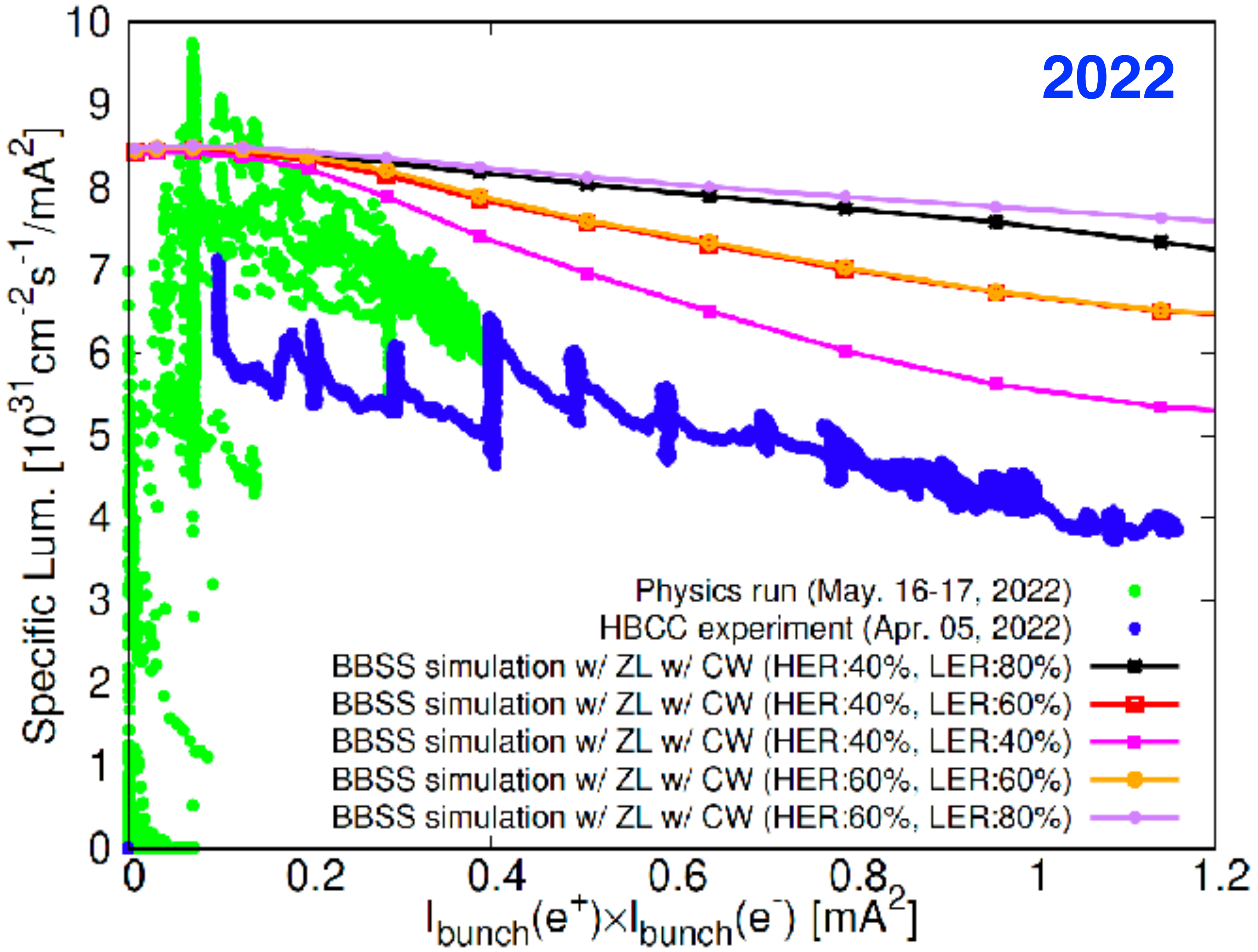
e+ beam BB parameter: ~0.04

Comparison of simulations and experiments

- HBCC (High Bunch Current Collision) machine studies with $\beta_y^* = 1$ mm in 2021 and 2022:
  - HBCC machine studies were done to extract the luminosity performance.
  - Lsp (specific luminosity) slope vs. product of beam currents improved in 2022 but still drops quickly due to vertical blowup.

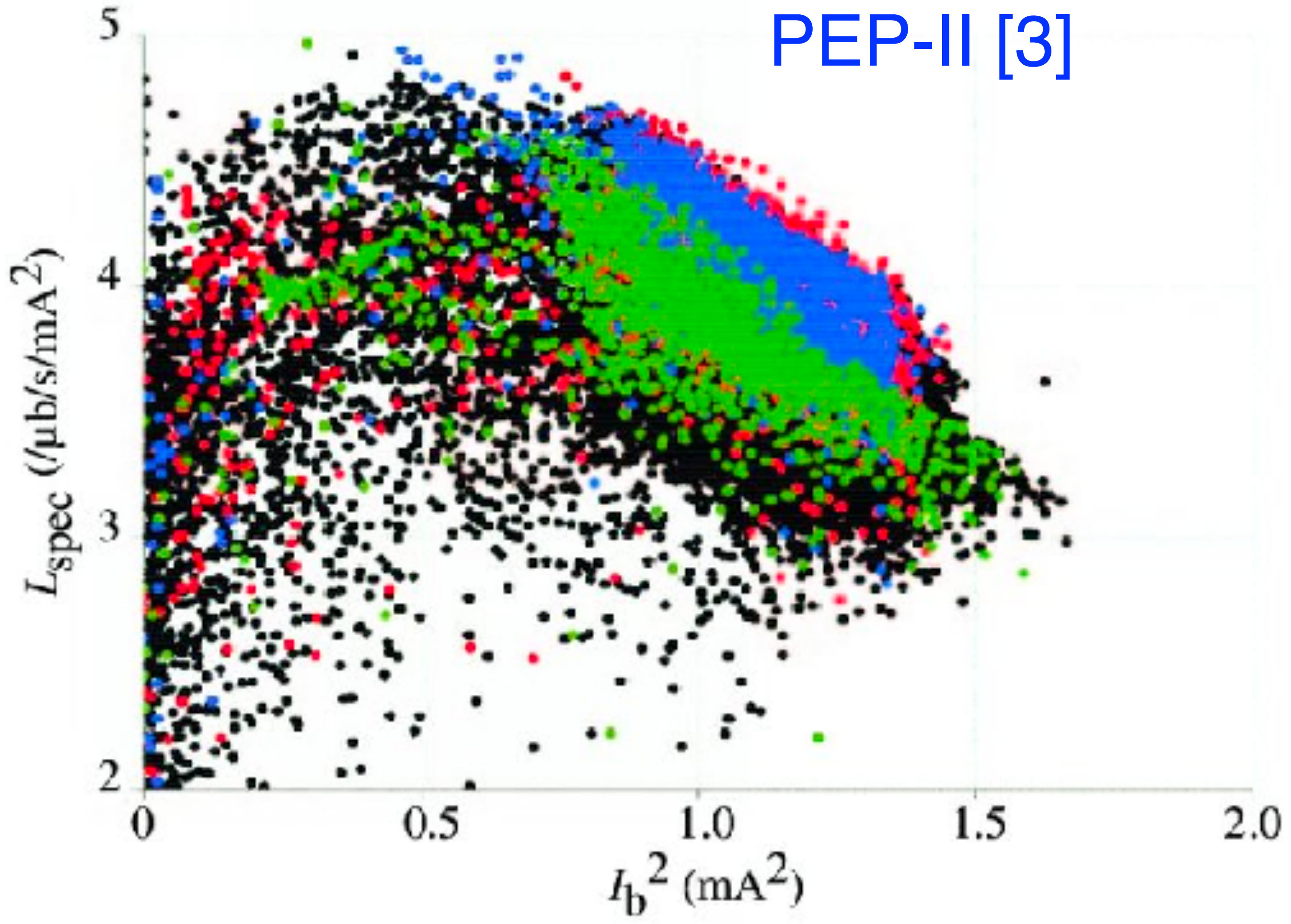
	2021.12.21		2022.04.05		Comments
	HER	LER	HER	LER	
I_{bunch} (mA)	Ie	1.25*Ie	Ie	1.25*Ie	
# bunch	393		393		Assumed value
ε_x (nm)	4.6	4.0	4.6	4.0	w/ IBS
ε_y (pm)	35	20	30	35	Estimated from XRM data
β_x (mm)	60	80	60	80	Calculated from lattice
β_y (mm)	1	1	1	1	Calculated from lattice
σ_{z0} (mm)	5.05	4.60	5.05	4.60	Natural bunch length (w/o MWI)
ν_x	45.53	44.524	45.532	44.524	Measured tune of pilot bunch
ν_y	43.572	46.589	43.572	46.589	Measured tune of pilot bunch
ν_s	0.0272	0.0233	0.0272	0.0233	Calculated from lattice
Crab waist	40%	80%	40%	80%	Lattice design





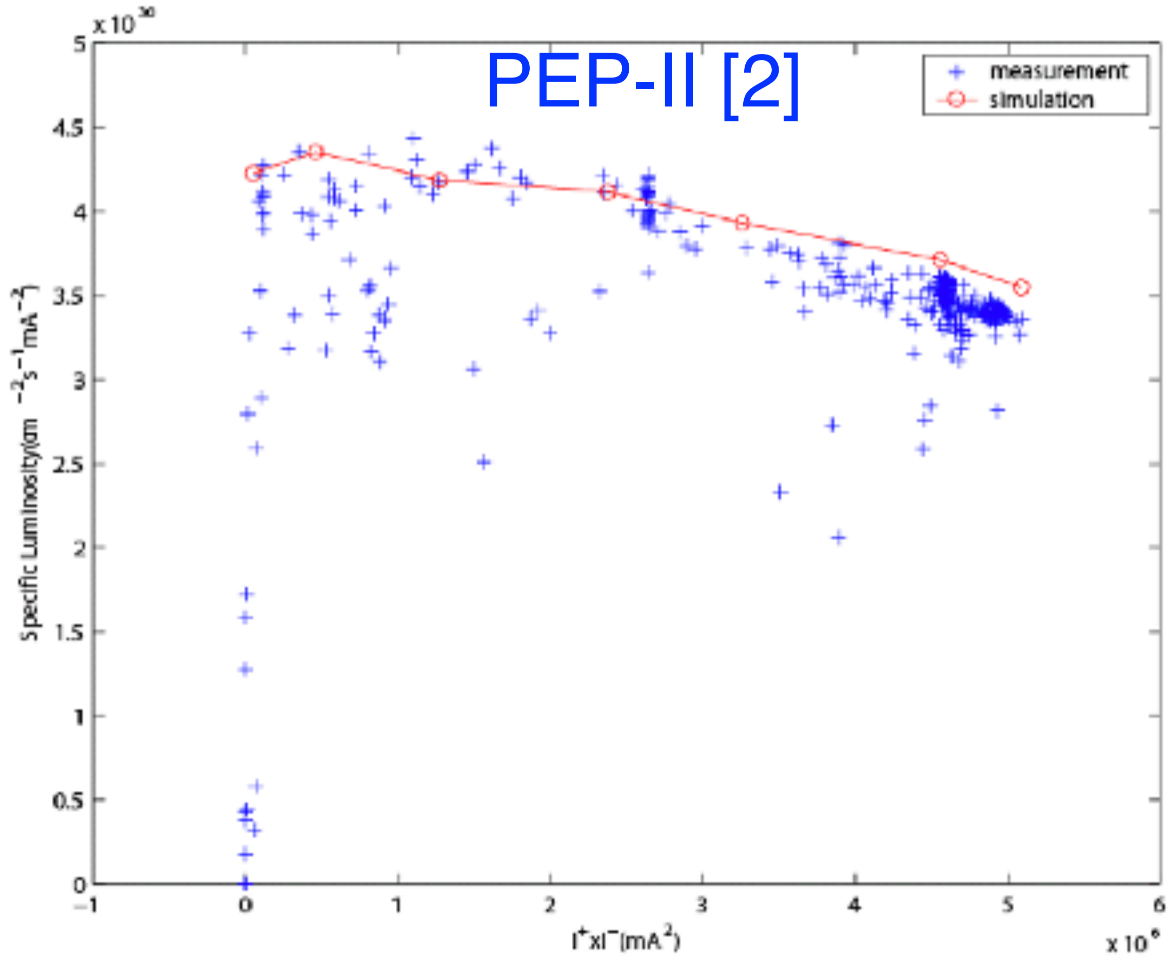
Comparison of simulations and experiments

- The “Lsp puzzle” (large discrepancy between beam-beam simulations and experiments)
  - The “Lsp puzzle” appeared in KEKB.
  - The “Lsp puzzle” re-appeared in SuperKEKB.
  - Was there such a “Lsp puzzle” in PEP-II?

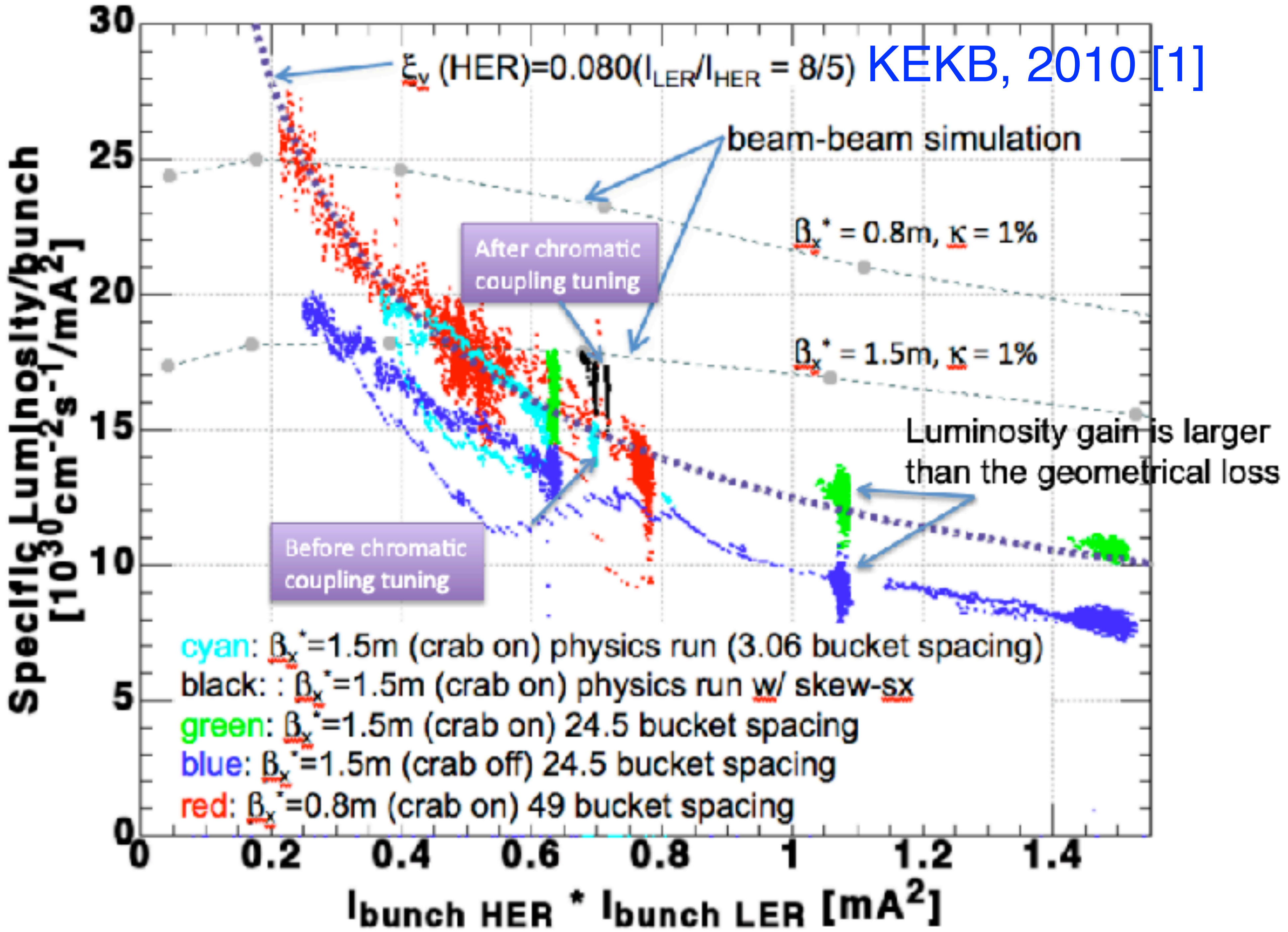


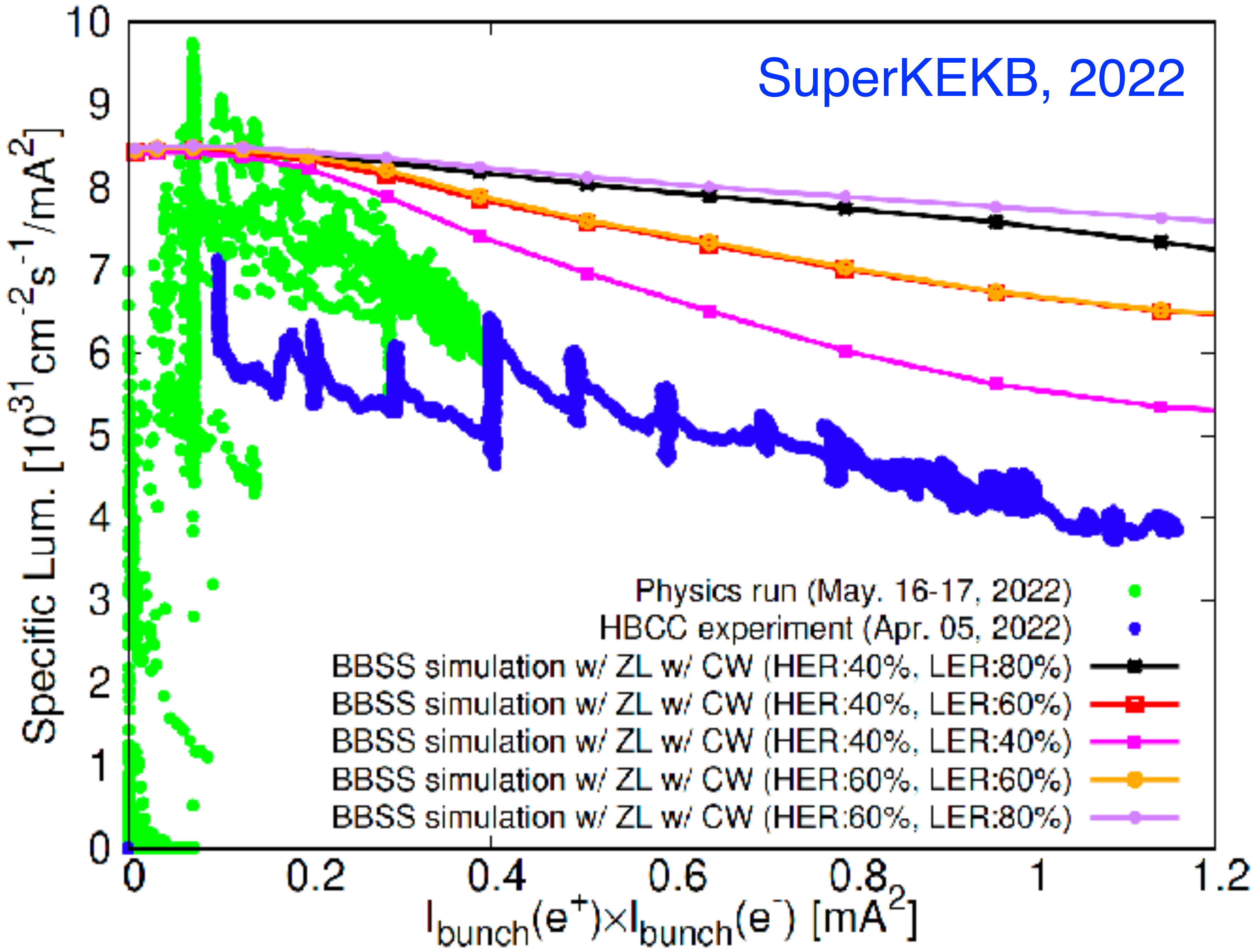
Specific Luminosity October 10, 2005

(I^+=2940mA, I^-=1733mA, n_b=1732)



Data taken in the history buffer for a 24 hours period.
The simulation used an approximately fixed current ratio.

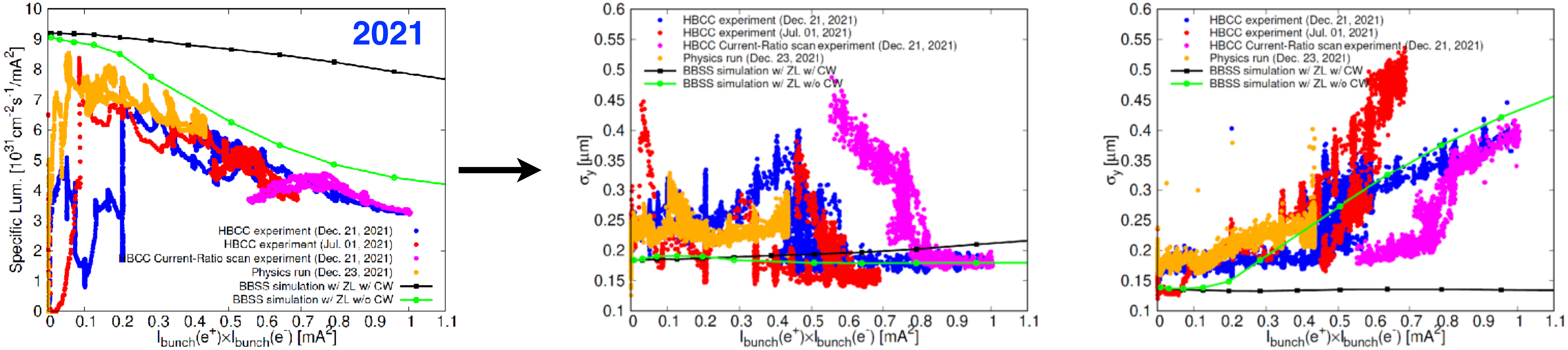




[1] Y. Funakoshi, KEKB MAC 2010; [2] Y. Cai, KEKB MAC 2006; [3] U. Wienands, PAC07.

Comparison of simulations and experiments

- HBCC machine studies with $\beta_y^* = 1$ mm in 2021 and 2022:
  - After fine-tuning of BxB FB system in 2022, the observed vertical beam sizes blowup became much more “normal” (a breakthrough in 2022) and closer to simulations. The origin of vertical blowup remains to be explained.

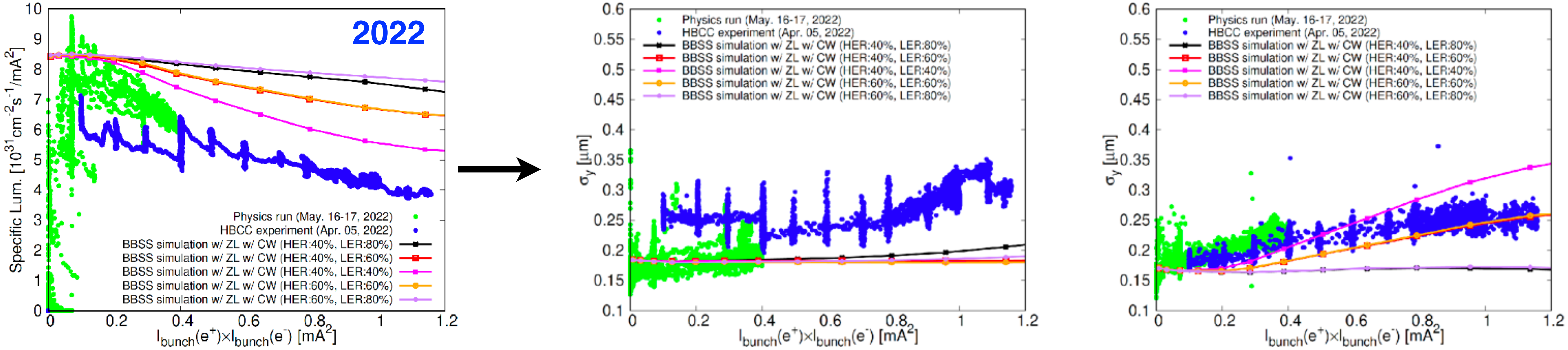


In 2021, there was a “flip-flop” correlation between σ_{y+}^* and σ_{y-}^*

Specific luminosity

Electron σ_y^*

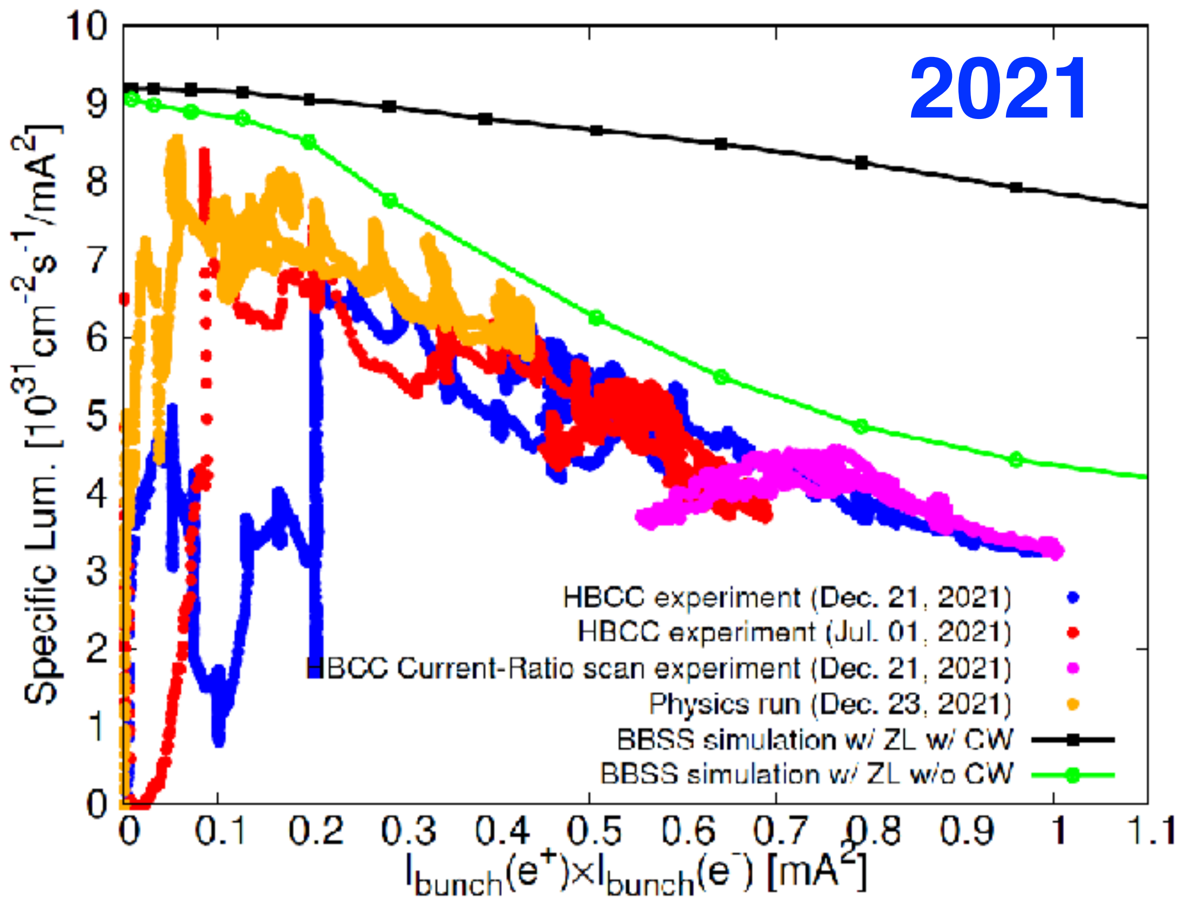
Positron σ_y^*

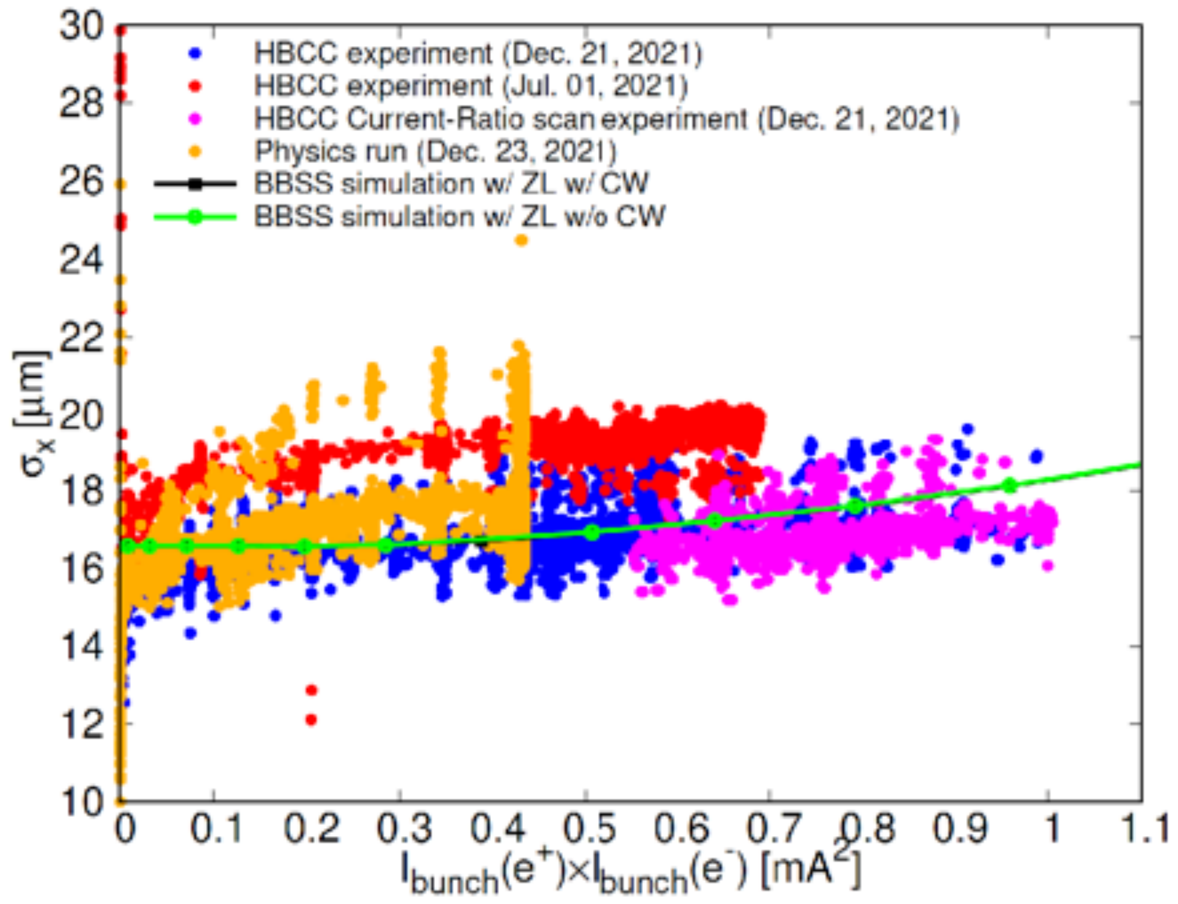


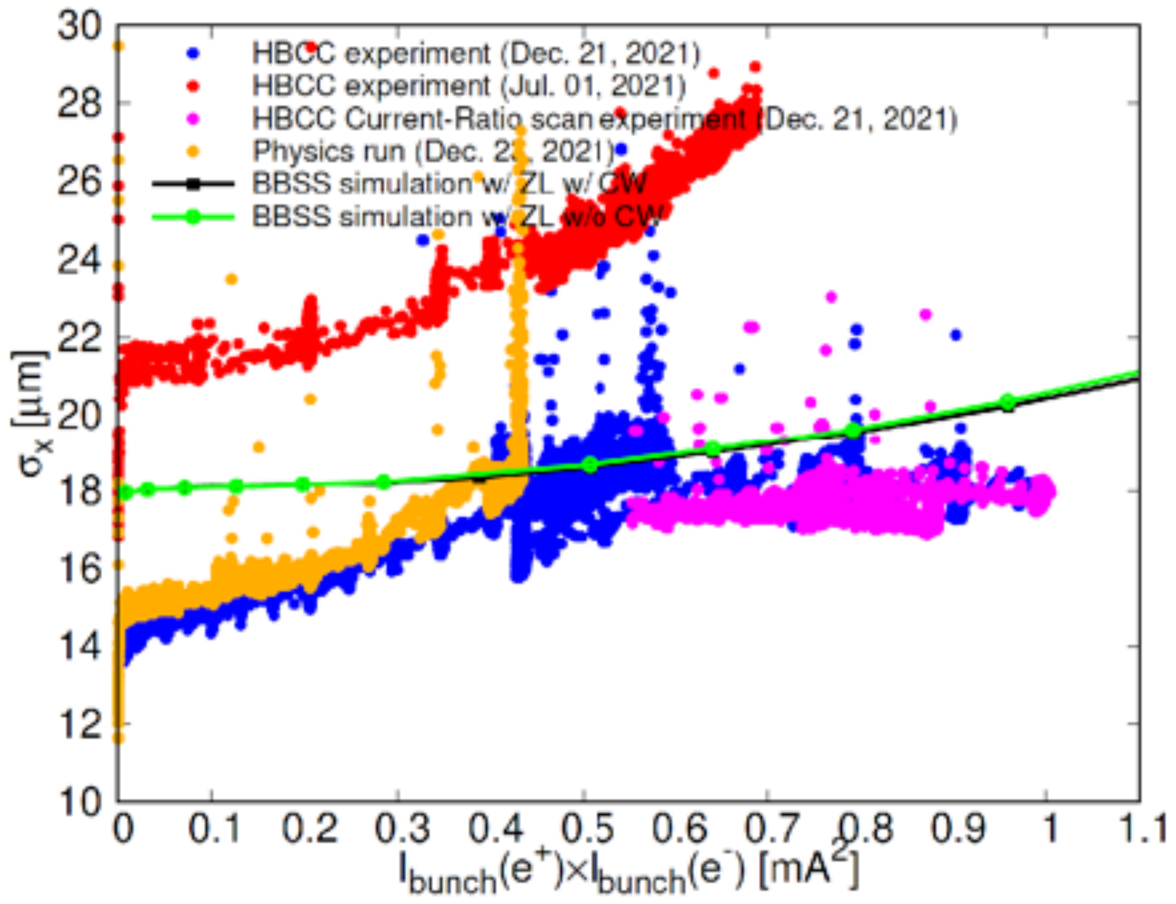
In 2022, this “flip-flop” problem was cured after fine-tuning of FB system

Comparison of simulations and experiments

- HBCC machine studies with $\beta_y^* = 1$ mm in 2021 and 2022:
  - Weak blowup of horizontal beam size: qualitative agreements between simulations and experiments
  - Horizontal blowup is sensitive to horizontal tune (see p.48 for simulation results)





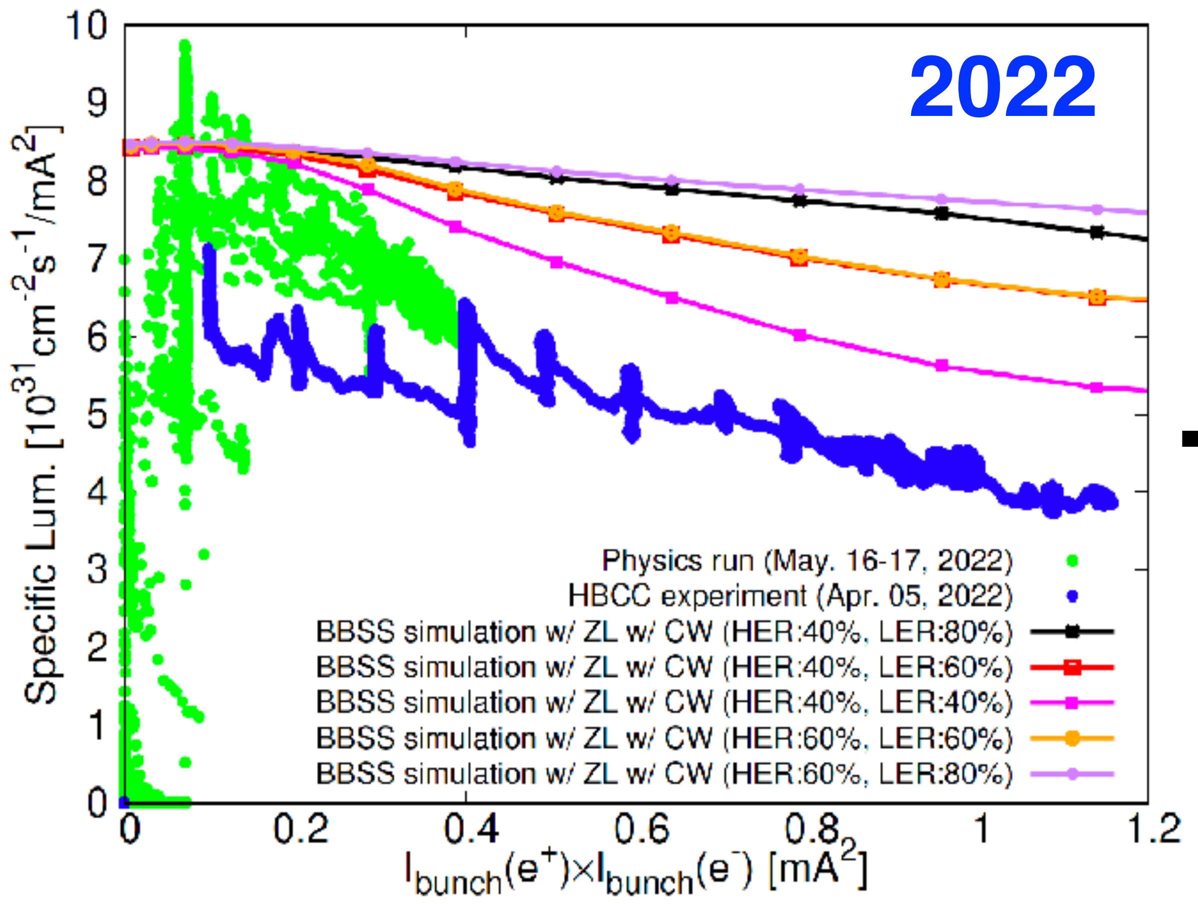


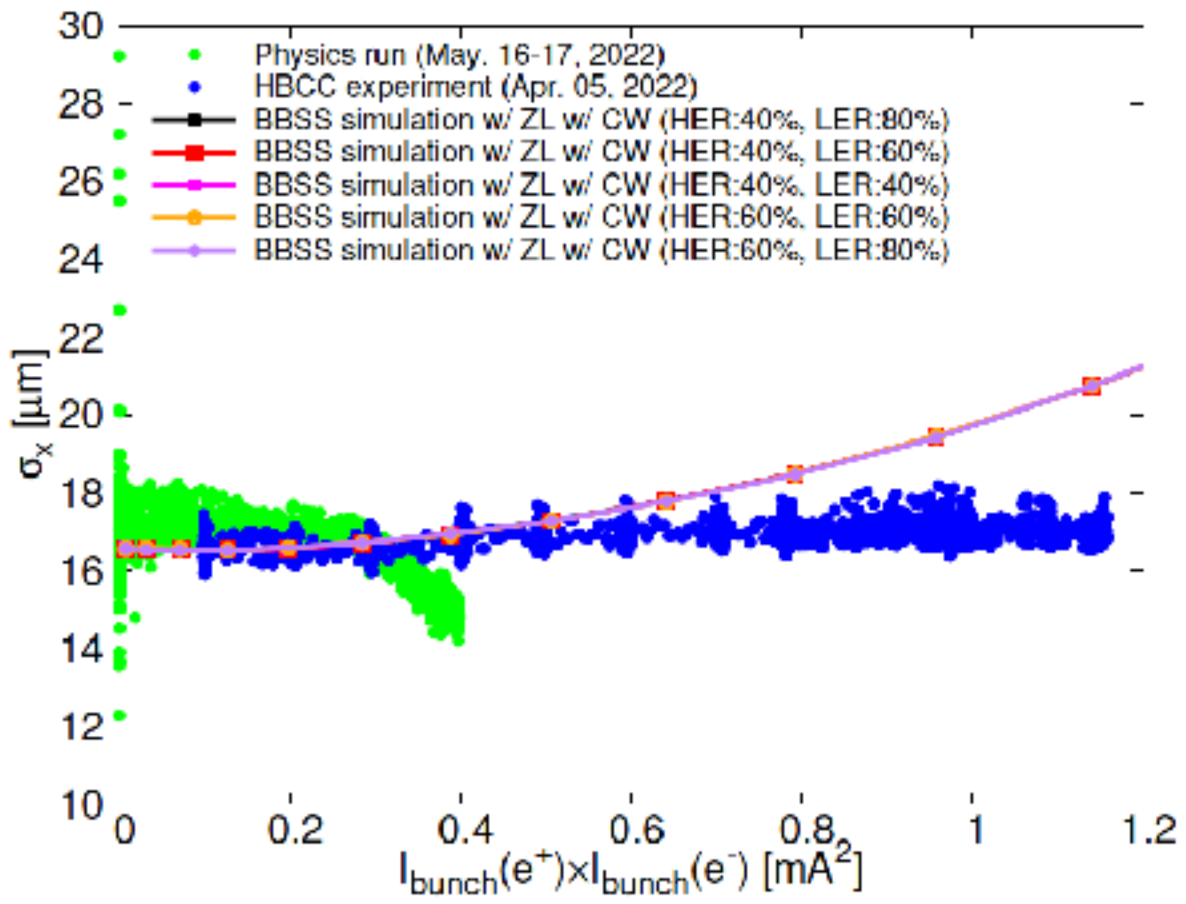
Measured data might have systematic offsets

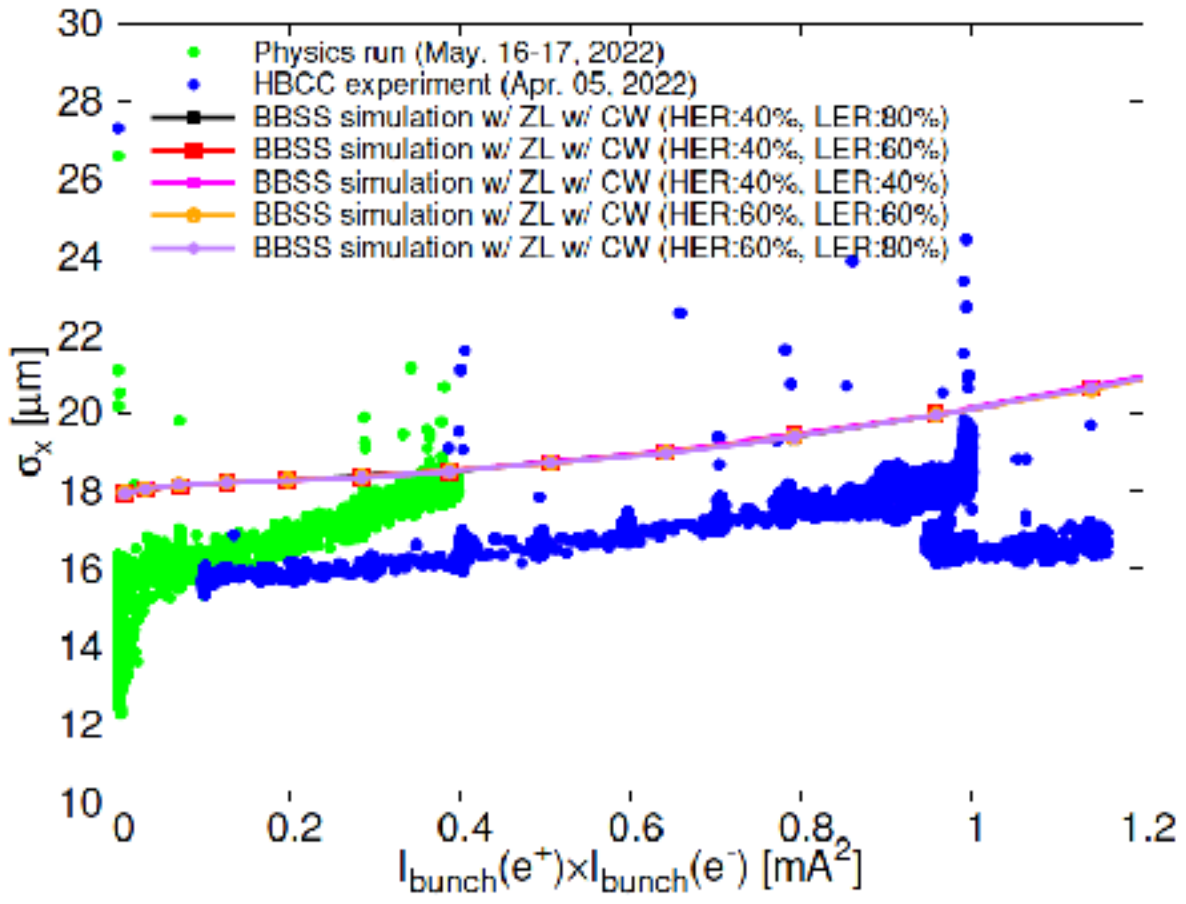
Specific luminosity

Electron σ_x^*

Positron σ_x^*

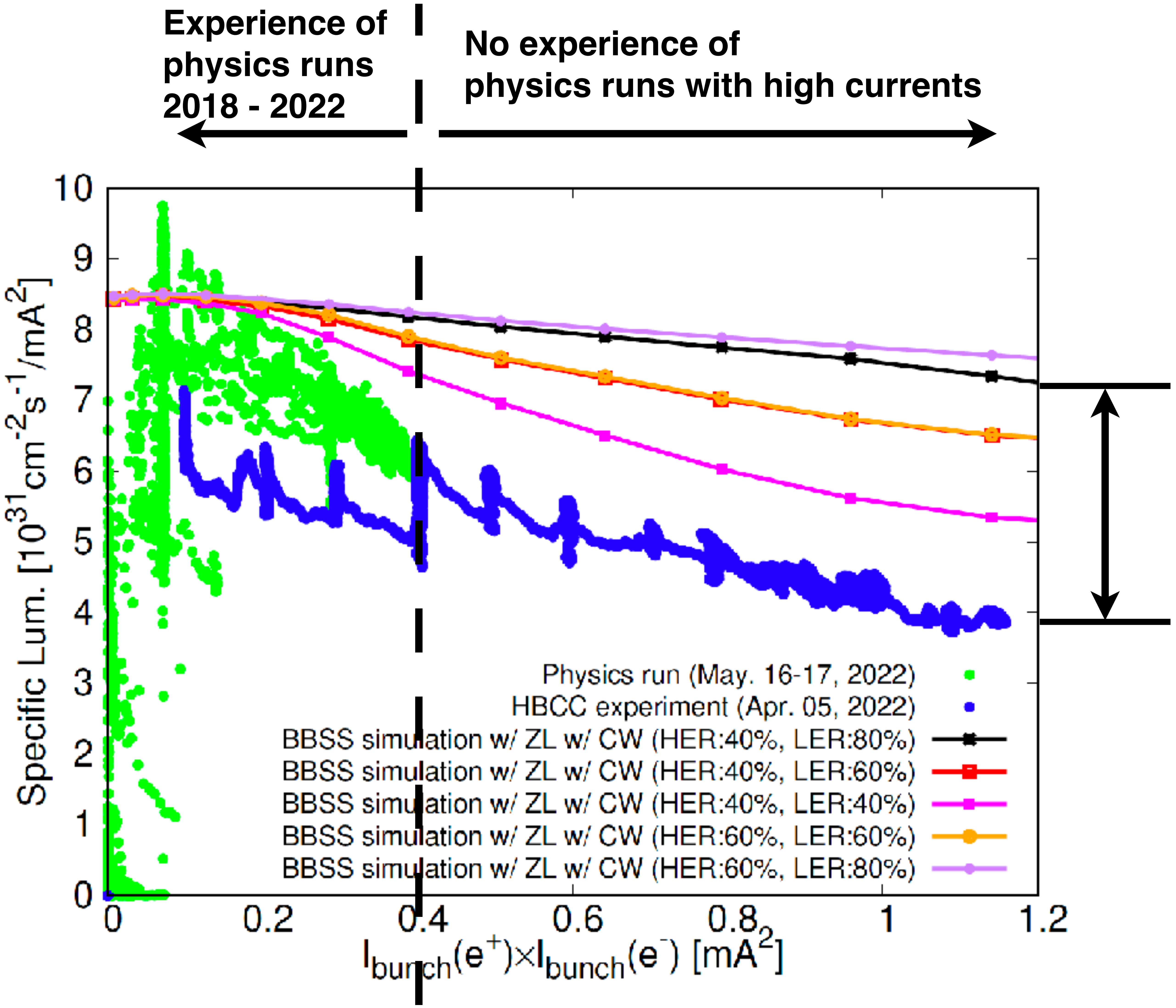






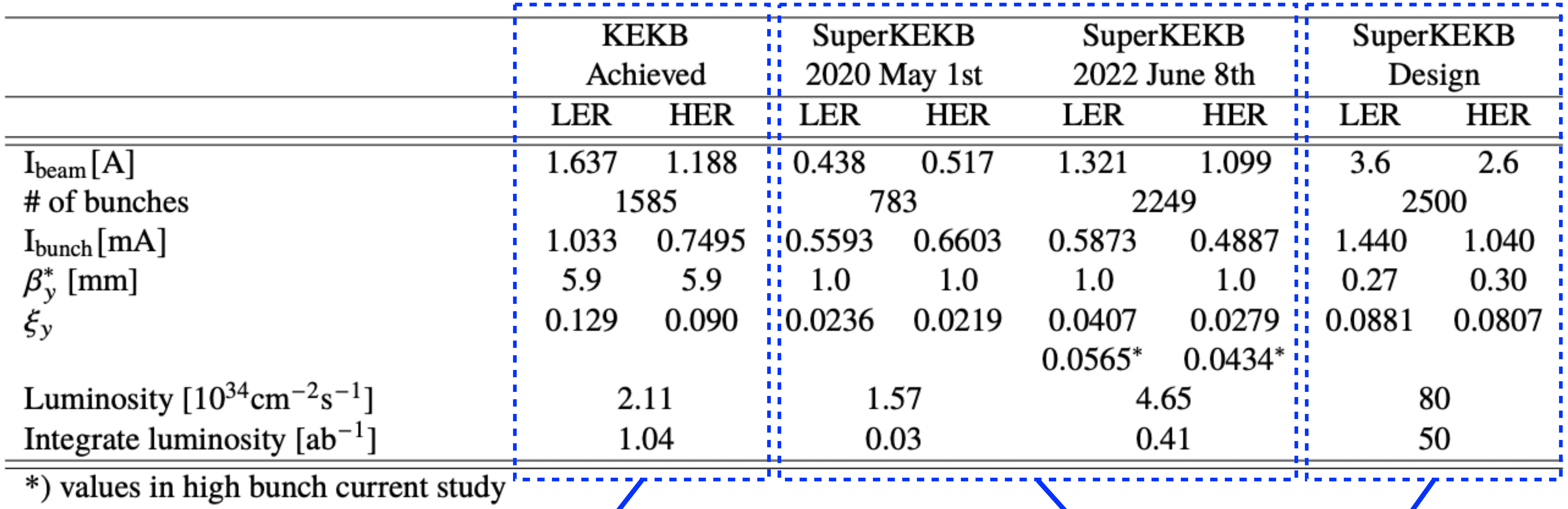
Comparison of simulations and experiments

- Filling the gap between simulated and measured Lsp
  - BBSS+PIC simulation showed 5% less Lsp at $I_{b+}I_{b-} = 0.8$ mA^2 [see p.12].
  - Impedance effects:
    - Simulations showed less bunch lengthening than measurements. If measured bunch lengthening is applied, it gives ~10% extra loss of Lsp at $I_{b+}I_{b-}$ =0.8 mA^2.
    - “-1 mode instability” due to the interplay of FB and vertical impedance [see K. Ohmi’s talk].
  - Found a large systematic in ECL luminosity at high injection background. This could explain a ~10% difference between simulation and measured data at $I_{b+}I_{b-}$=0.3 mA^2. There remains a difference of ~10% [see p.20-24]. No physics data was taken at high bunch currents, and this systematic’s impact is unknown.
  - The machine conditions for HBCC experiments were not optimal due to the limited beam time for machine studies.



Beam-beam parameters

- Overview of beam-beam parameters with crab waist [1]
  - The achieved beam-beam parameters during the physics run of SuperKEKB (i.e., the high voltage of Belle II was on.) in 2022 were 0.0407/0.0279 in LER/HER ($\gamma_+ I_{b+} \neq \gamma_- I_{b-}$, β_y^*=1 mm).
  - In 2022, 0.0565/0.0434 were achieved in LER/HER during HBCC machine studies (β_y^*=1 mm).
  - There was no clear evidence showing SuperKEKB had already reached the beam-beam limit.



	KEKB Achieved		SuperKEKB 2020 May 1st		SuperKEKB 2022 June 8th		SuperKEKB Design	
	LER	HER	LER	HER	LER	HER	LER	HER
I_{beam} [A]	1.637	1.188	0.438	0.517	1.321	1.099	3.6	2.6
# of bunches	1585		783		2249		2500	
I_{bunch} [mA]	1.033	0.7495	0.5593	0.6603	0.5873	0.4887	1.440	1.040
β_y^* [mm]	5.9	5.9	1.0	1.0	1.0	1.0	0.27	0.30
ξ_y	0.129	0.090	0.0236	0.0219	0.0407 0.0565*	0.0279 0.0434*	0.0881	0.0807
Luminosity [10^{34}cm^{-2}s^{-1}]	2.11		1.57		4.65		80	
Integrate luminosity [ab^{-1}]	1.04		0.03		0.41		50	

*) values in high bunch current study

$$\mathcal{L} = \frac{N_1 N_2 f}{4\pi\sigma_x^* \sigma_y^*} R_{\mathcal{L}} \left(\theta_x, \beta_x^*, \beta_y^*, \varepsilon_x, \varepsilon_y, \sigma_z\right) \ ,$$

$$\xi_{yk} = \frac{N_{3-k} r_e \beta_{yk}^*}{2\pi\gamma_k \left(\sigma_x^* + \sigma_y^*\right) \sigma_y^*} R_{\xi y} \left(\theta_x, \beta_x^*, \varepsilon_x, \varepsilon_y, \beta_y^*, \sigma_z\right) \ .$$

$$\mathcal{L} = \frac{\gamma_k I_k \xi_y}{2 e r_e \beta_y^*} \frac{R_{\mathcal{L}}}{R_{\xi y}} \ ,$$

$$L = \frac{1}{2 e r_e} \frac{\gamma_\pm I_\pm}{\beta_{y\pm}^*} \xi_{y\pm}^L$$

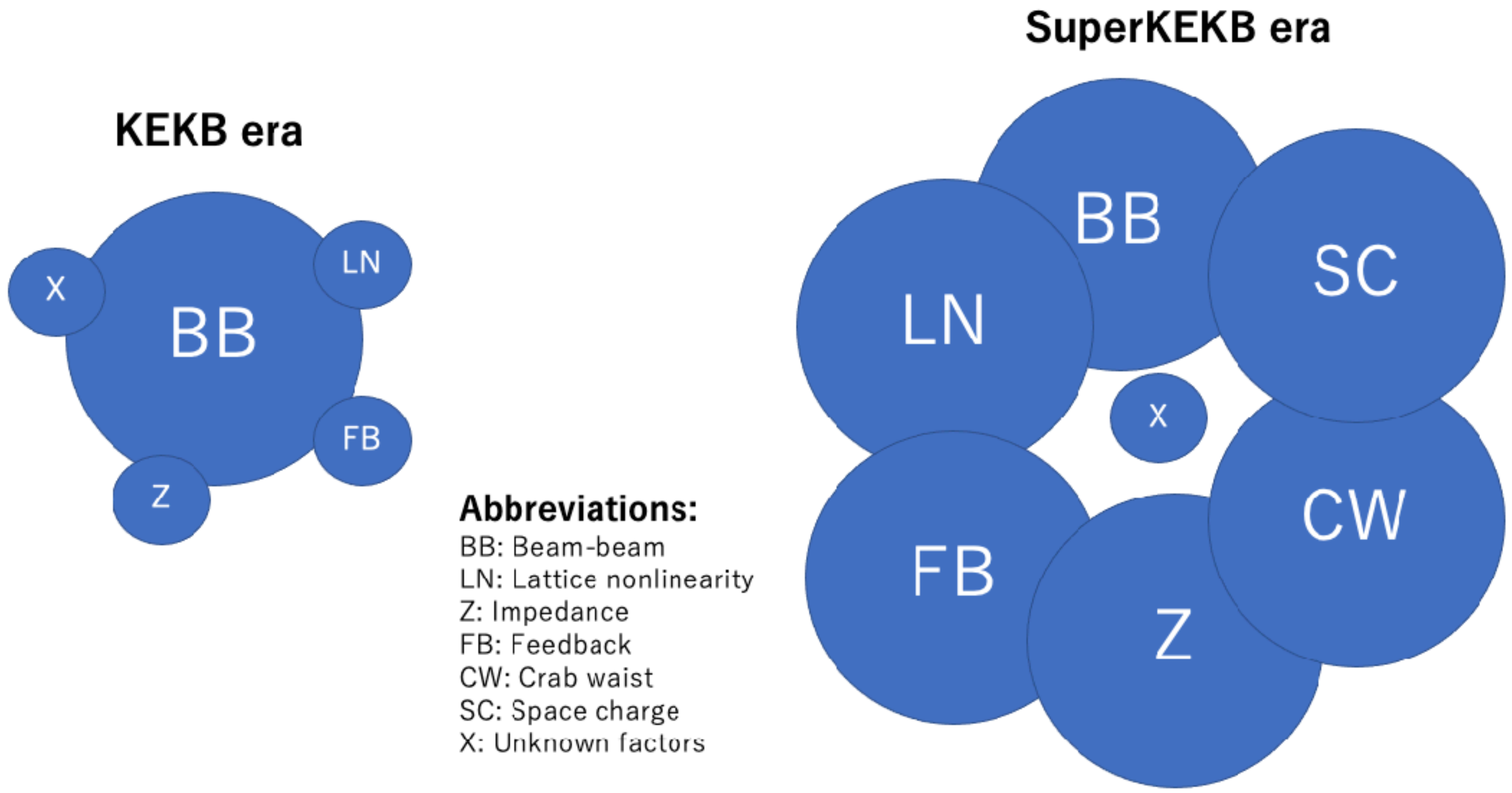
$$\xi_{y+}^{ih} = \frac{1}{4\pi p_0 c} \int_{-\infty}^{\infty} ds \beta_{y+}(s) \frac{\partial F_{y+}}{\partial y'}$$

[1] Y. Funakoshi, “The SuperKEKB Has Broken the World Record of the Luminosity”, IPAC’22.

Status of beam-beam simulations

- Beam-beam (BB) simulations
  - Available tools: BBWS (weak-strong BB model + simple one-turn map + perturbation maps); BBSS and IBB (strong-strong BB model + simple one-turn map + perturbation maps); SAD (BBWS’s BB model + complete lattice + perturbation maps).
  - SuperKEKB is challenging the predictability of BB simulations: It requires reliable models of multiple physics (BB, impedances, lattice nonlinearity, crab waist, realistic machine errors, space charge, etc.), not only BB.

$$L \approx \frac{N_bN_+N\_f}{2\pi\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^{2}+\sigma_{z-}^{2}}\tan\frac{\theta_c}{2}}e^{-\frac{\Delta^2}{2(\sigma_{y+}^{*2}+\sigma_{y-}^{*2})}}$$



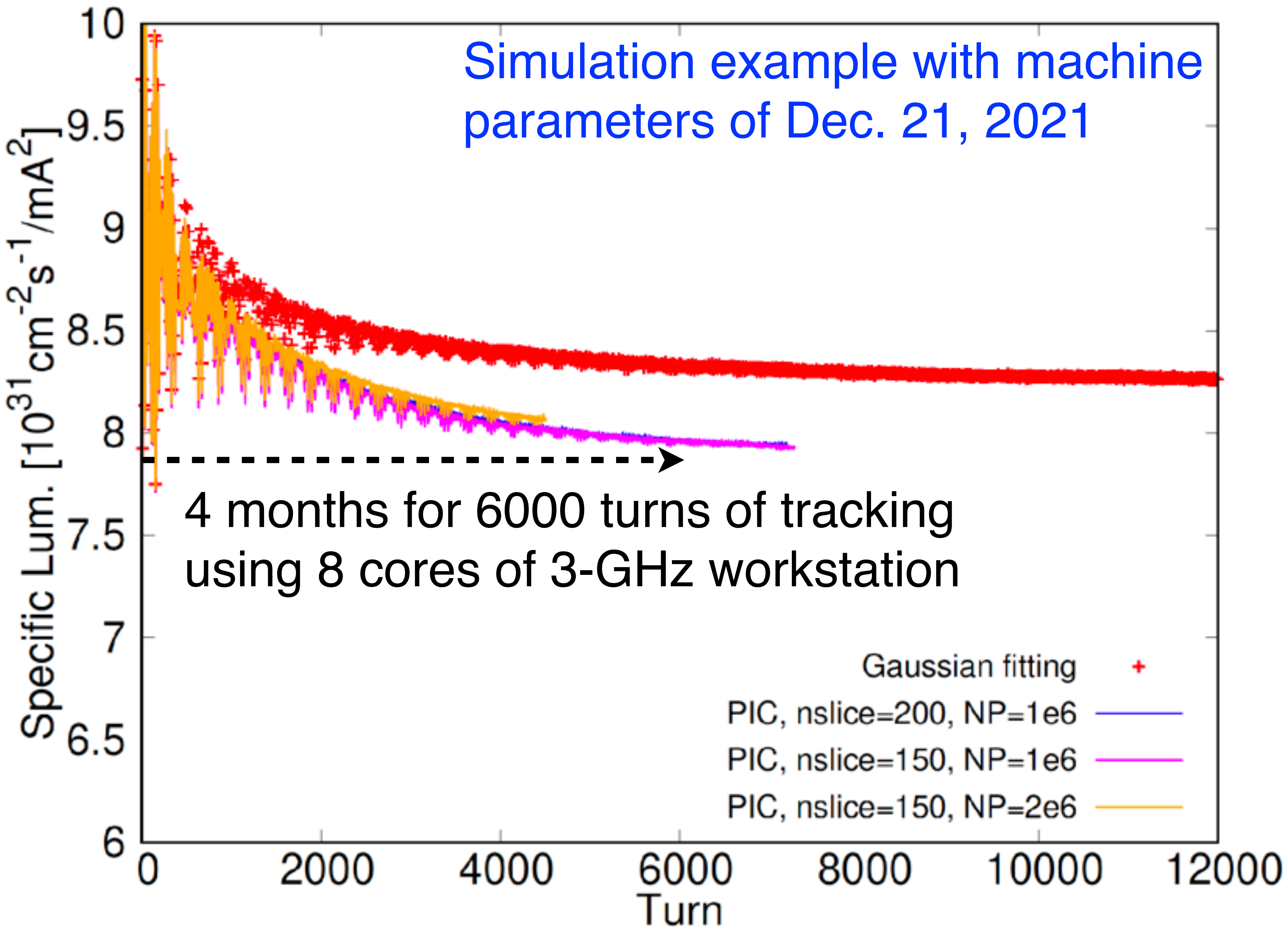
Abbreviations:
BB: Beam-beam
LN: Lattice nonlinearity
Z: Impedance
FB: Feedback
CW: Crab waist
SC: Space charge
X: Unknown factors

This schematic plot shows my private viewpoint

Status of beam-beam simulations

- Beam-beam (BB) simulations
  - Tools under development: SCTR-CUDA (K. Ohmi, BBSS + complete lattice + GPU acceleration); APES (Z. Li, Y. Zhang (IHEP), IBB + complete lattice + GPU acceleration); XSUITE (P. Kicsiny, X. Buffat (EPFL/CERN)).
  - The ultimate goal: PIC SS BB model + complete lattice + GPU acceleration (CPU-based parallel computing is not optimal for BB simulations).
  - Progress has been achieved in developing GPU-based BB codes. Preliminary tests showed a speed-up factor of ~25 for PIC BB simulations based on the CUDA compiler (K. Ohmi, in collaboration with Z. Li and Y. Zhang (IHEP), T. Yasui (J-PARC)).
- International collaboration
  - SuperKEKB, CEPC, and FCC-ee teams are working in the same direction: SS BB + multiple physics (US: BNL+SLAC are interested in joining).
  - We invite full international collaboration on beam-beam simulation and related physics.

$$L_{sp} \approx \frac{1}{2\pi e^2 f \underbrace{\sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}}_{\text{“Vertical blowup”}} \underbrace{\sqrt{\sigma_{z+}^{2} + \sigma_{z-}^{2}} \tan\frac{\theta_c}{2}}_{\text{“Longitudinal blowup”}}}$$



Simulation example with machine parameters of Dec. 21, 2021

4 months for 6000 turns of tracking using 8 cores of 3-GHz workstation

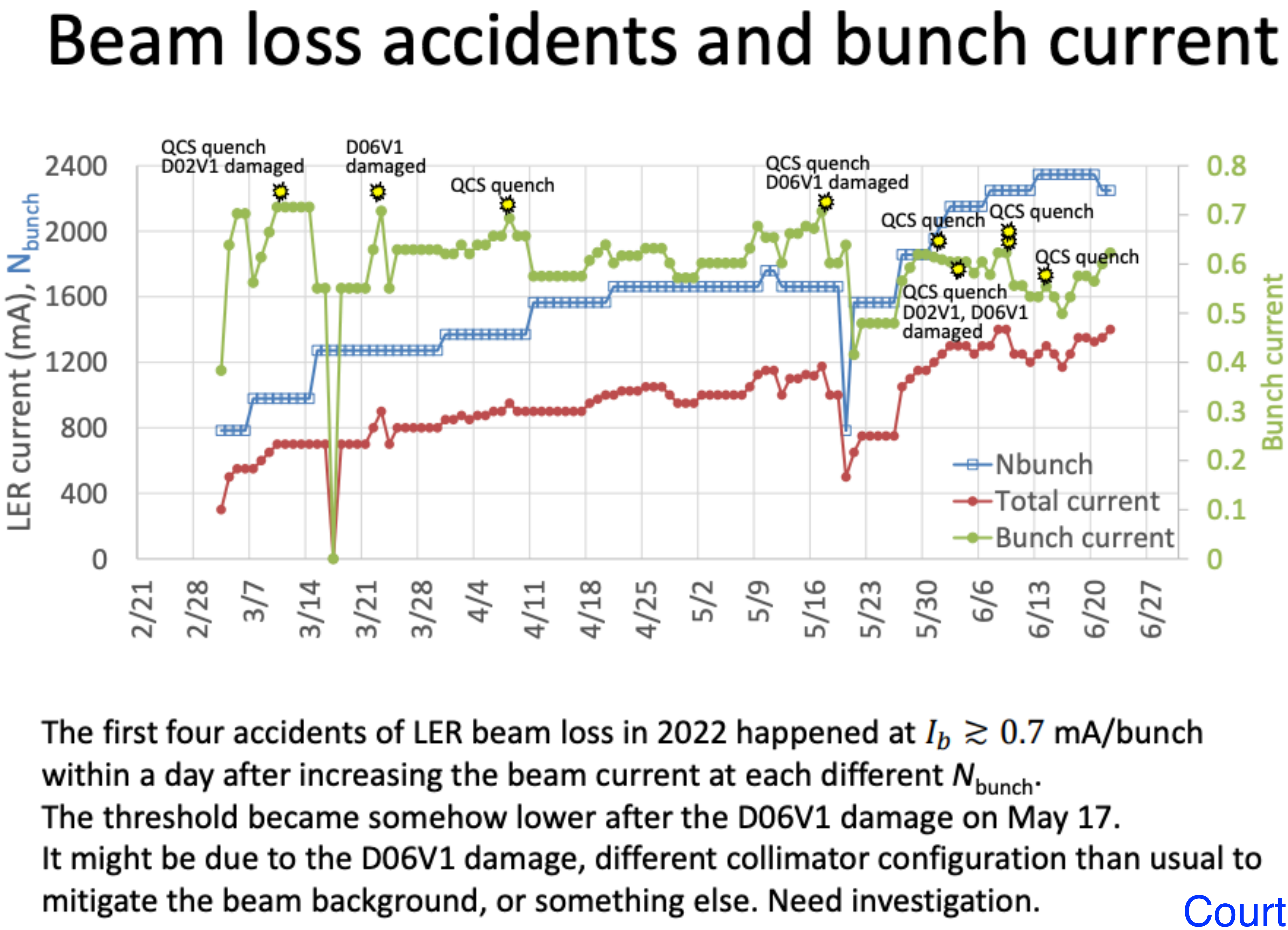
Limitations on current performance of SuperKEKB

- From the beam-beam perspective, we list some important issues:
  - Issue 1: Limits on bunch currents (see talks by Y. Ohnishi, H. Nakayama, H. Ikeda, and T. Ishibashi)
  - Issue 2: Multi-bunch effects
  - Issue 3: Optics distortion at high beam currents (see talks by Y. Ohnishi and H. Sugimoto)
  - Issue 4: Impedance effects (see K. Ohmi's talk)
  - Issue 5: Lsp injection correlation (see K. Matsuoka's talk)

$$\overset{\text{\#5}}{\boxed{L}} \approx \frac{\overset{\text{\#1,2,3,4,5}}{\boxed{N_b N_+ N_-}}\, f}{2\pi \sqrt{\underset{\text{\#1,2,3,4,5}}{\boxed{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}}}\, \sqrt{\underset{\text{\#4}}{\boxed{\sigma_{z+}^{2}+\sigma_{z-}^{2}}}}\, \tan \underset{\text{BB, CW, …}}{\boxed{\frac{\theta_c}{2}}}}\, e^{-\frac{\overset{\text{\#2,5}}{\boxed{\Delta^2}}}{2(\sigma_{y+}^{*2}+\sigma_{y-}^{*2})}}$$

Issue-1: Limit on bunch currents by Sudden Beam Losses (SBLs)

- Severe machine failures occurred at high beam currents when $I_{b+} > 0.7$ mA/bunch
- Bunch current $I_{b+} \lesssim 0.7$ mA (keeping $I_{b-}/I_{b+} = 0.8$) was respected in 2022ab run [1]



Beam loss accidents and bunch current

The first four accidents of LER beam loss in 2022 happened at $I_b \gtrsim 0.7$ mA/bunch within a day after increasing the beam current at each different N_{bunch}.
The threshold became somehow lower after the D06V1 damage on May 17.
It might be due to the D06V1 damage, different collimator configuration than usual to mitigate the beam background, or something else. Need investigation.

$$L \approx \frac{\boxed{N_b N_+ N_-} f}{2\pi\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^2+\sigma_{z-}^2}\tan\frac{\theta_c}{2}} e^{-\frac{\Delta^2}{2(\sigma_{y+}^{*2}+\sigma_{y-}^{*2})}}$$

For SBLs, see H. Ikeda's talk.

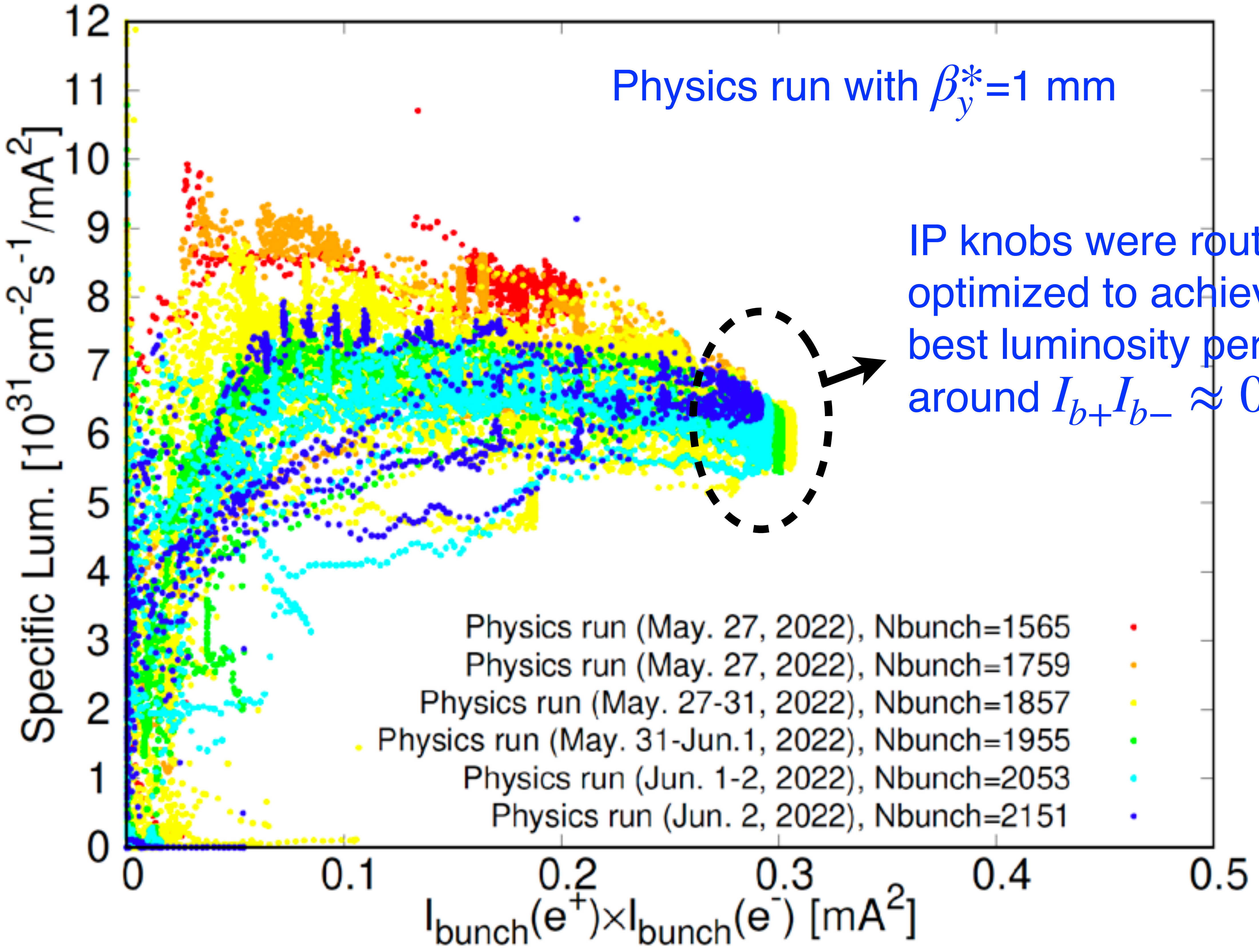
Courtesy of K. Matsuoka

[1] K. Matsuoka, "Belle II Report", SuperKEKB 2022ab summary meeting, https://kds.kek.jp/event/42954/.

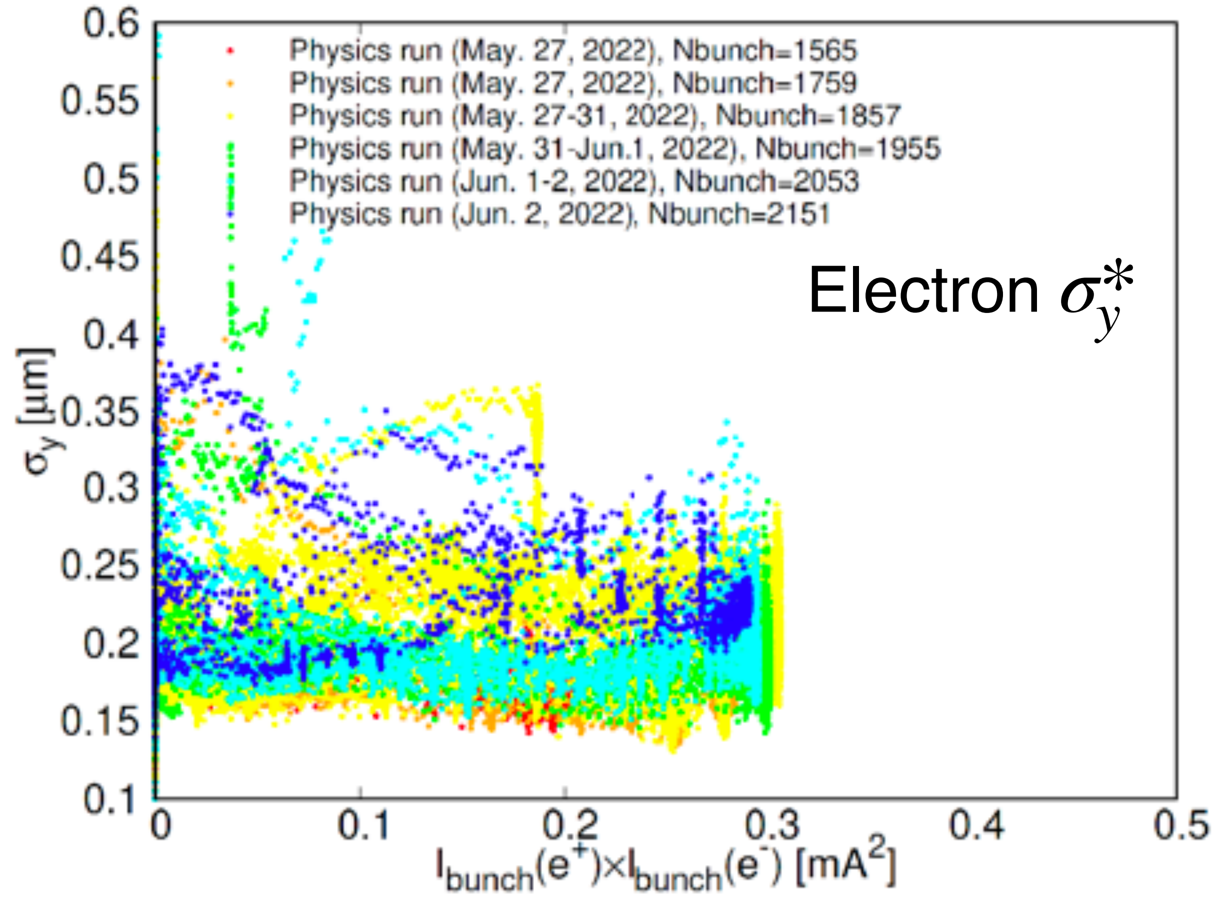
Issue-2: Multi-bunch effects

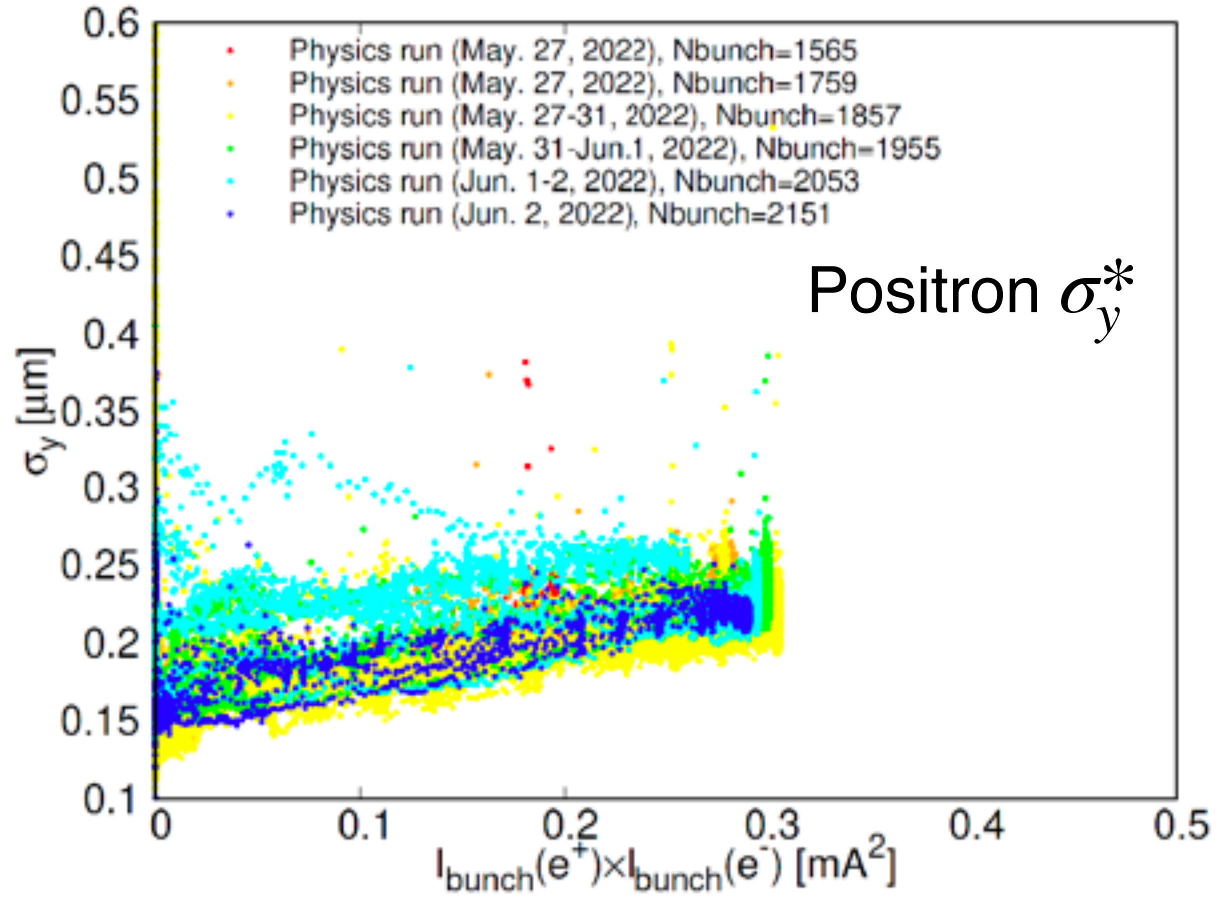
$$L \approx \frac{\boxed{N_b N_+ N_- f}}{2\pi\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^2+\sigma_{z-}^2}\tan\frac{\theta_c}{2}} e^{-\frac{\Delta^2}{2(\sigma_{y+}^{*2}+\sigma_{y-}^{*2})}}$$

- No clear evidence of Lsp degradation due to multi-bunch effects
  - Coupled-bunch instabilities were suppressed by the BxB FB system (M. Tobiyama).
  - Flat BxB luminosity was observed (S. Uehara).
  - Electron-cloud instability for e+ beam was not observed (Y. Suetsugu et al., see K. Shibata's talk).



IP knobs were routinely optimized to achieve the best luminosity performance around $I_{b+}I_{b-} \approx 0.3$ mA^2.



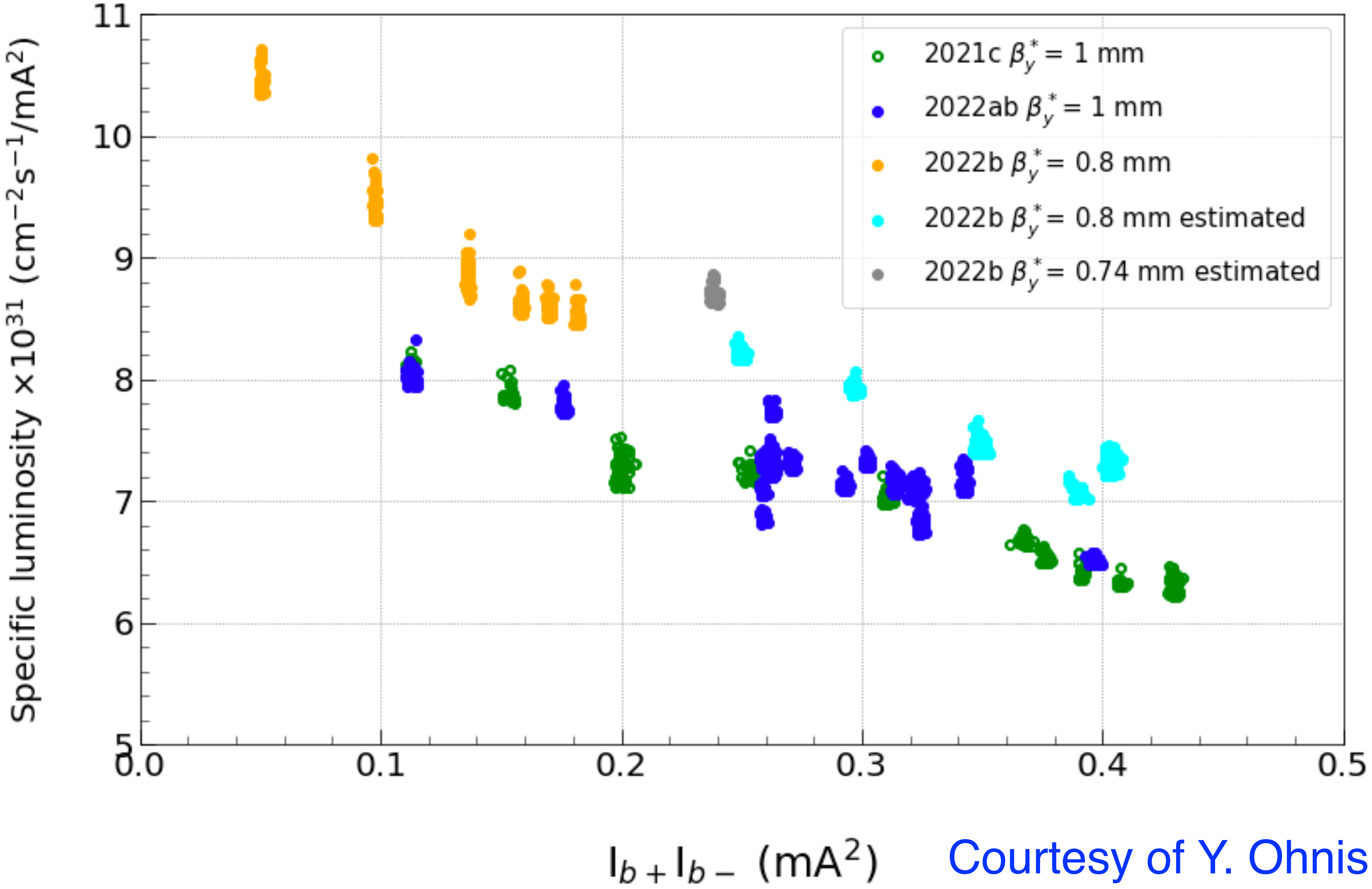


Issue-3: Optics distortion at high beam currents

$$L \approx \frac{\boxed{N_b N_+ N_- f}}{2\pi\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^{2}+\sigma_{z-}^{2}}\tan\frac{\theta_c}{2}}e^{-\frac{\Delta^2}{2(\sigma_{y+}^{*2}+\sigma_{y-}^{*2})}}$$

- Current-dependent optics distortion
  - Beta-beat and global coupling become worse at high currents.
  - An unexpected β_y^* squeeze explains the Lsp gain (see Y. Ohnishi's talk).

$$\xi_{y+}^{i} \approx \frac{r_e}{2\pi\gamma_+}\frac{N_-\boxed{\beta_{y+}^{*}}}{\sigma_{y-}^{*}\sqrt{\sigma_{z-}^{2}\tan^2\frac{\theta_c}{2}+\sigma_{x-}^{*2}}}$$

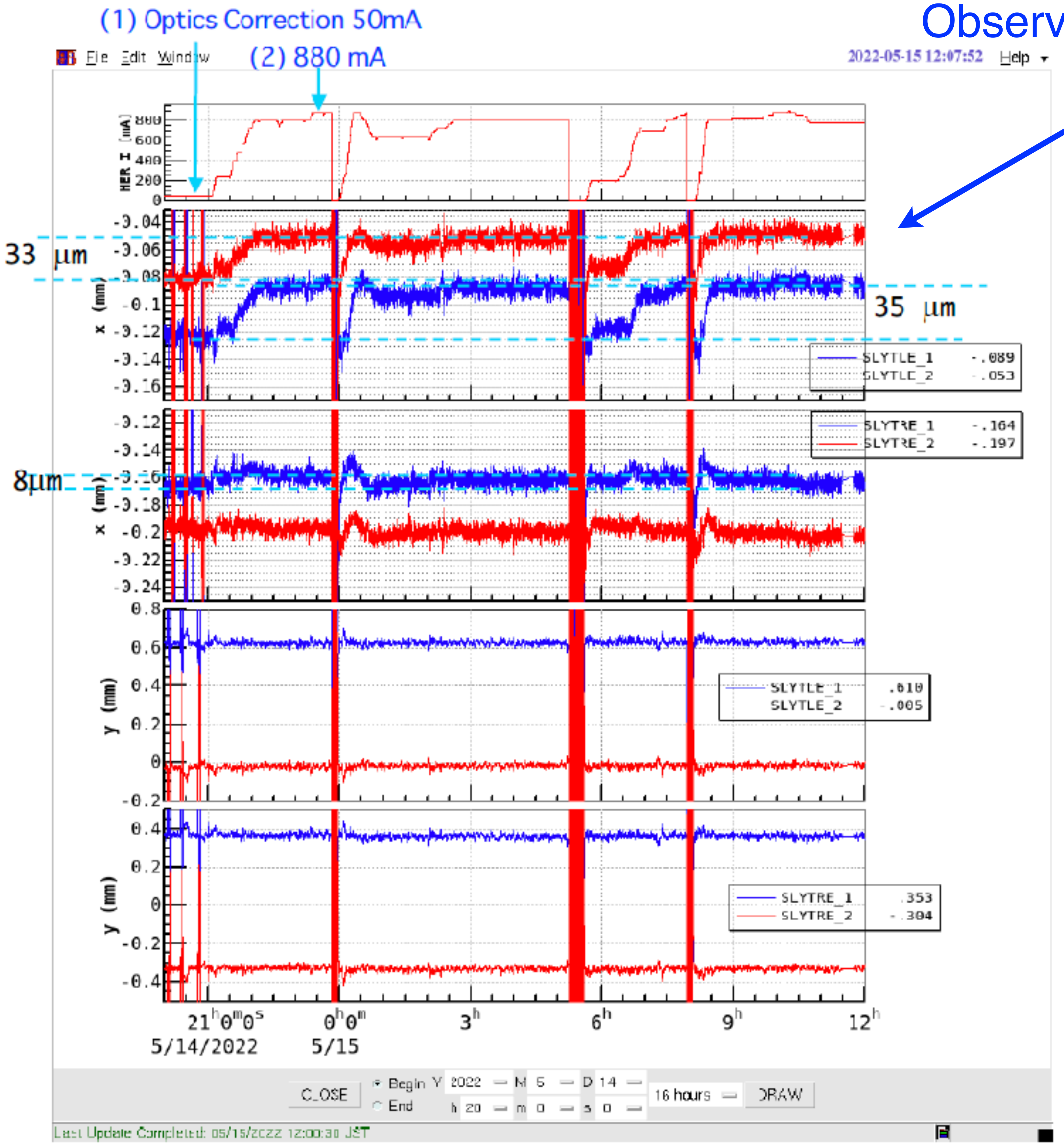


Courtesy of Y. Ohnishi

Issue-3: Optics distortion at high beam currents

$$\xi^i_{y+} \approx \frac{r_e}{2\pi\gamma_+}\frac{N_-\boxed{\beta^*_{y+}}}{\sigma^*_{y-}\sqrt{\sigma^2_{z-}\tan^2\frac{\theta_c}{2}+\sigma^{*2}_{x-}}}$$

- Current-dependent orbit offsets at SLY* magnets (see H. Sugimoto's talk)

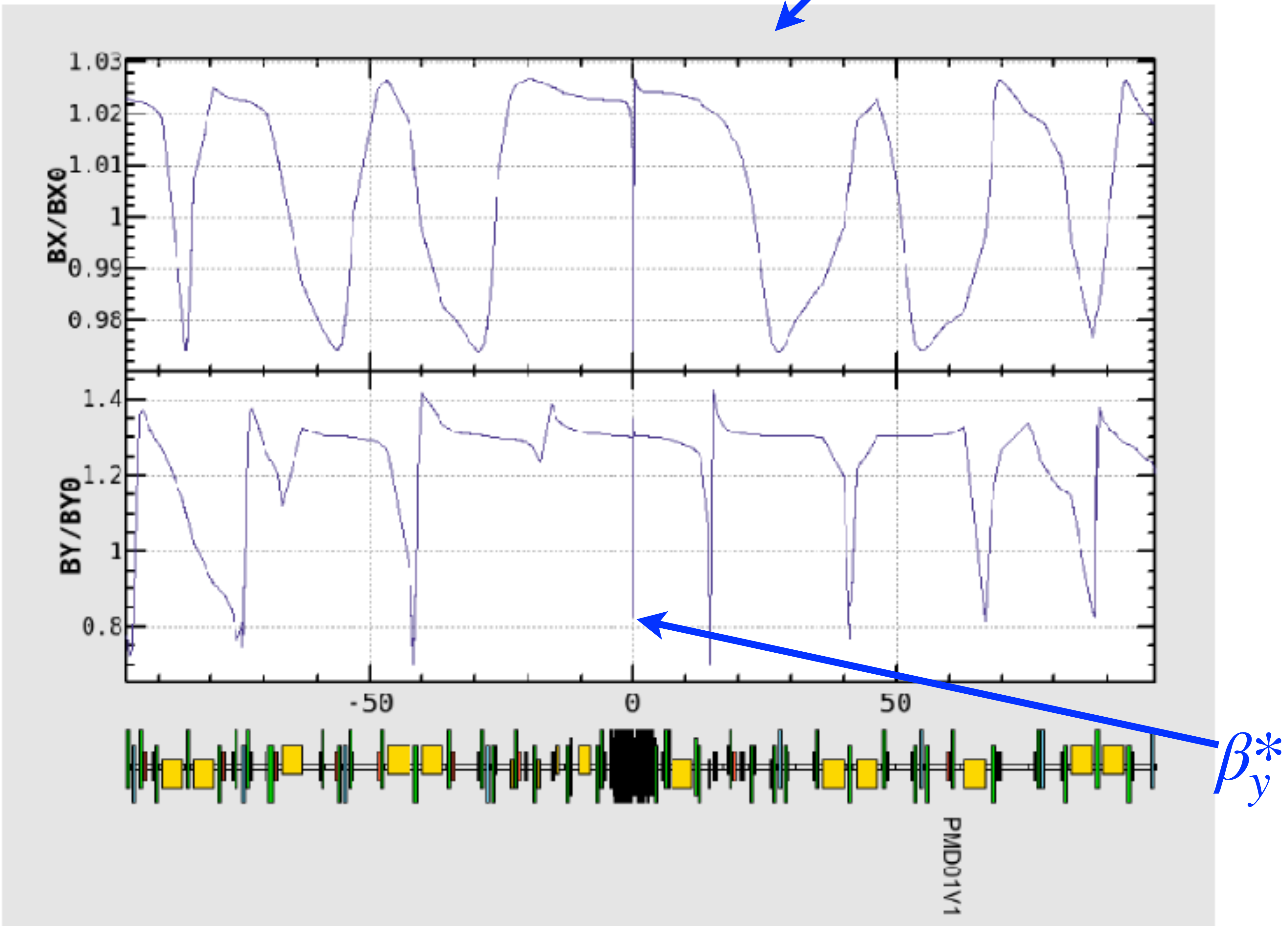


Observed offsets with beam

The following horizontal offsets of SLY* magnets are used to estimate the beta beats:

SLYTRE1	-8 µm
SLYTRE2	0
SLYTLE1	-35 µm
SLYTLE2	-33 µm

Resulting beta-beat (SAD simulation)



β^*_y squeezed

Courtesy of H. Koiso

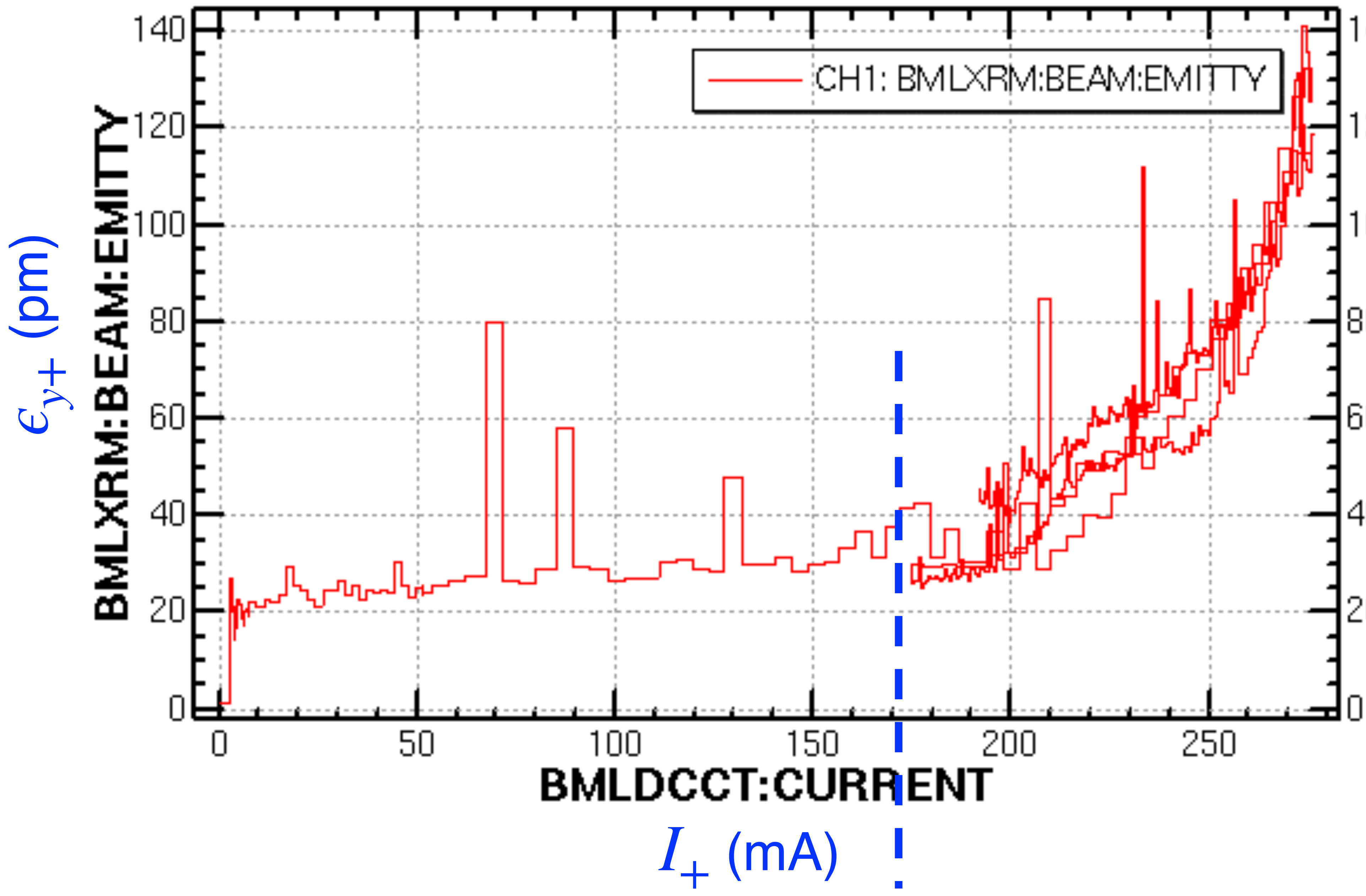
Issue-4: Impedance effects (LER)

$$L \approx \frac{N_b N_+ N_{-} f}{2\pi\sqrt{\boxed{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}}\sqrt{\sigma_{z+}^2 + \sigma_{z-}^2}\tan\frac{\theta_c}{2}} e^{-\frac{\Delta^2}{2(\sigma_{y+}^{*2} + \sigma_{y-}^{*2})}}$$

- Current-dependent single-beam blowup in LER
  - This problem was solved by fine-tuning the FB system in Mar. 2022. After new damage to collimators (D06V1 and D02V1), the LER beam blowup problem re-appeared.
  - On Jun. 21, 2022, tunings were done to improve the blowup threshold (from 0.5 mA/bunch to ~0.87 mA/bunch). This contributed to achieving the luminosity record 4.71×10^{34} cm^{-2}s^{-1} on Jun. 22, 2022.

Machine conditions:
Single-beam, 393 bunches

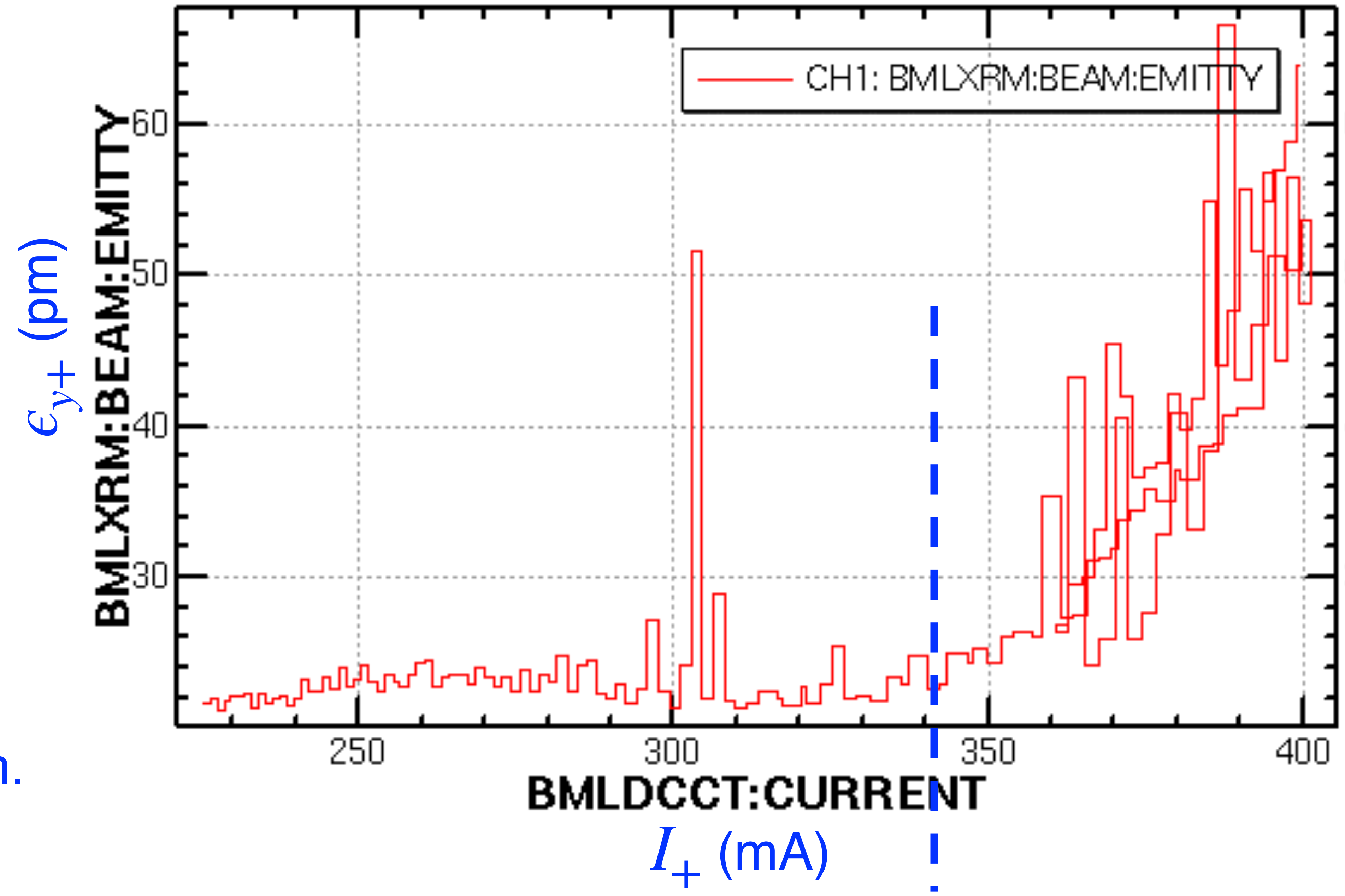
KCG shift report on LER vertical blowup study
By S. Terui, T. Ishibashi, K. Yoshihara, M. Nishiwaki
Jun. 21, 2022



Threshold <0.5mA/bunch

LER vertical tune:
ν_y .582 → .595;

D06V1 gap:
TOP 4.2 mm,
BTM -5.1 → -8.0 mm.



After tunings ~0.87mA/bunch

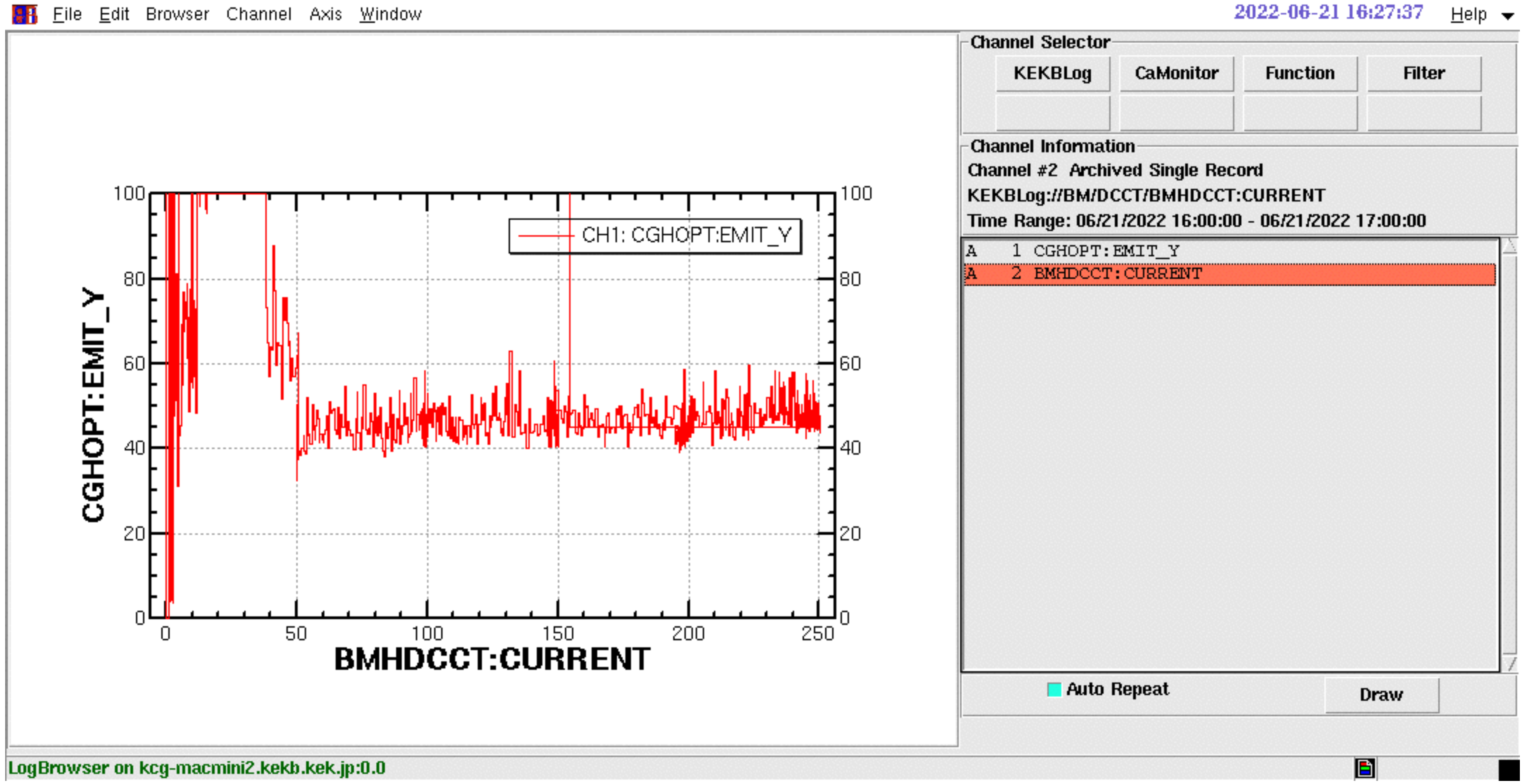
Issue-4: Impedance effects (HER)

$$L \approx \frac{N_b N_+ N_- f}{2\pi \boxed{\sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}} \sqrt{\sigma_{z+}^2 + \sigma_{z-}^2} \tan\frac{\theta_c}{2}} e^{-\frac{\Delta^2}{2(\sigma_{y+}^{*2} + \sigma_{y-}^{*2})}}$$

- Current-dependent single-beam vertical emittance in HER
  - No clear evidence of single-beam blowup (up to 0.64 mA/bunch) in HER

Machine conditions:
Single-beam, 393 bunches

KCG shift report on high bunch-current collision study
By D. Zhou, R. Ueki, M. Nishiwaki
Jun. 21, 2022



Channel Information
Channel #2 Archived Single Record
KEKBLog://BM/DCCT/BMHDCCT:CURRENT
Time Range: 06/21/2022 16:00:00 - 06/21/2022 17:00:00

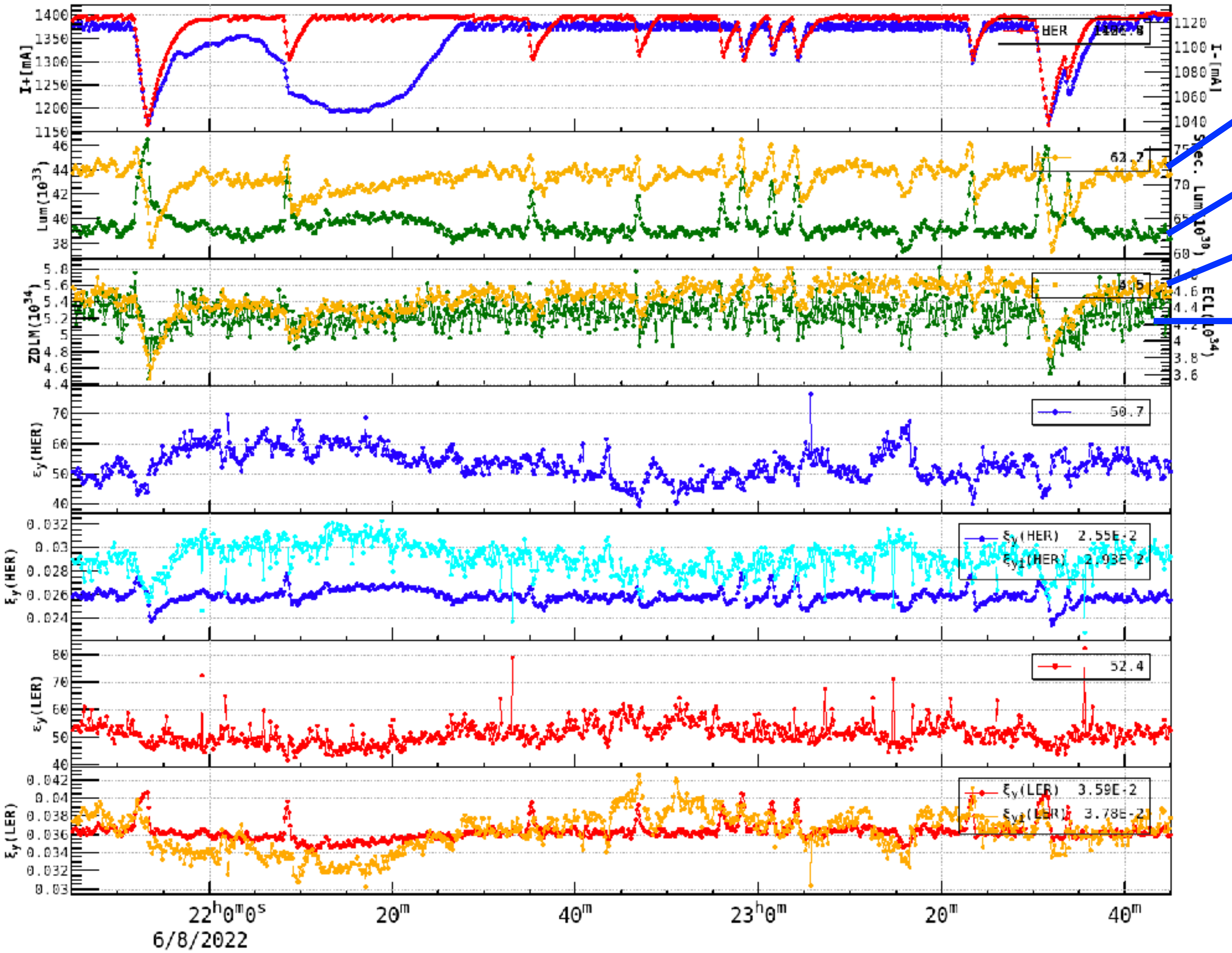
```
A   1 CGHOPT:EMIT_Y
A   2 BMHDCCT:CURRENT
```

LogBrowser on kcg-macmini2.kekb.kek.jp:0.0

Issue-5: Lsp-Injection correlation

$$\boxed{L} \approx \frac{N_b N_+ N_{-} f}{2\pi\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^{2}+\sigma_{z-}^{2}}\tan\frac{\theta_c}{2}}e^{-\frac{\Delta^2}{2(\sigma_{y+}^{*2}+\sigma_{y-}^{*2})}}$$

- Luminosity record of 4.65×10^{34} cm^{-2}s^{-1} was achieved with Belle II HV ON when the injection was intentionally stopped (Jun. 8, 2022).



Yellow: Total luminosity ECL (20-second average)

Green: Specific luminosity by ECL (Lsp)

Yellow: Total luminosity ZDLM

Green: Total luminosity ECL (updated per 2.5 second)

* Peak luminosity always appears after injection stopped

* Lsp always jump up after injection stopped

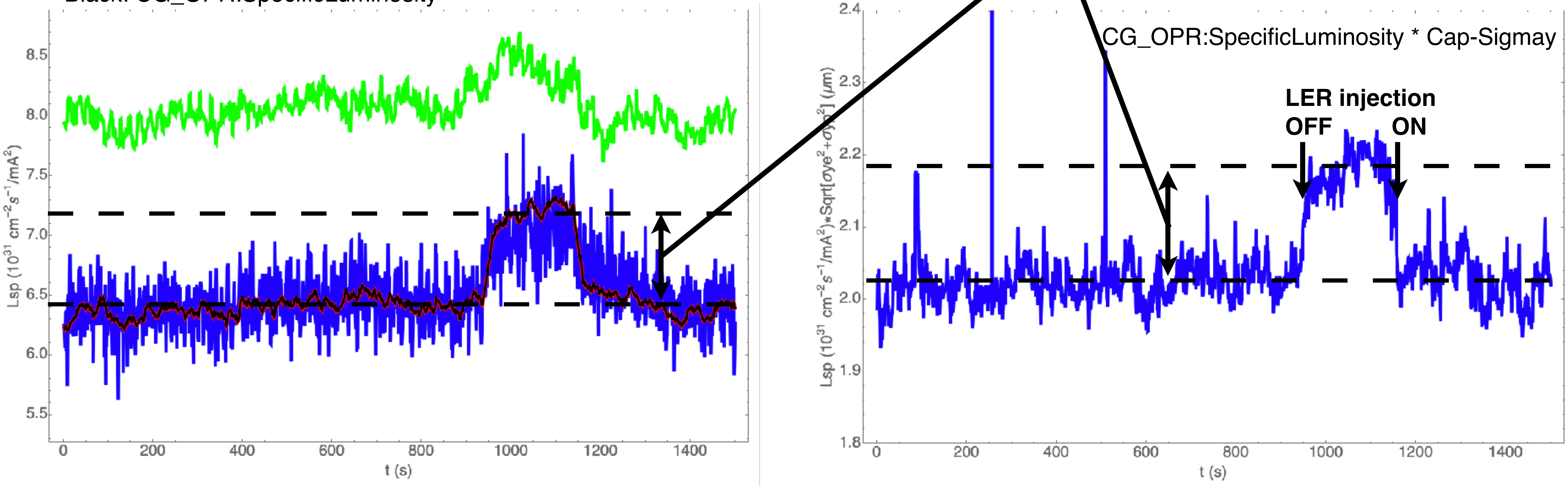
Issue-5: Lsp-Injection correlation

$$\boxed{L_{sp}} \approx \frac{1}{2\pi e^2 f\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^{2}+\sigma_{z-}^{2}}\,\tan\frac{\theta_c}{2}}e^{-\frac{\boxed{\Delta^2}}{2(\sigma_{y+}^{*2}+\sigma_{y-}^{*2})}}$$

- The phenomenon: 2022-06-02 21:05 PM
  - All luminosity PVs gave a similar jump response to injection stop/start.
  - $L_{sp}\cdot\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}$ still shows jump-response. It means there is a geometric loss of luminosity.

Blue: B2_nsm:get:ECL_LUM_MON:lum_acc_corrected
Red: B2_nsm:get:ECL_LUM_MON:lum_acc_20
Green: B2_nsm:get:MONZDLMINT:ZDLM_INTVAL:value
Black: CG_OPR:SpecificLuminosity

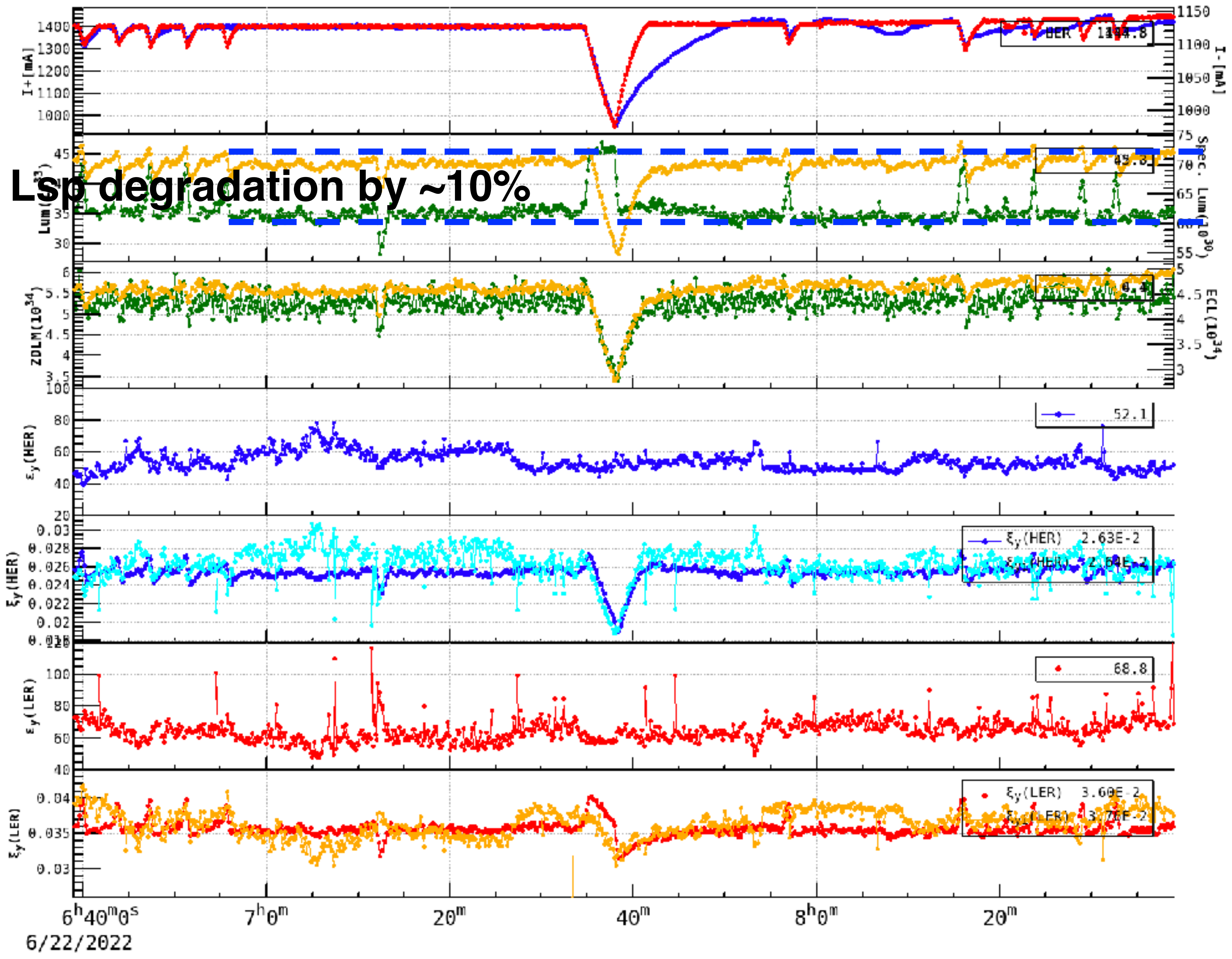
Lsp degradation by ~10%, independent to vertical emittances



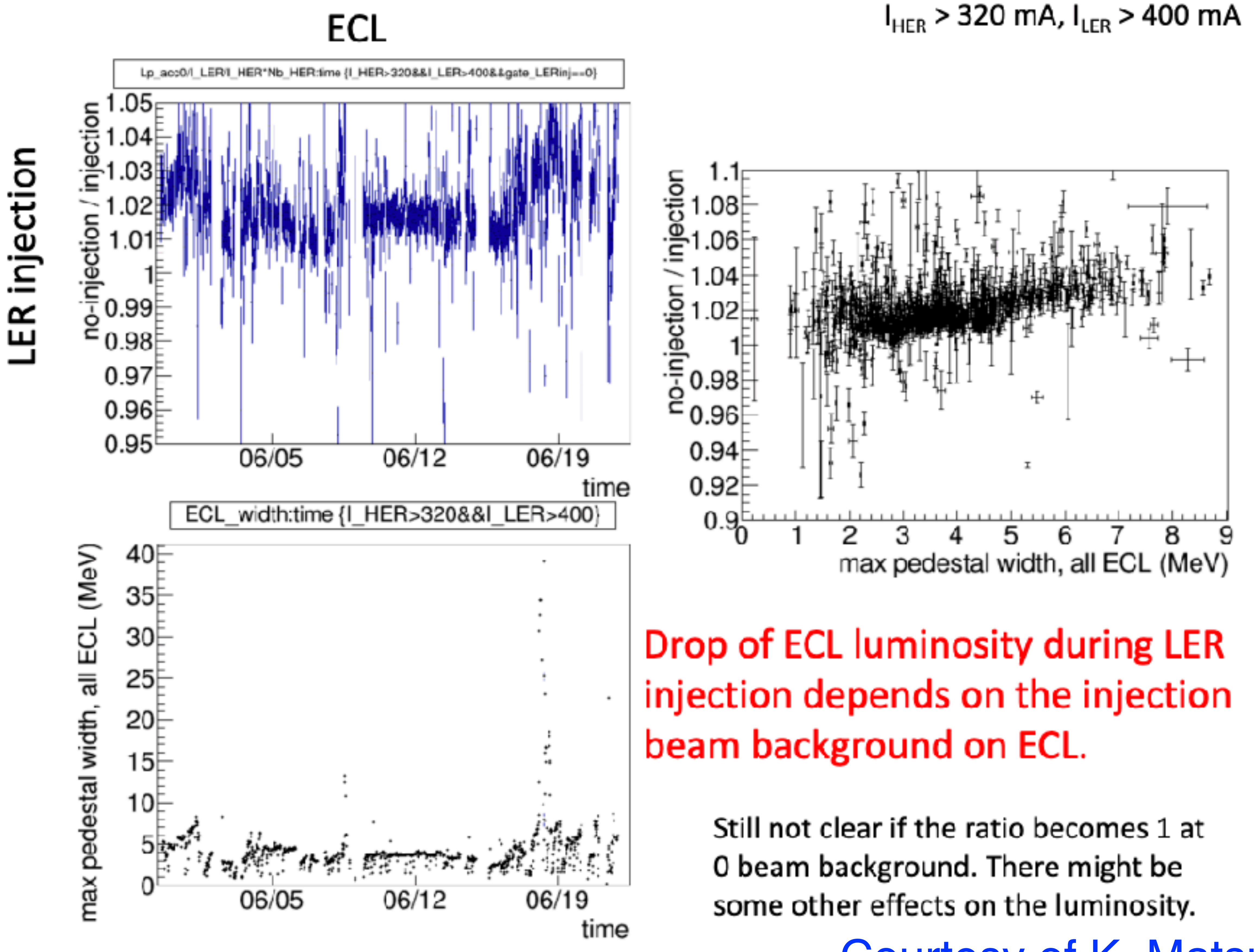
Issue-5: Lsp-Injection correlation

$$\boxed{L_{sp}} \approx \frac{1}{2\pi e^2 f\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^2+\sigma_{z-}^2}\tan\frac{\theta_c}{2}}e^{-\frac{\Delta^2}{2(\sigma_{y+}^{*2}+\sigma_{y-}^{*2})}}$$

- Injection background affected ECL luminosity [1]
- Data of Jun. 2022: Injection background contributed to ~5% luminosity “loss”



Lsp degradation by ~10%



Drop of ECL luminosity during LER injection depends on the injection beam background on ECL.

Still not clear if the ratio becomes 1 at 0 beam background. There might be some other effects on the luminosity.

Courtesy of K. Matsuoka

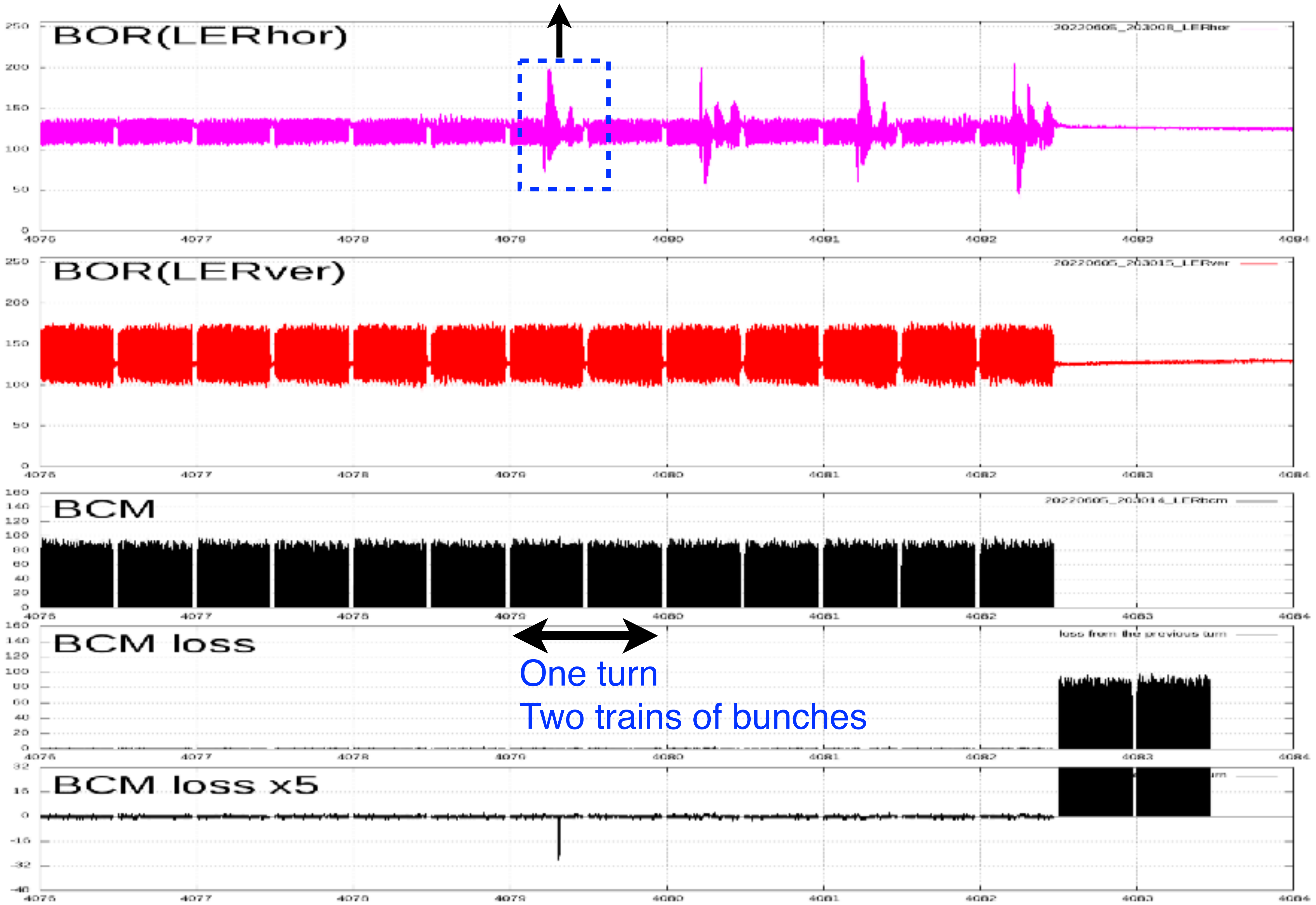
[1] K. Matsuoka, Oct. 14, 2022, https://kds.kek.jp/event/44070/.

Issue-5: Lsp-Injection correlation

$$L_{sp} \approx \frac{1}{2\pi e^2 f \sqrt{\boxed{\sigma^{*2}_{y+}+\sigma^{*2}_{y-}}}\sqrt{\sigma^2_{z+}+\sigma^2_{z-}}\tan\frac{\theta_c}{2}}\, e^{-\frac{\boxed{\Delta^2}}{2(\sigma^{*2}_{y+}+\sigma^{*2}_{y-})}}$$

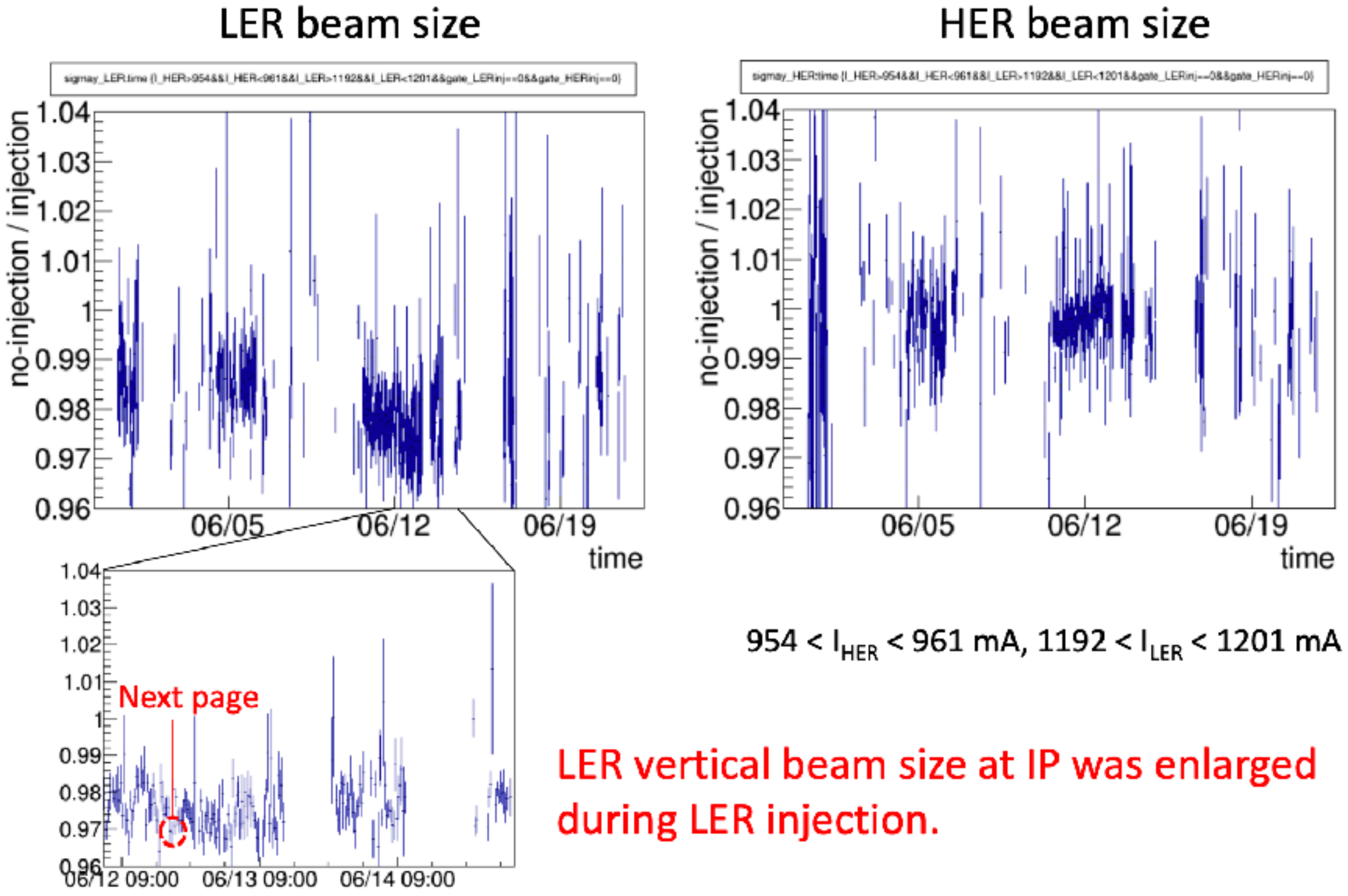
- Injection background on ECL was identified [1]
- Data of Jun. 2022: LER injection kicker (leakage fields) contributed to ~3% of luminosity loss

About 20% of the stored bunches are excited by the leakage fields of injection kickers in LER



One turn
Two trains of bunches

Vertical beam size at IP measured by XRM



954 < I_{HER} < 961 mA, 1192 < I_{LER} < 1201 mA

LER vertical beam size at IP was enlarged during LER injection.

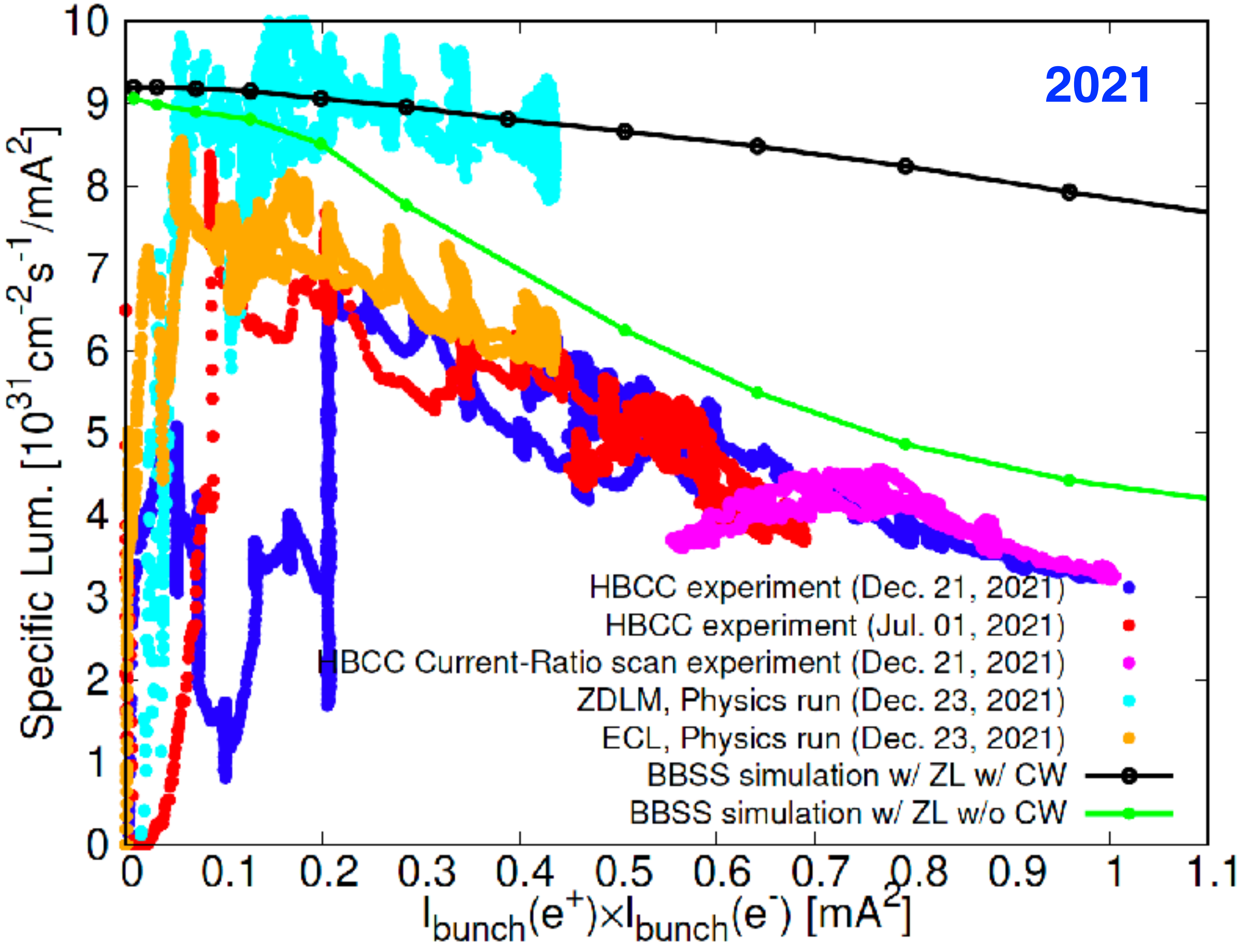
Courtesy of K. Matsuoka

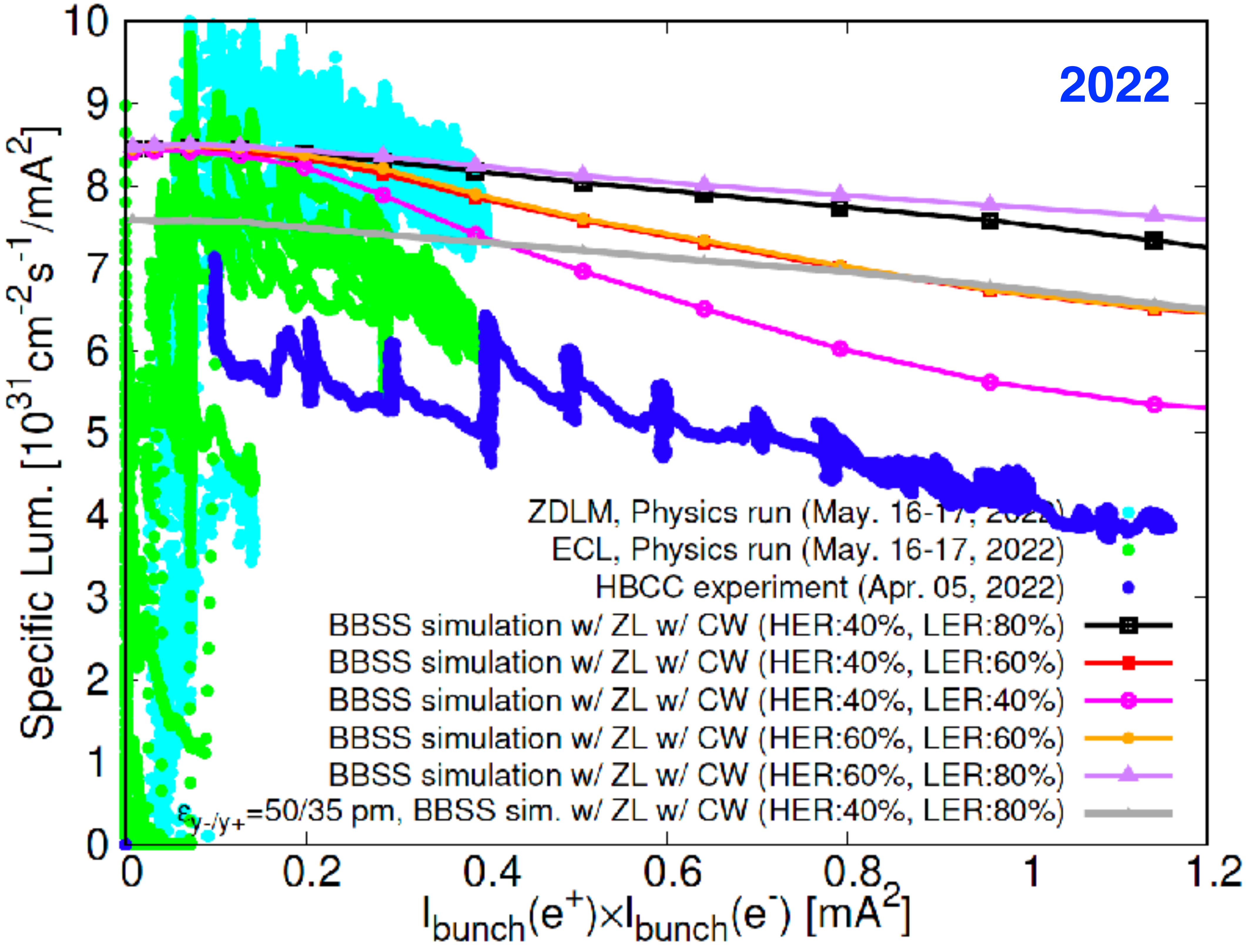
[1] K. Matsuoka, Oct. 14, 2022, https://kds.kek.jp/event/44070/.

Issue-5: Lsp-Injection correlation

$$\boxed{L_{sp}} \approx \frac{1}{2\pi e^2 f \sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}} \sqrt{\sigma_{z+}^2 + \sigma_{z-}^2} \tan \frac{\theta_c}{2}} e^{-\frac{\Delta^2}{2(\sigma_{y+}^{*2} + \sigma_{y-}^{*2})}}$$

- Lsp with $\beta_y^* = 1$ mm in 2021 and 2022:
  - The fast drop of measured Lsp vs. $I_{b+}I_{b-}$: ECL data of the physics run shows a slightly faster drop than ZDLM data.
  - It is consistent with K. Matsuoka's analysis: Injection background affects ECL luminosity more than ZDLM.





ZDLM gives relative luminosity, while ECL gives absolute luminosity.

Beam-beam perspective on achieving target luminosity

- Achieving 10^{35} cm^{-2}s^{-1}: SBLs, “-1 mode instability”, etc. → Non-Linear Collimator (NLC)
- Achieving 6×10^{35} cm^{-2}s^{-1}: DA (Dynamic aperture), lifetime, perfect CW, etc. → IR model (better understanding of the current IR) and upgrade (“Clean IR”)

$$L = \frac{1}{2er_e}\frac{\gamma_\pm I_\pm}{\beta^*_{y\pm}}\xi^L_{y\pm}$$

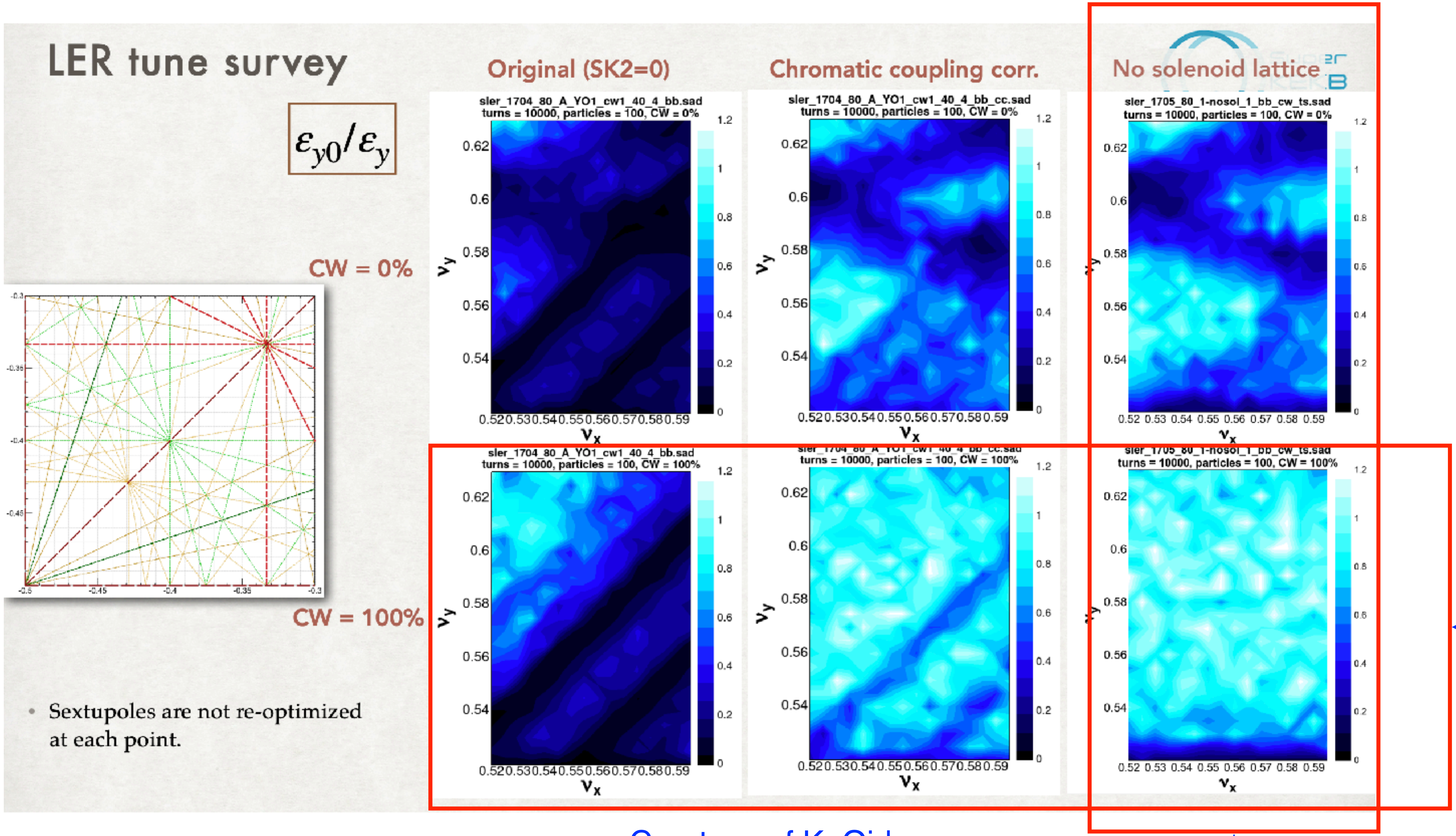
Luminosity measurement:
1) Fake luminosity loss in ECL
This is not a problem at all

Total beam currents:
We achieved 1.4 A in LER (Jun. 2022)
If we can achieve 3.6 A, we will gain by 2.5
Obstacles:
1) Sudden beam losses (SBLs)
2) Short lifetime (challenging injection power)

IR optics:
We achieved $\beta_y^* = 1$ mm
If we can achieve $\beta_y^* = 0.3$ mm, we will gain by 3.3
Obstacles:
1) DA and lifetime resulted from IR nonlinearity (+BB+CW)
2) Optics tuning at high currents

Beam-beam limit:
We achieved 0.04 in Jun. 2022
We expect the upper limit is ~0.1 (including the hourglass effect), then we will gain by 2.5
Obstacles:
1) Vertical blowup by “-1 mode instability” (NLC is the hoped solution)
2) Vertical blowup by BB (+Lattice nonlinearity+Impedance)
3) Imperfect crab waist (to be verified)

Beam-beam perspective on achieving target luminosity



LER tune survey

$\varepsilon_{y0}/\varepsilon_y$

- Sextupoles are not re-optimized at each point.

← **Crab waist**

Courtesy of K. Oide

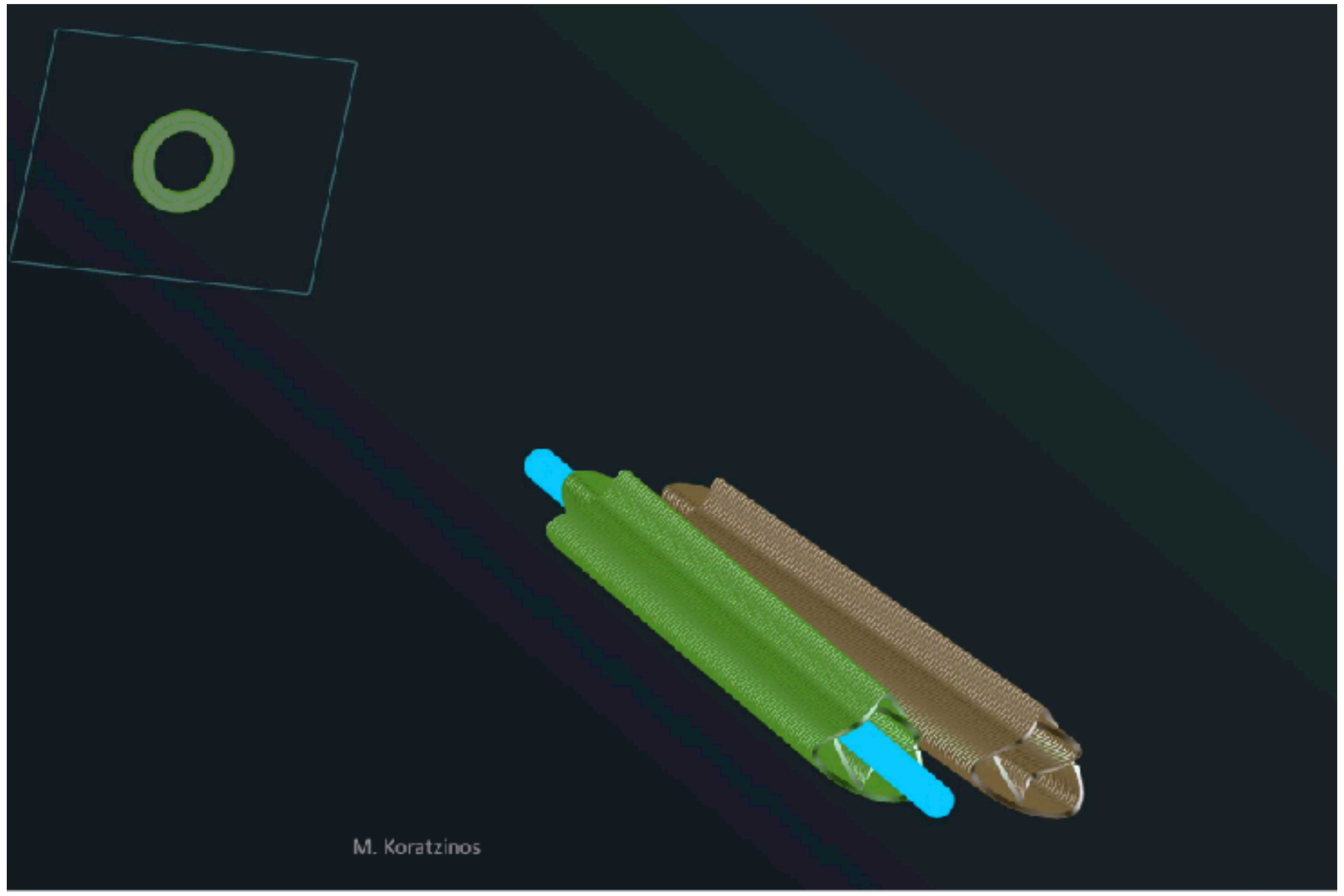
↑ **“Clean IR”**

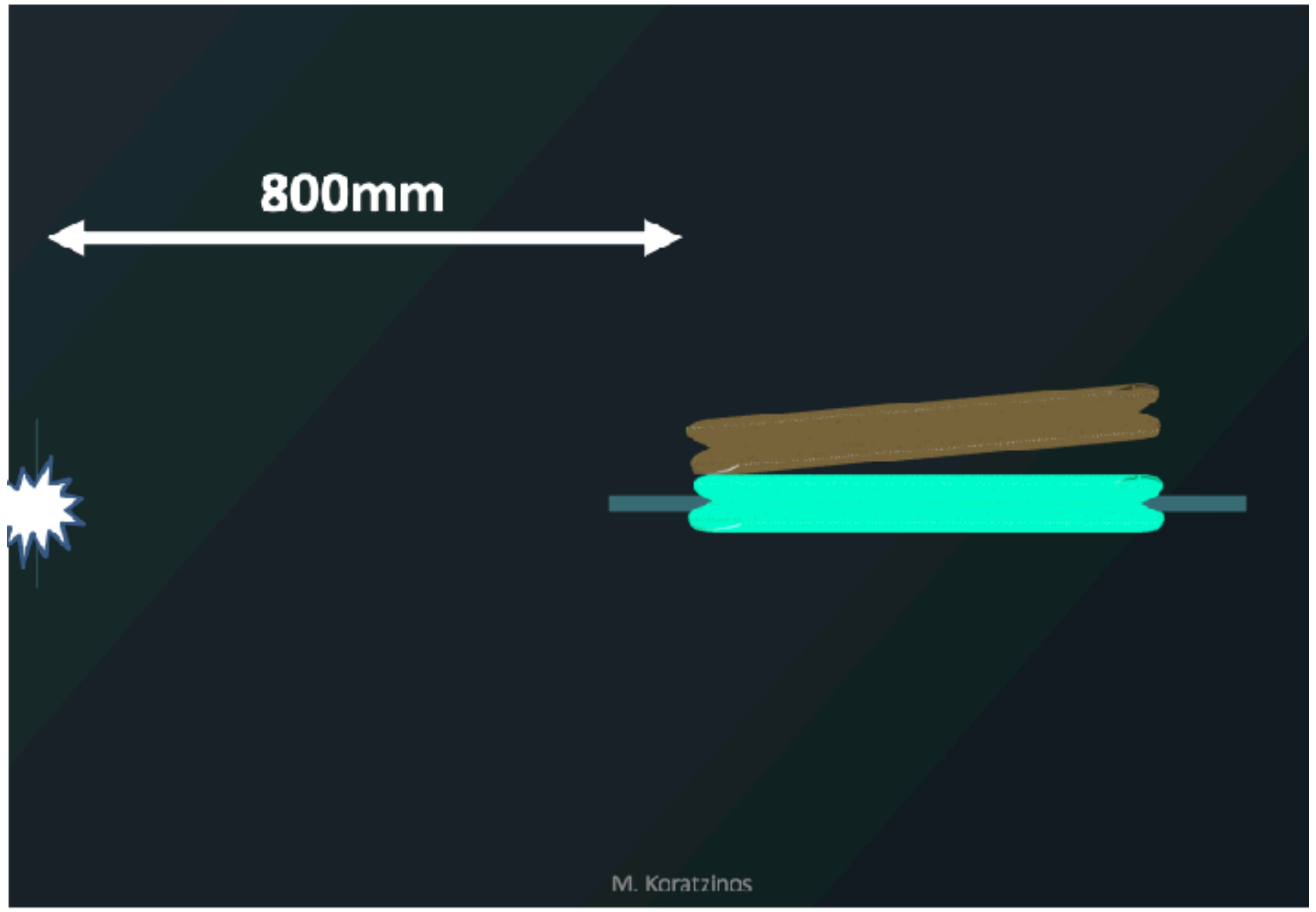
[1] K. Oide, https://kds.kek.jp/event/44644/.

“Clean IR”: A transparent IR with minimal amplitude-dependent and chromatic nonlinearities

Beam-beam perspective on achieving target luminosity

- How to achieve a “clean IR”
  - IR remodeling (the mainstream upgrade plan (see M. Masuzawa’s talk) under investigation)
  - Using CCT (Canted Cosine Theta) magnets: M. Koratzinos did the first exercise (considering constraints from the technology and infrastructure of SuperKEKB) and showed encouraging results. Using the CCT magnets, a compact and cleaner IR is conceivable (Idea: “The current distribution of any canted layer generates a pure harmonic field as well as a solenoid that can be canceled with a similar but oppositely canted layer.” [2]).





Courtesy of M. Koratzinos

  - From the beam-beam perspective, we invite full international collaboration on IR upgrades to achieve the target luminosity of SuperKEKB.

[1] M. Koratzinos, https://kds.kek.jp/event/44644/. [2] S. Caspi et al., “Canted-Cosine-Theta magnet (CCT)-A concept for high field accelerator magnets” , IEEE Trans. Appl. Supercond. 24, 1. (2014).

Summary

- With progress in machine tuning, the measured luminosity of SuperKEKB is approaching predictions of BB simulations (SS BB + Simple lattice model + Impedance models).
- Prediction of luminosity via beam-beam simulations requires reliable models of multiple dynamics, such as the beam-beam interaction, machine imperfections, impedance models, etc.
- Several sources of luminosity degradation in the current SuperKEKB have been well identified.
- Many subjects will be investigated via experiments (after LS1) and simulations.

- From the beam-beam perspective, with β_y^*=0.3 mm, a significant IR upgrade is required to achieve the target luminosity in SuperKEKB (after LS2).
- We invite full international collaboration on beam-beam simulations and an IR upgrade R&D program.

Backup

Luminosity

- Luminosity [1]:

$$L = N_+ N_- f_c K \int d^3\vec{x}\, ds_0\, \rho_+(\vec{x}, -s_0)\rho_-(\vec{x}, s_0)$$

- 3D Gaussian distribution:

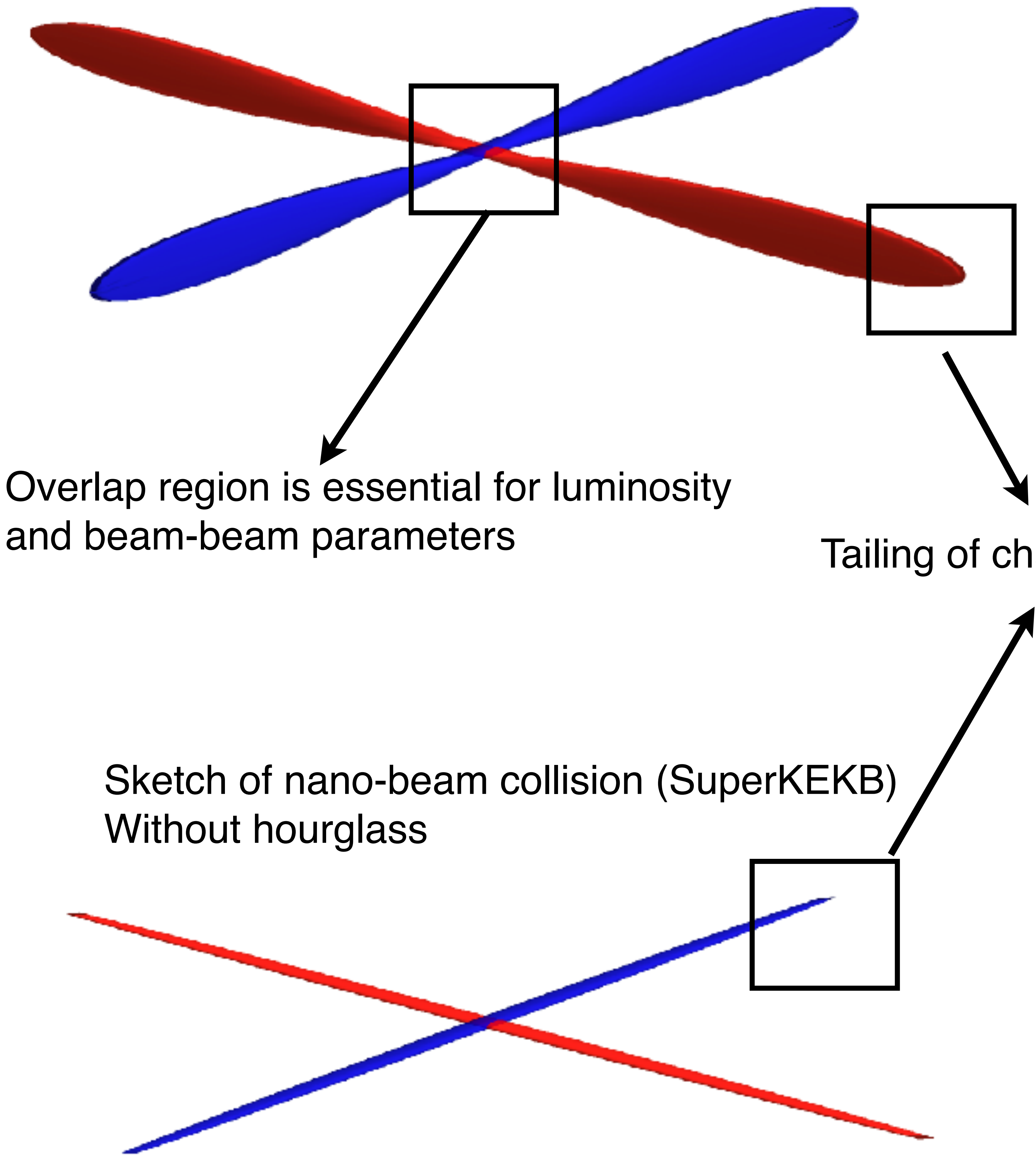
$$\rho(x, y, s, s_0) = \frac{e^{-\frac{x^2}{2\sigma_x^2(s)}-\frac{y^2}{2\sigma_y^2(s,x)}-\frac{(s-s_0)^2}{2\sigma_z^2}}}{(2\pi)^{3/2}\sigma_x(s)\sigma_y(s,x)\sigma_z}$$

- Hourglass effect:

$$\beta_{x,y}(s) = \beta^*_{x,y}\left(1 + s^2/\beta^{*2}_{x,y}\right) \qquad \sigma_{x,y}(s) = \sigma^*_{x,y}\sqrt{1 + s^2/\beta^{*2}_{x,y}}$$

- Crab waist:

$$\sigma_y(s, x) = \sigma^*_y\sqrt{1 + \left(s + R_{CW}x/\tan\theta_c\right)^2/\beta^{*2}_y}$$



Sketch of nano-beam collision (SuperKEKB)
With hourglass, without crab waist

Overlap region is essential for luminosity and beam-beam parameters

Tailing of charge density

Sketch of nano-beam collision (SuperKEKB)
Without hourglass

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[1] W. Herr and B. Muratori, Concept of luminosity, CAS lecture note (2006).

Luminosity

- Luminosity:

$$L = \frac{N_b I_{b+} I_{b-} R_{HC}}{2\pi e^2 f_0 \Sigma_x^* \Sigma_y^*} = L_0 R_{HC}$$

$$L_0 = \frac{N_b N_+ N_- f_c}{2\pi\sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}\sqrt{\sigma_{x+}^{*2} + \sigma_{x-}^{*2}}}$$

$$\Sigma_u^* = \sqrt{\sigma_{u+}^{*2} + \sigma_{u-}^{*2}}$$

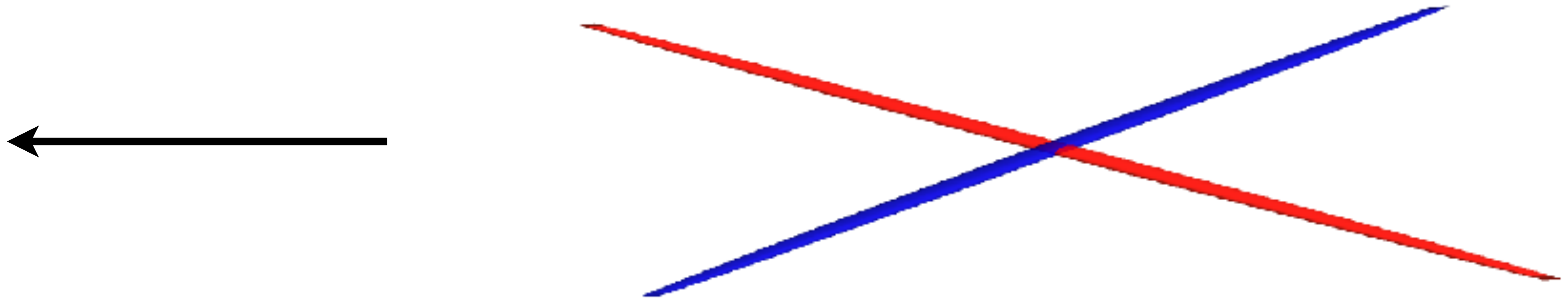
- R_{HC} is the geometric reduction factor including effects of crossing angle and hourglass
- If no hourglass effect, there is

$$R_{HC} = R_C = \frac{1}{\sqrt{1 + \frac{\sigma_{z+}^2 + \sigma_{z-}^2}{\sigma_{x+}^{*2} + \sigma_{x-}^{*2}} \tan^2 \frac{\theta_c}{2}}}$$

- R_C is defined as the reduction factor from the crossing angle
- The reduction factor from the hourglass effect can be defined as

$$R_H = R_{HC}/R_C \qquad L = L_0 R_H R_C = L_0' R_H$$

Sketch of nano-beam collision (SuperKEKB)
Without hourglass



Luminosity

- Assume no crab waist and flat beams ($\sigma_y^* \ll \sigma_x^*$); a very good approximation [1]:

$$R_{HC} \approx \sqrt{\frac{2}{\pi}} a e^{b} K_0(b)$$

$$a = \frac{\Sigma_y^*}{\Sigma_z \Sigma_\beta^*} \qquad b = a^2 \left(1 + \frac{\Sigma_z^2}{\Sigma_x^{*2}} \tan^2 \frac{\theta_c}{2}\right)$$

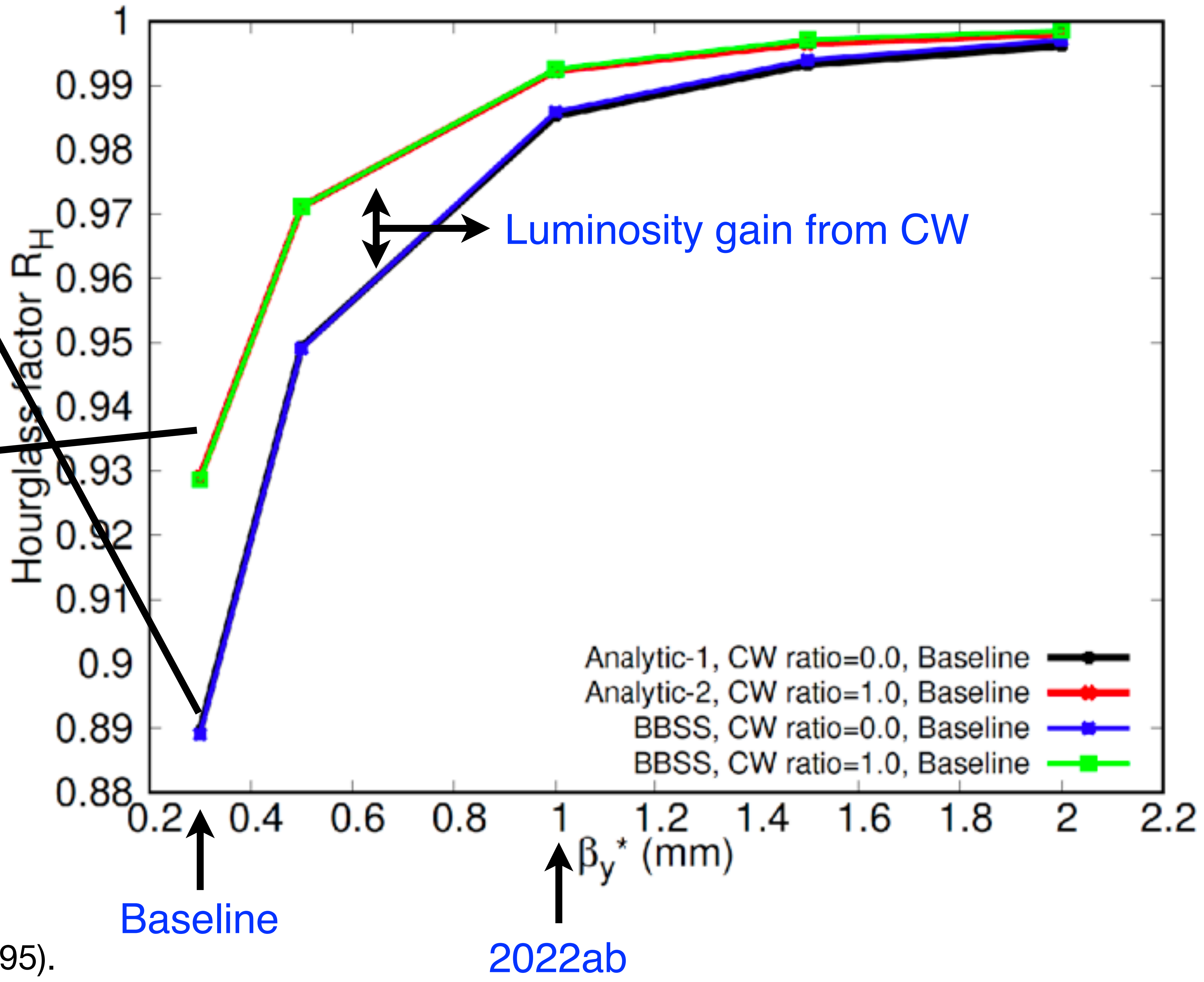
$$\Sigma_\beta^* = \sqrt{\sigma_{y+}^{*2}/\beta_{y+}^{*2} + \sigma_{y-}^{*2}/\beta_{y-}^{*2}} \qquad \Sigma_z = \sqrt{\sigma_{z+}^2 + \sigma_{z-}^2}$$

- With a full crab waist, flat beams, and a large Piwinski angle, I derived the analytic formula:

$$R_{HC}^{CW} \approx \frac{\Sigma_x^* \Sigma_z \tan \frac{\theta_c}{2}}{\Sigma_z^2 \tan^2 \frac{\theta_c}{2} + \sigma_{x+}^* \sigma_{x-}^*} f(d)$$

$$f(d) = \sqrt{\pi} d \cdot e^{d^2} \mathbf{Erfc}(d) \qquad d = \frac{\Sigma_y^* \Sigma_x^*}{\sqrt{2 \Sigma_\beta^* \sigma_{x+}^* \sigma_{x-}^*}} \sin \theta_c$$

- With the crossing angle factor R_C, we can calculate the hourglass factor $R_H = R_{HC}/R_C$
- Numerical tests showed great agreements between analytic formulae and BBSS (using SuperKEKB baseline design parameters)
- Luminosity gain from CW is less than 5%



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[1] K. Hirata, Analysis of beam-beam interactions with a large crossing angle, Physical review letters 74, 2228 (1995).

Luminosity

- Simple luminosity formula for “nano-beam scheme”:

$$L \approx L_0' = L_0 R_C$$

$$L_0 = \frac{N_b N_+ N_- f_c}{2\pi\sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}\sqrt{\sigma_{x+}^{*2} + \sigma_{x-}^{*2}}}$$

$$R_C = \frac{1}{\sqrt{1 + \frac{\sigma_{z+}^2 + \sigma_{z-}^2}{\sigma_{x+}^{*2} + \sigma_{x-}^{*2}} \tan^2 \frac{\theta_c}{2}}}$$

- The hourglass factor $L/L_0' = R_H$ is the order of 10% for the SuperKEKB baseline design [1] and about 2% for $\beta_y^* = 1$ mm (2022ab run)
- Conclusion: The simple formula L_0' is fairly good for discussions on the scaling laws of luminosity in SuperKEKB

- With large Piwinski angle, the formula for specific luminosity is simple:

$$L_{sp} = \frac{L}{N_b N_+ N_- (ef)^2}$$

$$L_{sp} \approx \frac{1}{2\pi e^2 f \sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^2 + \sigma_{z-}^2} \tan \frac{\theta_c}{2}}$$

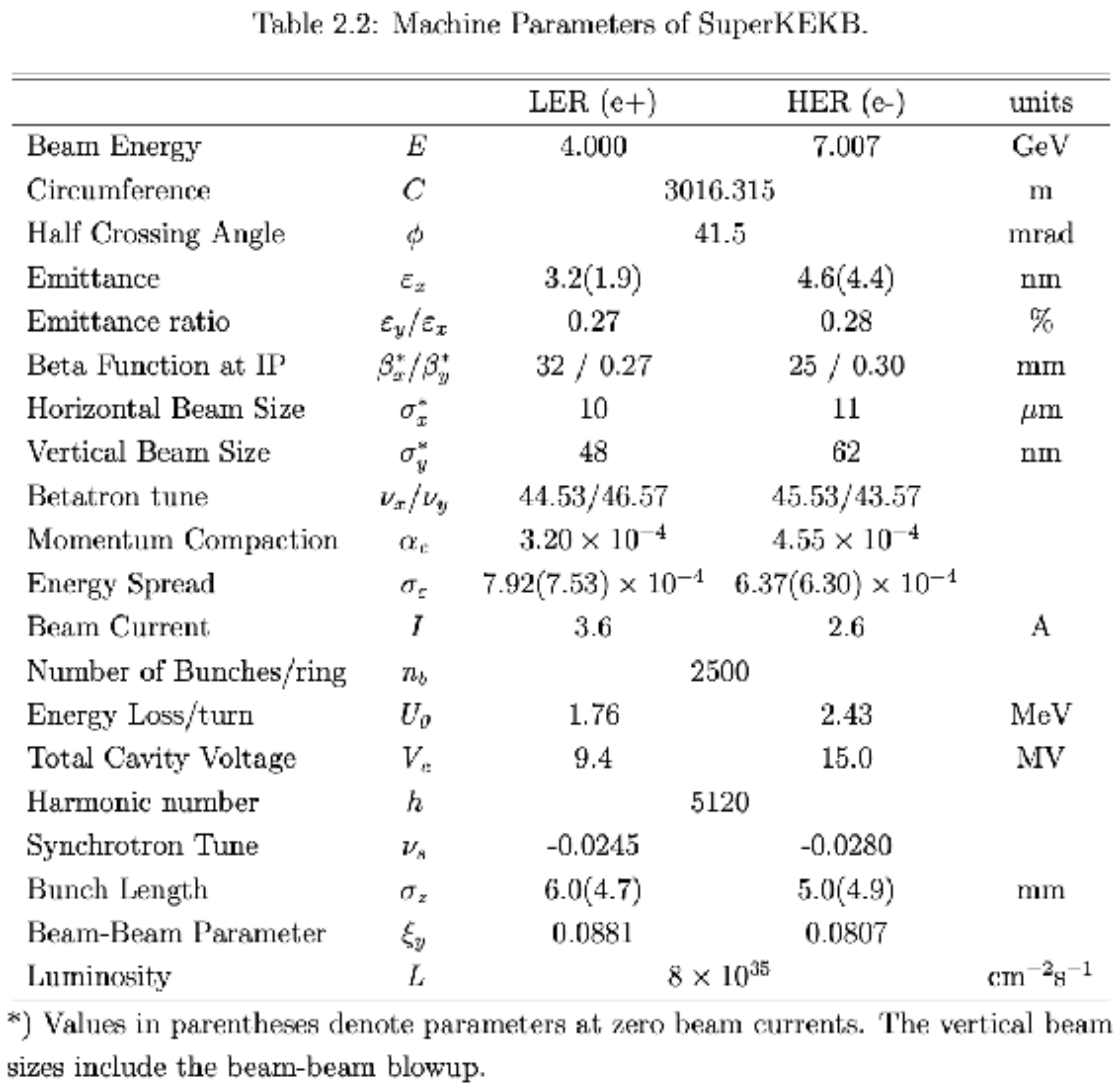


Table 2.2: Machine Parameters of SuperKEKB.

		LER (e+)	HER (e-)	units
Beam Energy	E	4.000	7.007	GeV
Circumference	C	3016.315		m
Half Crossing Angle	ϕ	41.5		mrad
Emittance	ε_x	3.2(1.9)	4.6(4.4)	nm
Emittance ratio	$\varepsilon_y/\varepsilon_x$	0.27	0.28	%
Beta Function at IP	β_x^*/β_y^*	32 / 0.27	25 / 0.30	mm
Horizontal Beam Size	σ_x^*	10	11	μm
Vertical Beam Size	σ_y^*	48	62	nm
Betatron tune	ν_x/ν_y	44.53/46.57	45.53/43.57	
Momentum Compaction	α_c	3.20×10^{-4}	4.55×10^{-4}	
Energy Spread	σ_ε	$7.92(7.53) \times 10^{-4}$	$6.37(6.30) \times 10^{-4}$	
Beam Current	I	3.6	2.6	A
Number of Bunches/ring	n_b	2500		
Energy Loss/turn	U_0	1.76	2.43	MeV
Total Cavity Voltage	V_c	9.4	15.0	MV
Harmonic number	h	5120		
Synchrotron Tune	ν_s	-0.0245	-0.0280	
Bunch Length	σ_z	6.0(4.7)	5.0(4.9)	mm
Beam-Beam Parameter	ξ_y	0.0881	0.0807	
Luminosity	L	8×10^{35}		$\mathrm{cm}^{-2}\mathrm{s}^{-1}$

*) Values in parentheses denote parameters at zero beam currents. The vertical beam sizes include the beam-beam blowup.

[1] SuperKEKB TDR, https://kds.kek.jp/event/15914/.

Beam-beam parameters

- The beam-beam parameters (=incoherent BB tune shifts) can be calculated from the electromagnetic fields of 3D Gaussian beam:

$$E_{x-}(x,y,z,t) = \frac{eN\_\gamma\_x}{2\epsilon_0 \pi^{3/2}} \int_0^\infty dw \frac{e^{-\frac{x^2}{2\sigma_{x-}^2(s)+w} - \frac{y^2}{2\sigma_{y-}^2(s)+w} - \frac{\gamma_-^2 (z-s)^2}{2\gamma_-^2 \sigma_{z-}^2 + w}}}{\left(2\sigma_{x-}^2(s)+w\right)^{3/2} \left(2\sigma_{y-}^2(s)+w\right)^{1/2} \left(2\gamma_-^2 \sigma_{z-}^2 + w\right)^{1/2}}$$

$$E_{y-}(x,y,z,t) = \frac{eN\_\gamma\_y}{2\epsilon_0 \pi^{3/2}} \int_0^\infty dw \frac{e^{-\frac{x^2}{2\sigma_{x-}^2(s)+w} - \frac{y^2}{2\sigma_{y-}^2(s)+w} - \frac{\gamma_-^2 (z-s)^2}{2\gamma_-^2 \sigma_{z-}^2 + w}}}{\left(2\sigma_{x-}^2(s)+w\right)^{1/2} \left(2\sigma_{y-}^2(s)+w\right)^{3/2} \left(2\gamma_-^2 \sigma_{z-}^2 + w\right)^{1/2}}$$

$$B_{x-} = -\frac{1}{c} E_{y-} \qquad\qquad B_{y-} = \frac{1}{c} E_{x-}$$

$$\xi_{x+}^{ih} = \frac{1}{4\pi p_0 c} \int_{-\infty}^{\infty} ds \beta_{x+}(s) \frac{\partial F_{x+}}{\partial x'} \qquad\qquad \xi_{y+}^{ih} = \frac{1}{4\pi p_0 c} \int_{-\infty}^{\infty} ds \beta_{y+}(s) \frac{\partial F_{y+}}{\partial y'}$$

Beam-beam parameters

- Explicit formulae exist [1]:

$$\xi^{ih}_{x+} = \frac{\Lambda_+}{\sqrt{\pi}} \int_0^\infty dw \int_{-\infty}^{\infty} ds \frac{\gamma_- \left(1 + \cos\theta_c\right) \beta_{x+}(s) g_{x+}(s) e^{-\frac{s^2 \sin^2\theta_c}{2\sigma^2_{x-}(s) + w} - \frac{\gamma_-^2 s^2 (1+\cos\theta_c)^2}{2\gamma_-^2 \sigma^2_{z-} + w}}}{\left(2\sigma^2_{x-}(s) + w\right)^{3/2} \left(2\sigma^2_{y-}(s) + w\right)^{1/2} \left(2\gamma_-^2\sigma^2_{z-} + w\right)^{1/2}}$$

$$\xi^{ih}_{y+} = \frac{\Lambda_+}{\sqrt{\pi}} \int_0^\infty dw \int_{-\infty}^{\infty} ds \frac{\gamma_- \left(1 + \cos\theta_c\right) \beta_{y+}(s) e^{-\frac{s^2 \sin^2\theta_c}{2\sigma^2_{x-}(s) + w} - \frac{\gamma_-^2 s^2 (1+\cos\theta_c)^2}{2\gamma_-^2 \sigma^2_{z-} + w}}}{\left(2\sigma^2_{x-}(s) + w\right)^{1/2} \left(2\sigma^2_{y-}(s) + w\right)^{3/2} \left(2\gamma_-^2\sigma^2_{z-} + w\right)^{1/2}}$$

$$\Lambda_+ = \frac{r_e N_-}{2\pi\gamma_+}$$

$$g_{x+}(s) = \cos\theta_c + 2s^2 \sin\theta_c^2 \left[\frac{\gamma_-^2 \left(1 + \cos\theta_c\right)}{2\gamma_-^2\sigma^2_{z-} + w} - \frac{\cos\theta_c}{2\sigma^2_{x-}(s) + w}\right]$$

- If no hourglass effect, exact analytic formulae can be derived:

$$\xi^i_{x+} = \frac{\Lambda_+ \beta^*_{x+}}{\sigma^{*2}_{x-} \sqrt{1 + \phi_-^2} \sqrt{\alpha_+} \left(\sqrt{1 + \phi_-^2} + \kappa_- \sqrt{\alpha_+}\right)}$$

$$\xi^i_{y+} = \frac{\Lambda_+ \beta^*_{y+}}{\sigma^*_{x-} \sigma^*_{y-} \left(\sqrt{1 + \phi_-^2} + \kappa_- \sqrt{\alpha_+}\right)}$$

$$\phi_- = \frac{\sigma_{z-}}{\sigma^*_{x-}} \tan\frac{\theta_c}{2}$$ ⟶ Piwinski angle

$$\kappa_- = \frac{\sigma^*_{y-}}{\sigma^*_{x-}}$$ ⟶ Flat beams: $\kappa_\pm \ll 1$

$$\alpha_+ = 1 + \frac{1}{\gamma_+^2} \tan^2\frac{\theta_c}{2}$$ ⟶ Lorentz factors

$$\bar{\sigma}_{x-} = \sqrt{\sigma^{*2}_{x-} + \sigma^2_{z-} \tan^2\frac{\theta_c}{2}}$$ ⟶ Projected horizontal beam sizes

[1] A. Valishev, Practical Beam-Beam Tune Shift Formulae for Simulation Cross-Check, Tech. Rep. (Fermi National Accelerator Lab.(FNAL), Batavia, IL (United States), 2013).

Beam-beam parameters

- With hourglass effect, it is difficult to find the analytic solutions of ξ_{x+}^{ih} and ξ_{y+}^{ih}:

$$\xi_{x+}^{ih} = \frac{\Lambda_+}{\sqrt{\pi}} \int_0^{\infty} dw \int_{-\infty}^{\infty} ds \frac{\gamma_- \left(1 + \cos\theta_c\right) \beta_{x+}(s) g_{x+}(s) e^{-\frac{s^2 \sin^2\theta_c}{2\sigma_{x-}^2(s)+w} - \frac{\gamma_-^2 s^2 (1+\cos\theta_c)^2}{2\gamma_-^2\sigma_{z-}^2+w}}}{\left(2\sigma_{x-}^2(s)+w\right)^{3/2} \left(2\sigma_{y-}^2(s)+w\right)^{1/2} \left(2\gamma_-^2\sigma_{z-}^2+w\right)^{1/2}}$$

$$\xi_{y+}^{ih} = \frac{\Lambda_+}{\sqrt{\pi}} \int_0^{\infty} dw \int_{-\infty}^{\infty} ds \frac{\gamma_- \left(1 + \cos\theta_c\right) \beta_{y+}(s) e^{-\frac{s^2 \sin^2\theta_c}{2\sigma_{x-}^2(s)+w} - \frac{\gamma_-^2 s^2 (1+\cos\theta_c)^2}{2\gamma_-^2\sigma_{z-}^2+w}}}{\left(2\sigma_{x-}^2(s)+w\right)^{1/2} \left(2\sigma_{y-}^2(s)+w\right)^{3/2} \left(2\gamma_-^2\sigma_{z-}^2+w\right)^{1/2}}$$

- For flat beams and $\beta_x^* \gg \sigma_z$ (this is the case of SuperKEKB), approximate formulae can be found:

$$\xi_{x+}^{ih} \approx \xi_{x+}^{i} = \frac{r_e}{2\pi\gamma_+} \frac{N_- \beta_{x+}^*}{\bar{\sigma}_{x-}^2}$$

$$\xi_{y+}^{ih} \approx \frac{r_e N_-}{2\pi\gamma_+} \frac{\beta_{y+}^*}{\bar{\sigma}_{x-}\sigma_{y-}^*} \left[\sqrt{\frac{2}{\pi}} r_- e^{r_-^2} K_0(r_-^2) + \frac{\beta_{y-}^{*2}}{2\sqrt{2}\beta_{y+}^{*2} r_-} U\left(\frac{1}{2}, 0, 2r_-^2\right) \right]$$

$$r_- = \frac{\beta_{y-}^* \bar{\sigma}_{x-} \cos^2\frac{\theta_c}{2}}{\sigma_{z-}\sigma_{x-}^*} \longrightarrow$$ Important parameter for evaluation of hourglass effect

- The hourglass factor for beam-beam parameters can be defined as:

$$R_{\xi u\pm} = \xi_{u\pm}^{ih} / \xi_{u\pm}^{i}$$

- $R_{\xi u\pm}$ can be calculated via numerical integrations. But we must be careful: convergence issues appear, especially for the SuperKEKB case. Inconsistency might appear when we use Mathematica, SAD, BBWS (Numerical evaluation of complex error function), etc.

Beam-beam parameters

- For the hourglass factor of luminosity, there always is $R_H < 1$
- For the hourglass factors of BB parameters, there can be $\xi^{ih}_{y+} < 1$, or $\xi^{ih}_{y+} > 1$:

$$\xi^{ih}_{y+} = \frac{\Lambda_+}{\sqrt{\pi}} \int_0^\infty dw \int_{-\infty}^{\infty} ds \frac{\gamma_- \left(1 + \cos\theta_c\right) \beta_{y+}(s) e^{-\frac{s^2 \sin^2\theta_c}{2\sigma_{x-}^2(s) + w} - \frac{\gamma_-^2 s^2 (1+\cos\theta_c)^2}{2\gamma_-^2\sigma_{z-}^2 + w}}}{\left(2\sigma_{x-}^2(s) + w\right)^{1/2} \left(2\sigma_{y-}^2(s) + w\right)^{3/2} \left(2\gamma_-^2\sigma_{z-}^2 + w\right)^{1/2}}$$

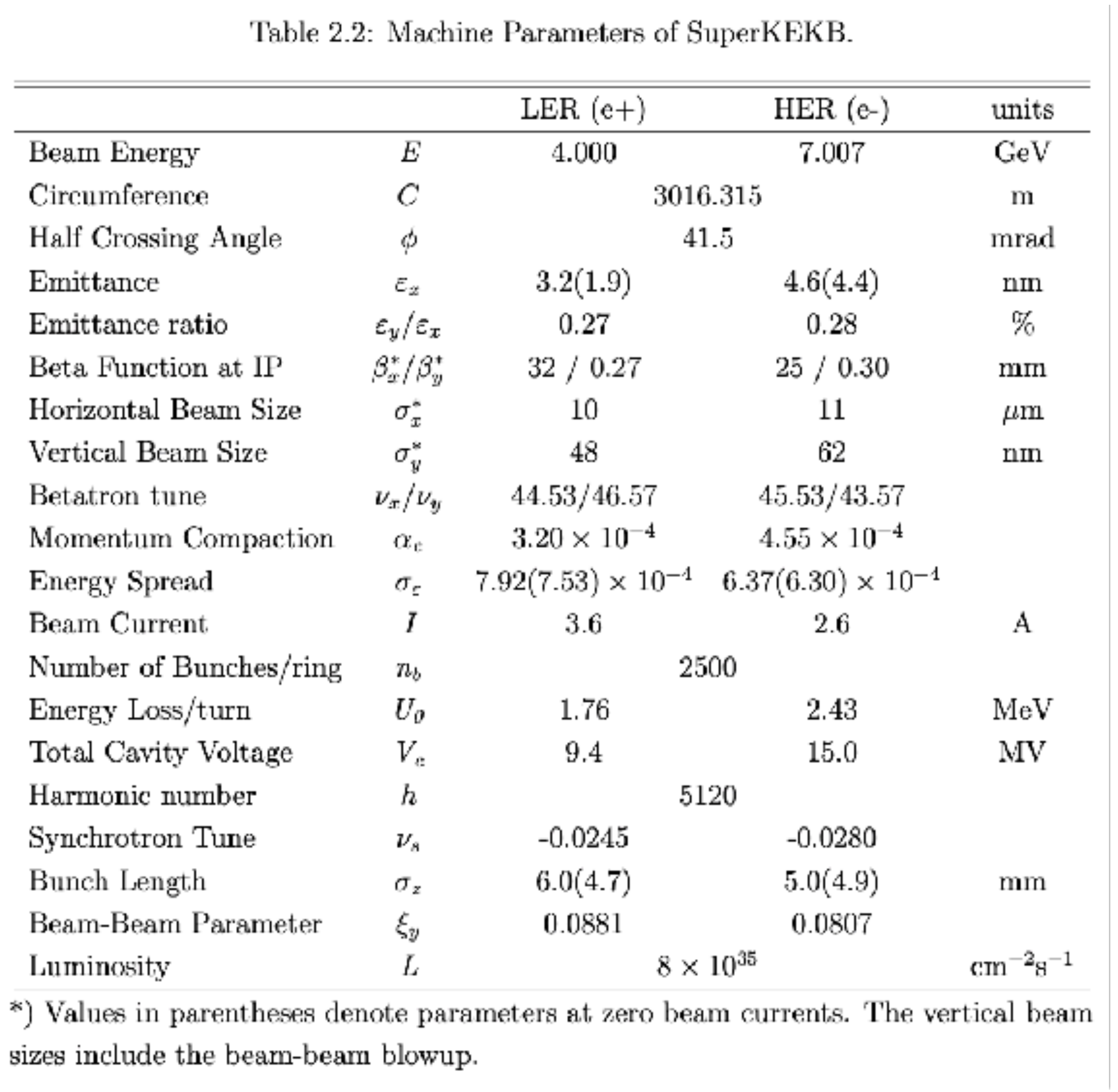
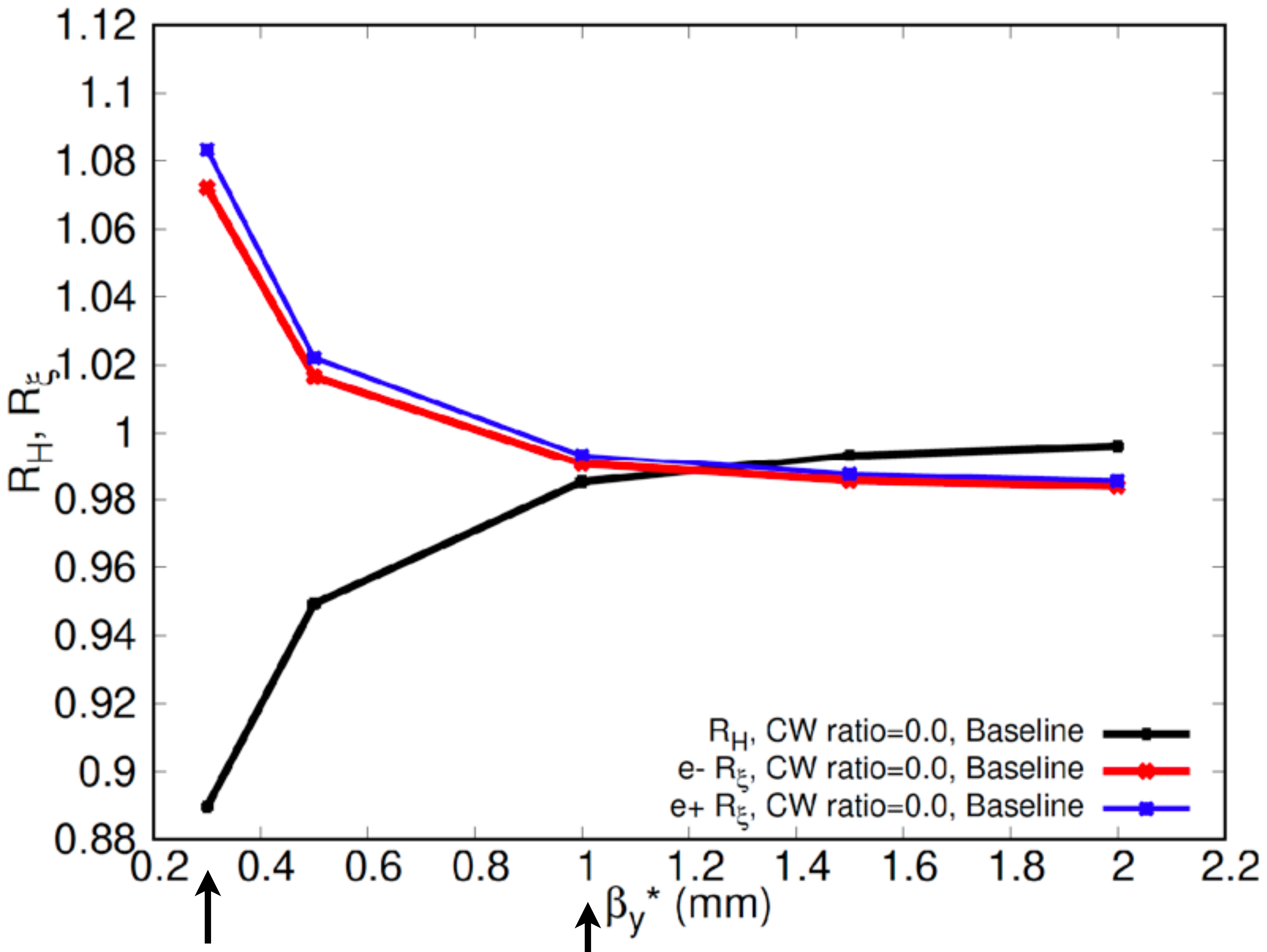


Table 2.2: Machine Parameters of SuperKEKB.

		LER (e+)	HER (e-)	units
Beam Energy	E	4.000	7.007	GeV
Circumference	C	3016.315		m
Half Crossing Angle	ϕ	41.5		mrad
Emittance	ε_x	3.2(1.9)	4.6(4.4)	nm
Emittance ratio	$\varepsilon_y/\varepsilon_x$	0.27	0.28	%
Beta Function at IP	β_x^*/β_y^*	32 / 0.27	25 / 0.30	mm
Horizontal Beam Size	σ_x^*	10	11	μm
Vertical Beam Size	σ_y^*	48	62	nm
Betatron tune	ν_x/ν_y	44.53/46.57	45.53/43.57	
Momentum Compaction	α_c	3.20×10^{-4}	4.55×10^{-4}	
Energy Spread	σ_ε	$7.92(7.53) \times 10^{-4}$	$6.37(6.30) \times 10^{-4}$	
Beam Current	I	3.6	2.6	A
Number of Bunches/ring	n_b	2500		
Energy Loss/turn	U_0	1.76	2.43	MeV
Total Cavity Voltage	V_c	9.4	15.0	MV
Harmonic number	h	5120		
Synchrotron Tune	ν_s	-0.0245	-0.0280	
Bunch Length	σ_z	6.0(4.7)	5.0(4.9)	mm
Beam-Beam Parameter	ξ_y	0.0881	0.0807	
Luminosity	L	8×10^{35}		cm^{-2}s^{-1}

*) Values in parentheses denote parameters at zero beam currents. The vertical beam sizes include the beam-beam blowup.

- The hourglass $R_{\xi y\pm} = \xi^{ih}_{y\pm}/\xi^{i}_{y\pm}$ is the order of 8% for the SuperKEKB baseline design and about 1% for $\beta_y^* = 1$ mm (2022ab run)
- Conclusion: The simple formula $\xi^{i}_{y\pm}$ is fairly good for discussions on the scaling laws of beam-beam parameters in SuperKEKB



Baseline

2022ab

Relation of luminosity and beam-beam parameters

- The luminosity L and beam-beam parameters $\xi^{ih}_{y\pm}$ can be correlated:

$$L = \frac{1}{2er_e}\frac{\gamma_\pm I_\pm}{\beta^*_{y\pm}}\underbrace{\xi^{ih}_{y\pm}}_{\text{Beam-beam parameter}}\underbrace{\frac{2\sigma^*_{y\pm}\overline{\sigma}_{x\pm}}{\Sigma_y\overline{\Sigma}_x}}_{\text{Asymmetry factor}}\underbrace{\frac{R_H}{R_{\xi y\pm}}}_{\text{Hourglass factor}}$$

- There are three “methods” for calculating the beam-beam parameters from luminosity.
- If the beta functions and beam sizes at the IP are well known, this is the standard method:

$$\xi^{ih}_{y+} = \frac{\Lambda_+}{\sqrt{\pi}}\int_0^\infty dw\int_{-\infty}^{\infty} ds\frac{\gamma_-\left(1+\cos\theta_c\right)\beta_{y+}(s)e^{-\frac{s^2\sin^2\theta_c}{2\sigma^2_{x-}(s)+w}-\frac{\gamma^2_- s^2\left(1+\cos\theta_c\right)^2}{2\gamma^2_-\sigma^2_{z-}+w}}}{\left(2\sigma^2_{x-}(s)+w\right)^{1/2}\left(2\sigma^2_{y-}(s)+w\right)^{3/2}\left(2\gamma^2_-\sigma^2_{z-}+w\right)^{1/2}}$$

- If we ignore both asymmetries of the colliding beams and hourglass effects, this is Ohmi-method [1]:

$$L = \frac{1}{2er_e}\frac{\gamma_\pm I_\pm}{\beta^*_{y\pm}}\xi^L_{y\pm}$$

- If we use “measured” beam sizes at the IP to estimate the hourglass factor and use it as a calibration factor, this is the KEKB method [As Oide-san proposed](Assumed: equal beam parameters and flat beams) [2]:

$$\mathcal{L} = \frac{N_1N_2f}{4\pi\sigma^*_x\sigma^*_y}R_{\mathcal{L}}\left(\theta_x,\beta^*_x,\beta^*_y,\varepsilon_x,\varepsilon_y,\sigma_z\right) \ ,$$

$$\xi_{yk} = \frac{N_{3-k}r_e\beta^*_{yk}}{2\pi\gamma_k\left(\sigma^*_x+\sigma^*_y\right)\sigma^*_y}R_{\xi y}\left(\theta_x,\beta^*_x,\varepsilon_x,\varepsilon_y,\beta^*_y,\sigma_z\right) \ .$$

$$\mathcal{L} = \frac{\gamma_k I_k\xi_y}{2er_e\beta^*_y}\frac{R_{\mathcal{L}}}{R_{\xi y}} \ ,$$

[1] K. Ohmi et al., PRST-AB 7, 104401 (2004).

[2] KEKB B-Factory Design Report, KEK Report 95-7 (1995).

Relation of luminosity and beam-beam parameters

- Note that my notation has certain differences with the formulations in KEKB design report:

$$\mathcal{L} = \frac{N_1 N_2 f}{4\pi\sigma_x^*\sigma_y^*} R_{\mathcal{L}}\left(\theta_x, \beta_x^*, \beta_y^*, \varepsilon_x, \varepsilon_y, \sigma_z\right) \ ,$$

$$\xi_{yk} = \frac{N_{3-k} r_e \beta_{yk}^*}{2\pi\gamma_k\left(\sigma_x^* + \sigma_y^*\right)\sigma_y^*} R_{\xi y}\left(\theta_x, \beta_x^*, \varepsilon_x, \varepsilon_y, \beta_y^*, \sigma_z\right) \ .$$

Crossing angle factors cancel each other with symmetric beams

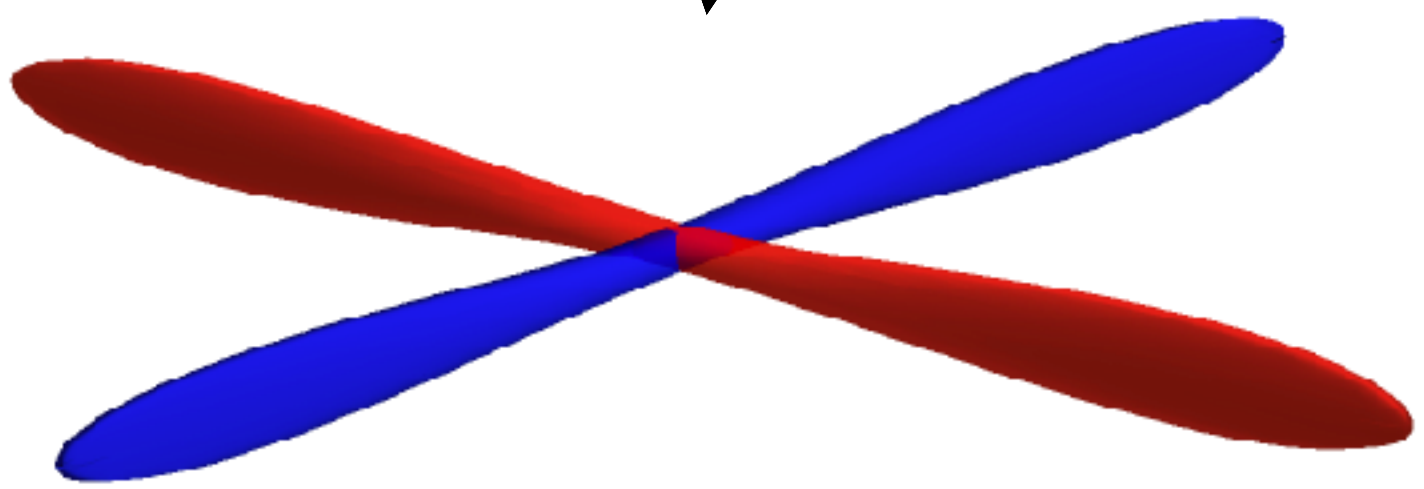
$$R_L = R_{HC} = R_H R_C = R_H \boxed{\frac{1}{\sqrt{1 + \frac{\sigma_{z+}^2 + \sigma_{z-}^2}{\sigma_{x+}^{*2} + \sigma_{x-}^{*2}} \tan^2 \frac{\theta_c}{2}}}}$$

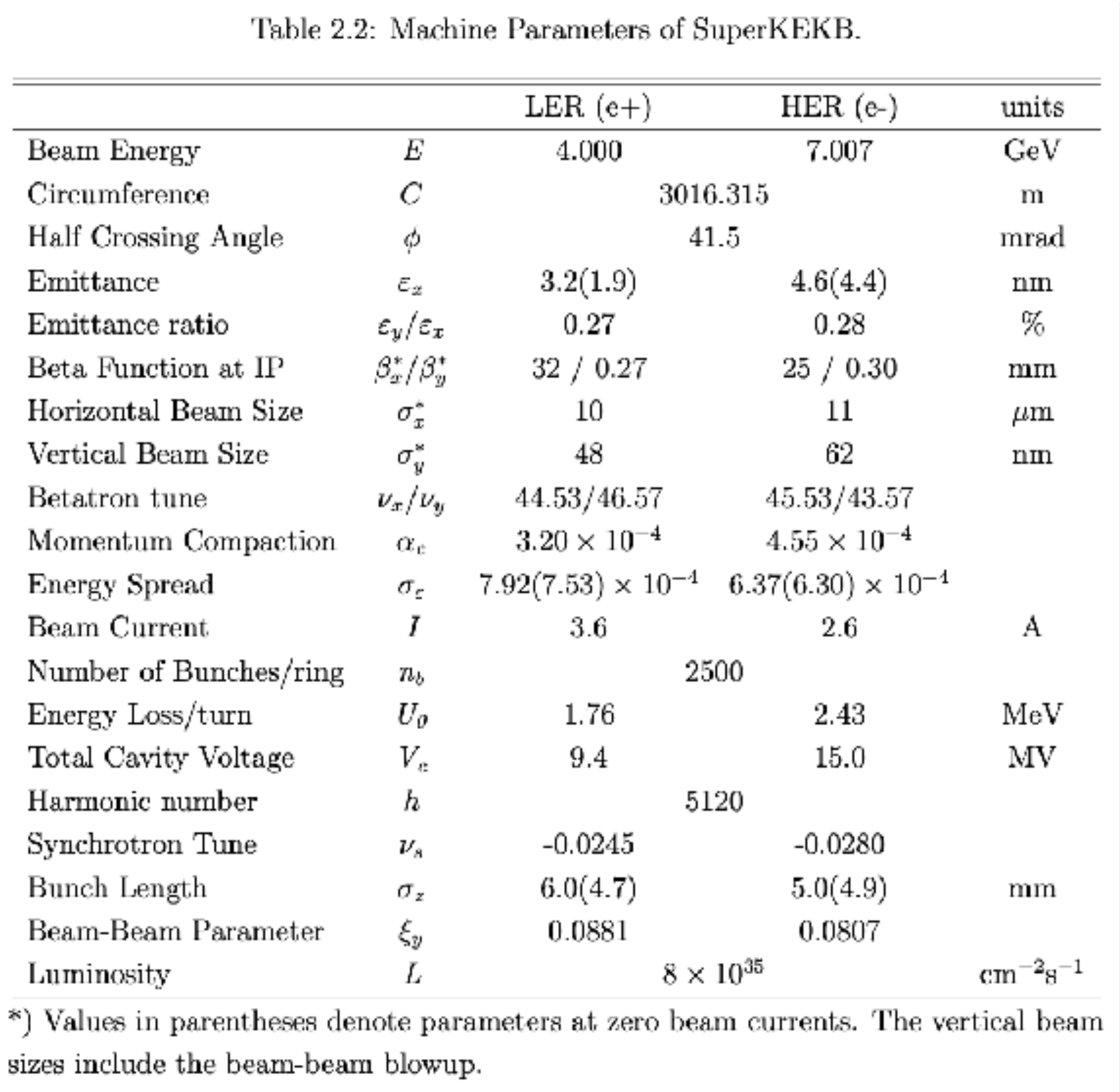
$$\mathcal{L} = \frac{\gamma_k I_k \xi_y}{2 e r_e \beta_y^*} \frac{\boxed{R_{\mathcal{L}}}}{\boxed{R_{\xi y}}},$$

$$R_{\xi y}(\textbf{KEKB}) = R_{\xi y\pm}(\textbf{DZ}) \boxed{\frac{1}{\sqrt{1 + \frac{\sigma_{z\pm}^2}{\sigma_{x\pm}^{*2}} \tan^2 \frac{\theta_c}{2}}}}$$

Relation of luminosity and beam-beam parameters

- For the SuperKEKB baseline design, the hourglass factor $R_L/R_{\xi y}$ is 0.8.
- For the KEKB case, $R_L/R_{\xi y}$ was about 0.7 [1].
- For the 2022ab run ($\beta_y^* = 1$ mm) of SuperKEKB, $R_L/R_{\xi y}$ is very close to 1. It should be safe to use simple formulae (w/o hourglass) for luminosity and beam-beam parameters.





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Betatron tune	ν_x/ν_y	44.53/46.57	45.53/43.57	
Momentum Compaction	α_c	3.20×10^{-4}	4.55×10^{-4}	
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Beam Current	I	3.6	2.6	A
Number of Bunches/ring	n_b	2500		
Energy Loss/turn	U_0	1.76	2.43	MeV
Total Cavity Voltage	V_c	9.4	15.0	MV
Harmonic number	h	5120		
Synchrotron Tune	ν_s	-0.0245	-0.0280	
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Beam-Beam Parameter	ξ_y	0.0881	0.0807	
Luminosity	L	8×10^{35}		$\mathrm{cm}^{-2}\mathrm{s}^{-1}$

*) Values in parentheses denote parameters at zero beam currents. The vertical beam sizes include the beam-beam blowup.

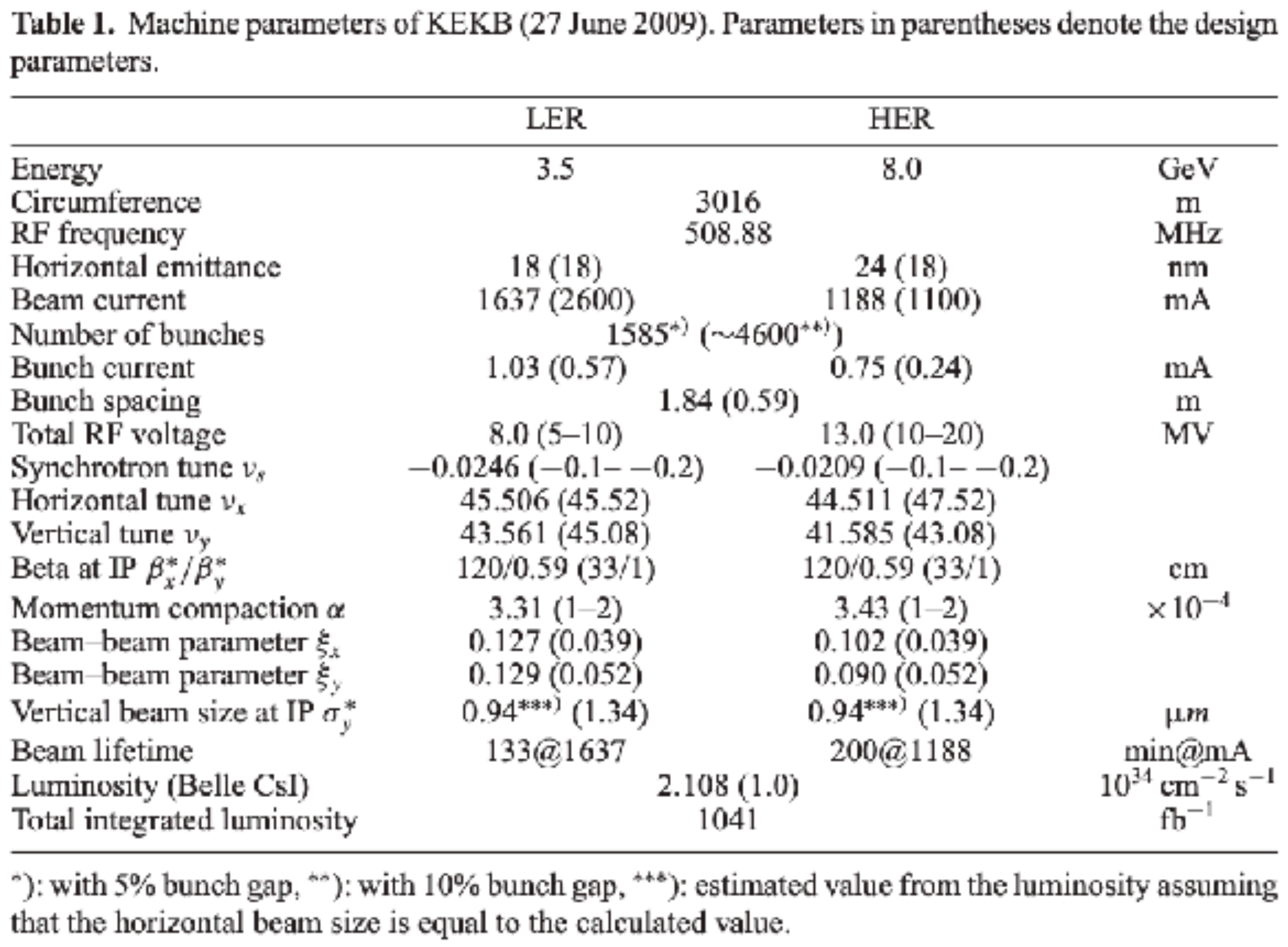


Table 1. Machine parameters of KEKB (27 June 2009). Parameters in parentheses denote the design parameters.

	LER	HER	
Energy	3.5	8.0	GeV
Circumference	3016		m
RF frequency	508.88		MHz
Horizontal emittance	18 (18)	24 (18)	nm
Beam current	1637 (2600)	1188 (1100)	mA
Number of bunches	$1585^{*)}$ ($\sim 4600^{**)}$)		
Bunch current	1.03 (0.57)	0.75 (0.24)	mA
Bunch spacing	1.84 (0.59)		m
Total RF voltage	8.0 (5–10)	13.0 (10–20)	MV
Synchrotron tune ν_s	−0.0246 (−0.1– −0.2)	−0.0209 (−0.1– −0.2)	
Horizontal tune ν_x	45.506 (45.52)	44.511 (47.52)	
Vertical tune ν_y	43.561 (45.08)	41.585 (43.08)	
Beta at IP β_x^*/β_y^*	120/0.59 (33/1)	120/0.59 (33/1)	cm
Momentum compaction α	3.31 (1–2)	3.43 (1–2)	$\times 10^{-4}$
Beam–beam parameter ξ_x	0.127 (0.039)	0.102 (0.039)	
Beam–beam parameter ξ_y	0.129 (0.052)	0.090 (0.052)	
Vertical beam size at IP σ_y^*	$0.94^{***)}$ (1.34)	$0.94^{***)}$ (1.34)	μm
Beam lifetime	133@1637	200@1188	min@mA
Luminosity (Belle CsI)	2.108 (1.0)		$10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$
Total integrated luminosity	1041		fb^{-1}

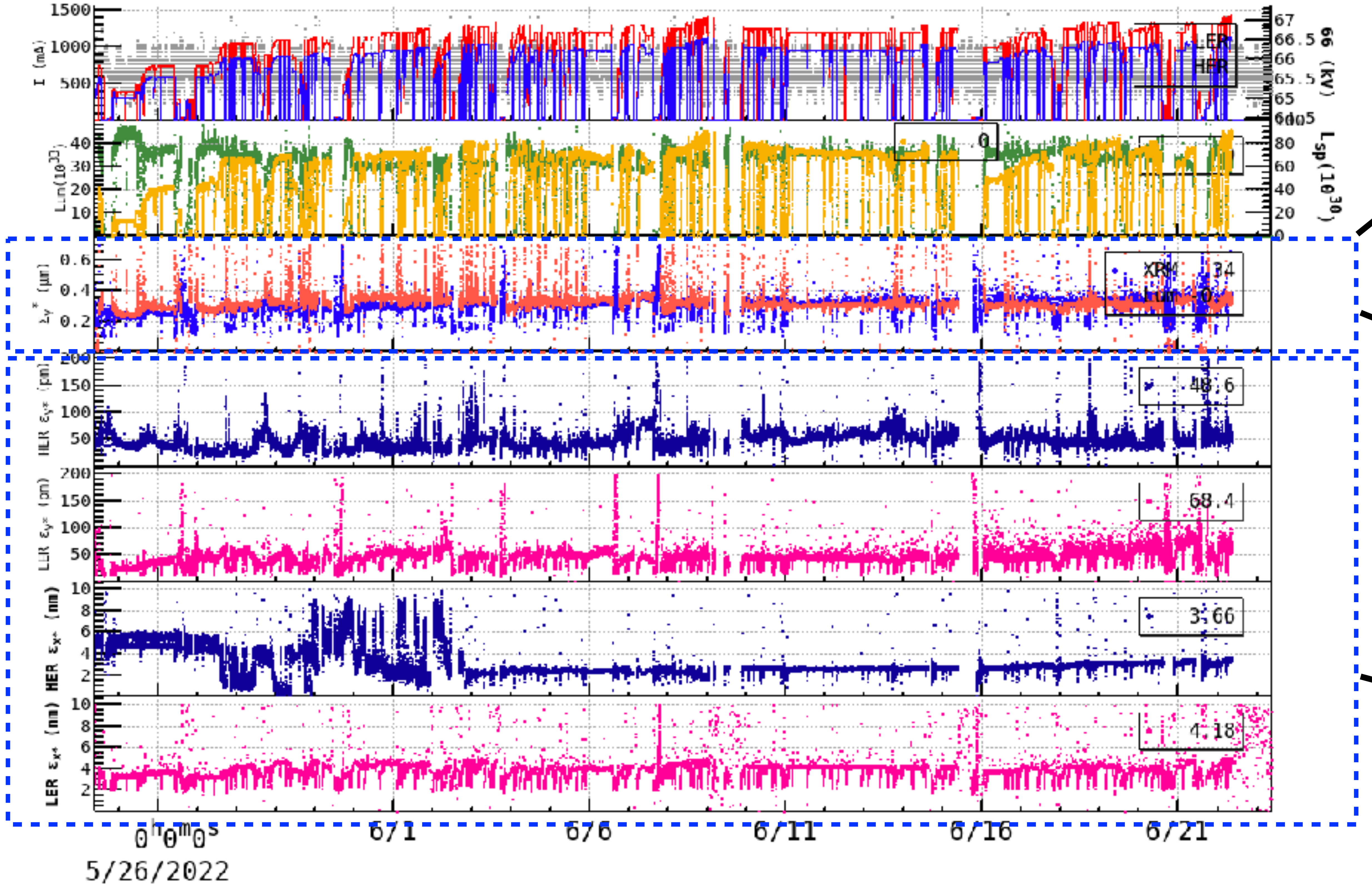
*): with 5% bunch gap, **): with 10% bunch gap, ***): estimated value from the luminosity assuming that the horizontal beam size is equal to the calculated value.

[1] Y. Funakoshi, Achievements of KEKB, Prog. Theor. Exp. Phys. (2013) 03A001.

Applications to SuperKEKB

- Cap sigma Σ_y^* (SAD panels for SuperKEKB operation)

$$L \approx \frac{N_b N_+ N\_f}{2\pi\sqrt{\sigma_{y+}^{*2}+\sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^2+\sigma_{z-}^2}\,\tan\frac{\theta_c}{2}} \qquad \textbf{(5)}$$

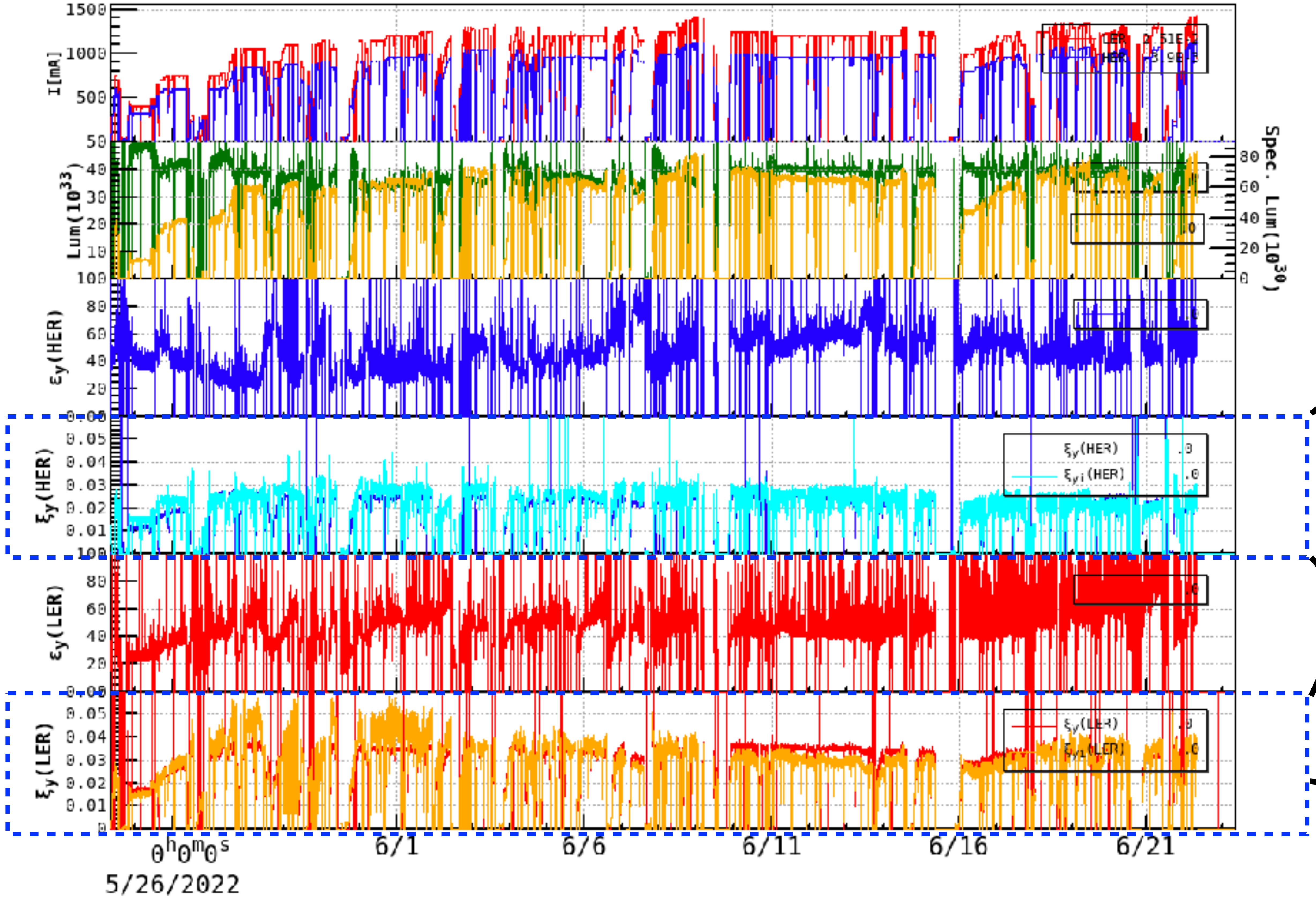


When machine conditions are good, the cap sigma Σ_y^* estimated from luminosity and XRM data agree with each other.

Data from XRMs and SRMs

Applications to SuperKEKB

- Beam-beam parameters



Two ways to calculate beam-beam parameters:
Using XRM data and using luminosity

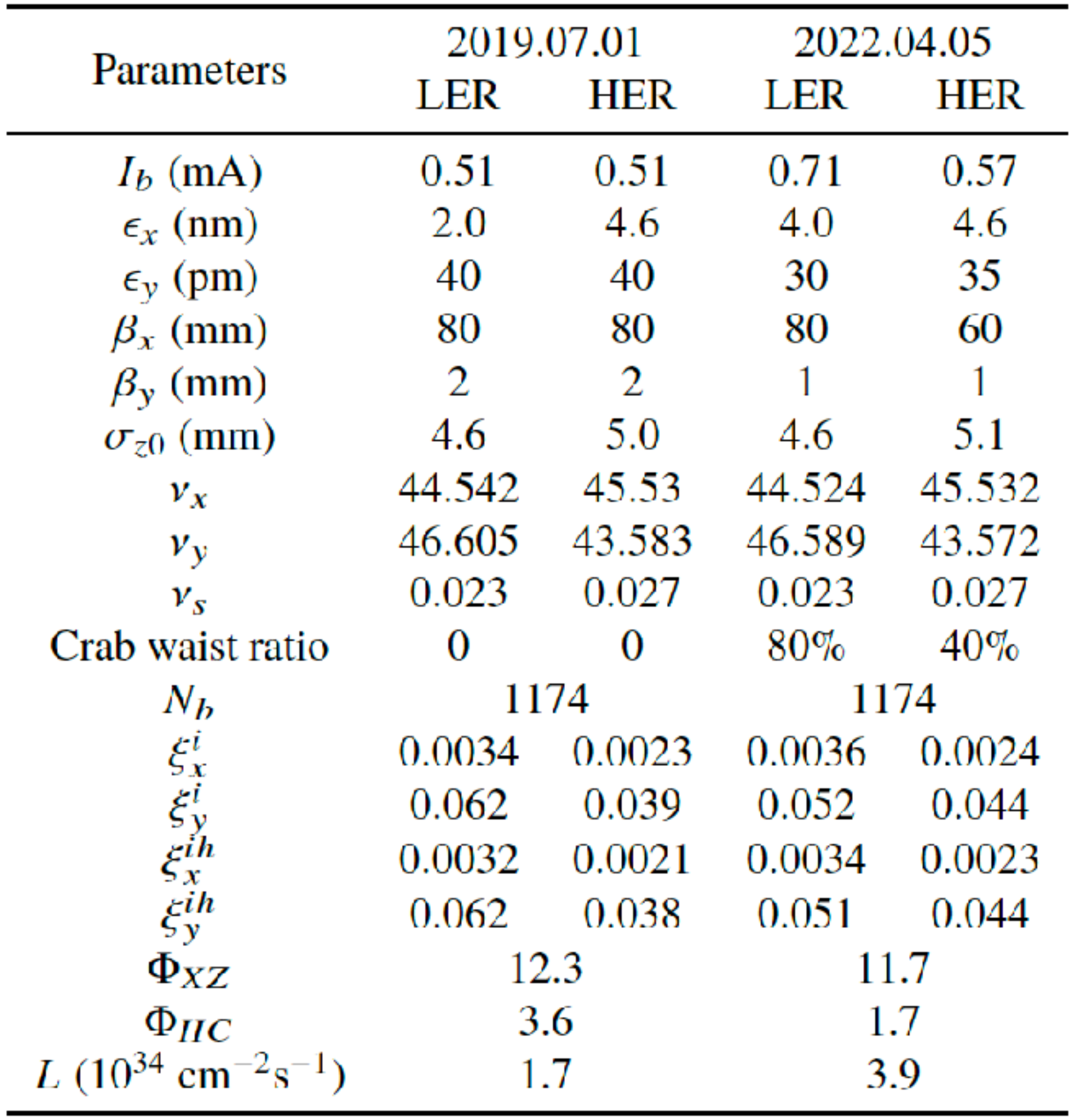
$$\xi_{yi+} \approx \frac{r_e}{2\pi\gamma_+} \frac{N_- \beta^*_{y+}}{\sigma^*_{y-}\left(\sqrt{\sigma_{z-}^2 \tan^2 \frac{\theta_c}{2} + \sigma_{x-}^{*2}} + \sigma^*_{y-}\right)}$$

$$L = \frac{1}{2er_e} \frac{\gamma_\pm I_\pm}{\beta^*_{y\pm}} \xi_{y\pm}$$

When machine conditions are good, the beam-beam parameters estimated from luminosity and XRM data agree with each other.

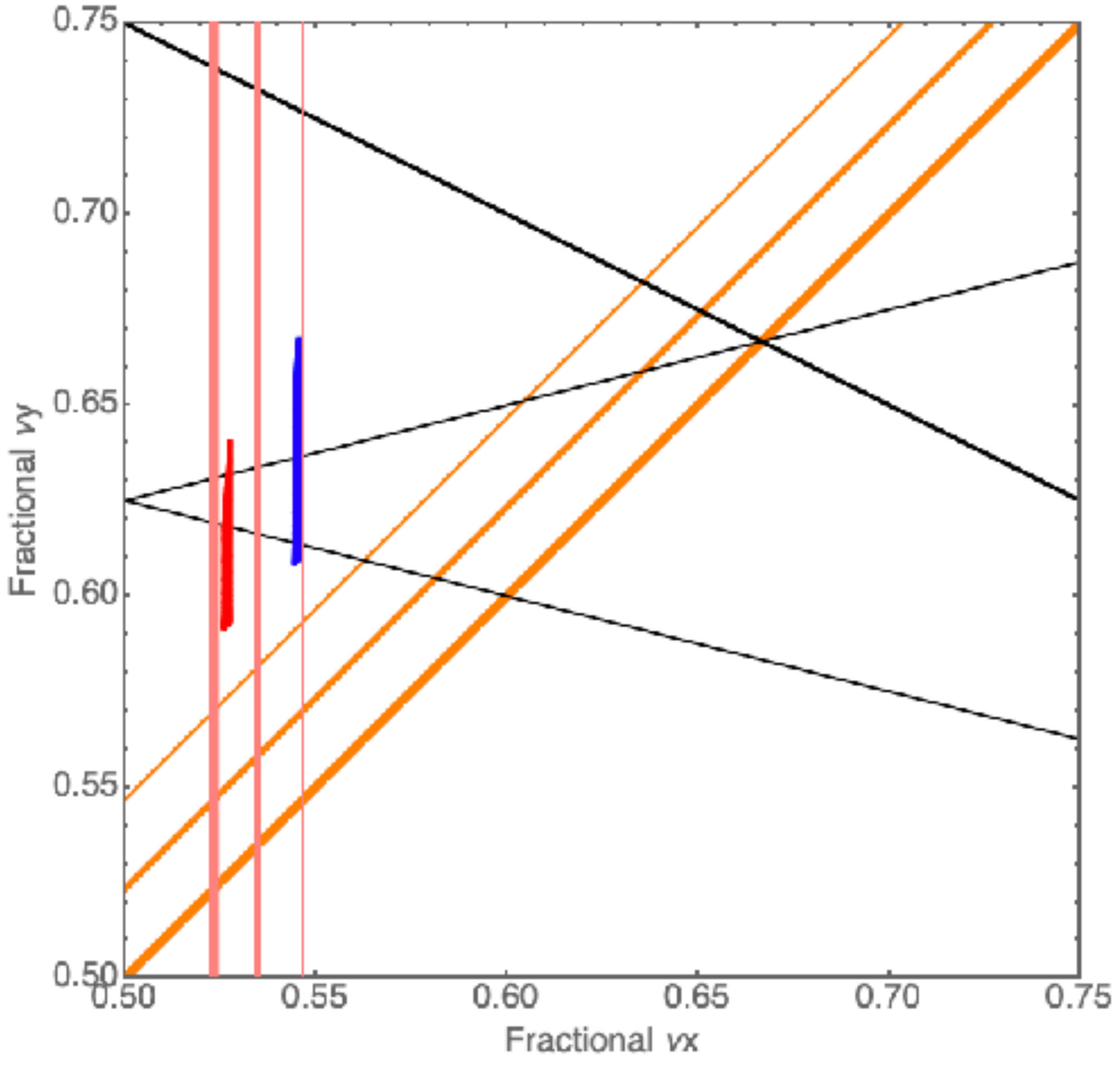
Luminosity and beam-beam tune shifts

- “Nano-beam scheme” for SuperKEKB
  - Beam-beam-driven footprint in tune space is useful for understanding beam-beam effects.
  - The choice of working point dynamically depends on machine conditions.



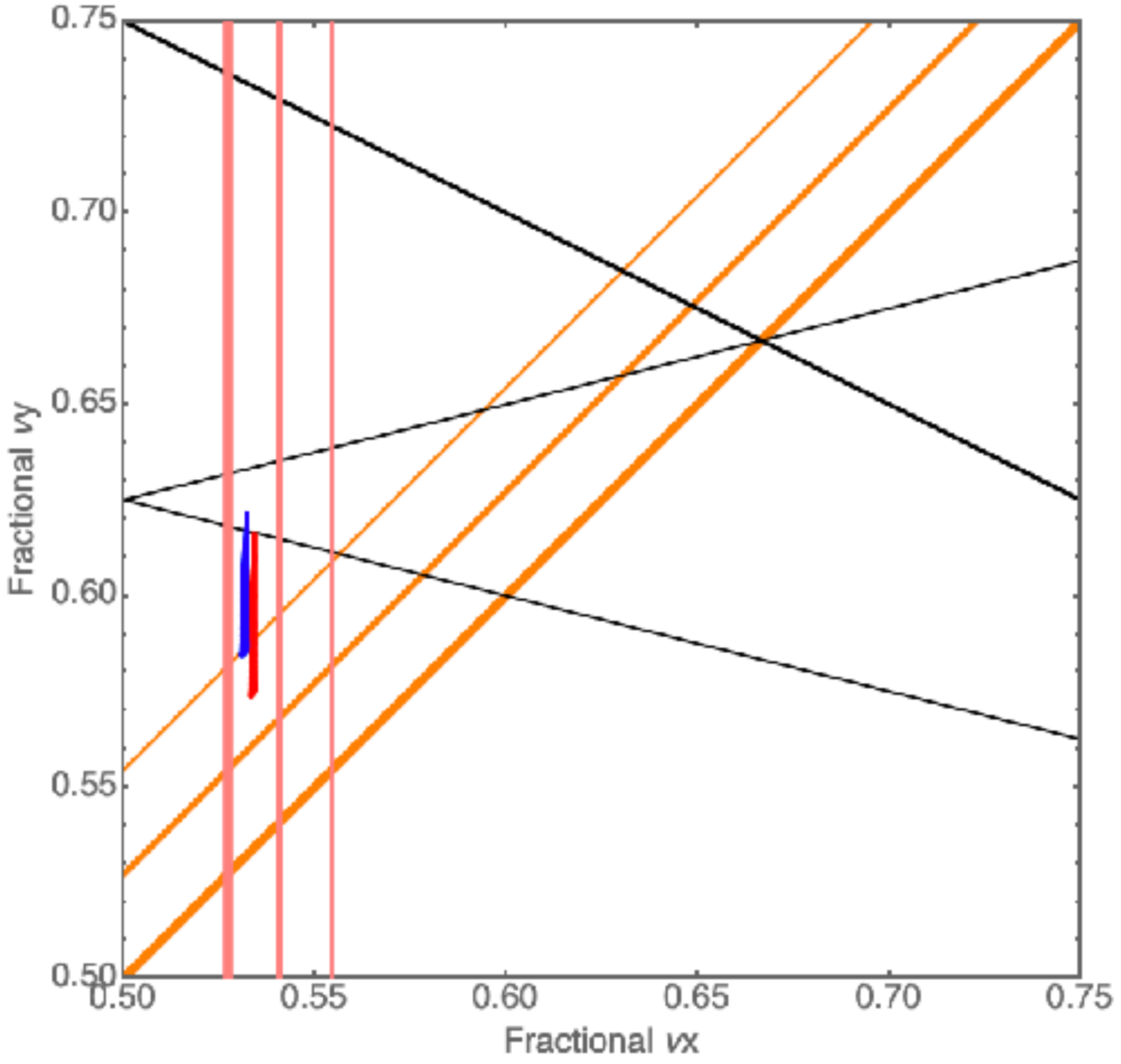
Parameters	2019.07.01 LER	2019.07.01 HER	2022.04.05 LER	2022.04.05 HER
I_b (mA)	0.51	0.51	0.71	0.57
ϵ_x (nm)	2.0	4.6	4.0	4.6
ϵ_y (pm)	40	40	30	35
β_x (mm)	80	80	80	60
β_y (mm)	2	2	1	1
σ_{z0} (mm)	4.6	5.0	4.6	5.1
ν_x	44.542	45.53	44.524	45.532
ν_y	46.605	43.583	46.589	43.572
ν_s	0.023	0.027	0.023	0.027
Crab waist ratio	0	0	80%	40%
N_b	1174		1174	
ξ_x^i	0.0034	0.0023	0.0036	0.0024
ξ_y^i	0.062	0.039	0.052	0.044
ξ_x^{ih}	0.0032	0.0021	0.0034	0.0023
ξ_y^{ih}	0.062	0.038	0.051	0.044
Φ_{XZ}	12.3		11.7	
Φ_{HC}	3.6		1.7	
L (10^{34} cm^{-2}s^{-1})	1.7		3.9	

LER
Red: 2022.04.05, w/ CW
Blue: 2019.07.01, w/o CW



Notes:
* Hourglass effect ignored in calculation of BB footprint
* Resonances $m\nu_x \pm n\nu_y = N$ not plotted
* Collective effects dynamically shift the resonances

HER
Red: 2022.04.05, w/ CW
Blue: 2019.07.01, w/o CW



Status of beam-beam simulations

- Weak-strong model + simple one-turn map: BBWS code [1]
  - The weak beam is represented by N macro-particles (statistical errors ~ $1/\sqrt{N}$). The strong beam has a rigid charge distribution with its EM fields expressed by the Bassetti-Erskine formula.
  - The simple one-turn map contains lattice transformation (Tunes, alpha functions, beta functions, X-Y couplings, dispersions, etc.), chromatic perturbation, synchrotron radiation damping, quantum excitation, crab waist, etc.

$$M = M_{rad} \circ M_{chr} \circ M_{bb} \circ M_{cw} \circ M_0$$

$$M_0 = R \cdot M_{lin} \cdot R^{-1}$$

- Weak-strong model + full lattice: SAD code
  - The BBWS code was implemented into SAD as a type of BEAMBEAM element, where the beam-beam map is called during particle tracking.
  - Tracking using SAD: 1) Symplectic maps for elements of BEND, QUAD, MULT, CAVI, etc. 2) Element-by-element SR damping/excitation; 3) Distributed weak-strong space-charge; 4) MAP element for arbitrary perturbation maps (such as crab waist, wakefields, artificial SR damping/excitation, etc.); …

```
 ;
 BEAMBEAM    BMBMP  =(NP=3.63776D10
                     BETAX=0.06 BETAY=0.001
                     EX=0.D0 EY=0.D0
                     EMIX=4.6D-9 EMIY=40.D-12
                     SIGZ=6.D-3  DP=6.30427D-4
                     ALPHAX=0.D0 ALPHAY=0.D0
                     DX=0.E-6 DZ=0.0
                     SLICE=200.D0  XANGLE=41.5D-3
                     STURN=1000)
;
```

- Strong-strong model + simple one-turn map: BBSS code [1]
  - Both beams are represented by N macro-particles
  - The one-turn map is the same as weak-strong code. The Beamstrahlung model is also available. Choices of numerical techniques: PIC, Gaussian fitting for each slice, …
  - For SuperKEKB, it is hard to include lattice.
- GPU-powered strong-strong model + full lattice: SCTR code
  - Under development (K. Ohmi)
  - KEK/IHEP/J-PARC collaboration

[1] K. Ohmi, Talk presented at the 2019 SAD workshop, https://conference-indico.kek.jp/event/75/.

Status of beam-beam simulations

- Beam-beam simulations have shown that multiple factors can strongly interplay with beam-beam interaction.
  - Imperfections in linear optics: beta beat, linear couplings, dispersions, etc. at the IP
  - Geometric nonlinearities: It is crucial when $\beta_y^* < 1$ mm
  - Coupling impedances: Longitudinal and transverse
  - Space charge
  - BxB feedback
- Predictability of beam-beam simulations: The case of SuperKEKB sets demands on
  - Accurate modeling of linear optics
  - Strong-strong model of beam-beam interaction
    - X-Z instability (i.e. Beam-beam head-tail instability)
    - Synchro-betatron resonances with working points near half integers
  - Reliable impedance modeling
    - Longitudinal impedance: potential-well distortion and synchrotron tune spread
    - Transverse impedance: Betatron tune shift and spread
    - Monopolar (longitudinal potential-well distortion and transverse beam tilt), dipole (TMCI), and quadrupolar (tune shift)

Status of beam-beam simulations

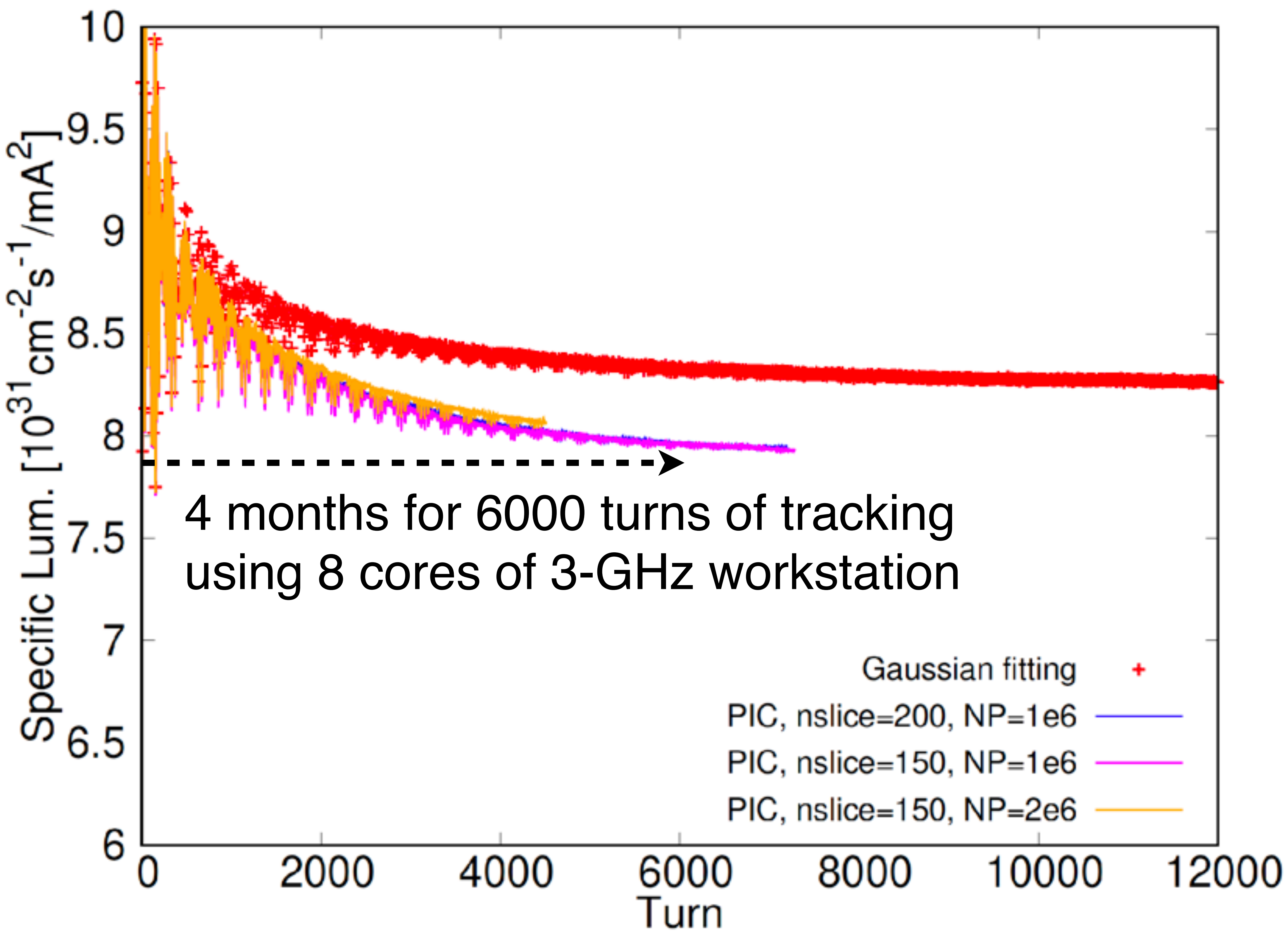
- Weak-strong model + simple one-turn map: BBWS code
  - Pros: Fast simulation of luminosity and beam-beam effects. Not require much computing resources. Used for tune survey, fast luminosity calculation, etc..
  - Cons: Strong beam frozen. Crab waist of strong beam not implemented. Not sensitive to coherent beam-beam head-tail (BBHT) instability (BBHTI).
- Weak-strong model + full lattice: SAD code
  - Pros: Relatively fast to allow tracking with lattice. Interplay of beam-beam and lattice nonlinearity. Space-charge modeling possible. Localized geometric wakes possible.
  - Cons: Same as BBWS code. Tune survey possible but relatively slow.
- Strong-strong model + simple one-turn map: BBSS code
  - Pros: Allow dynamic evolution of 3D distribution of two beams. Detect BBHTI.
  - Cons: Tracking quite slow. Not feasible for tune survey. No effective method of parallelization.

Status of beam-beam simulations

$$L_{sp} \approx \frac{1}{2\pi e^2 f \underbrace{\sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}}_{\text{“Vertical blowup”}} \underbrace{\sqrt{\sigma_{z+}^2 + \sigma_{z-}^2} \tan\frac{\theta_c}{2}}_{\text{“Longitudinal blowup”}}}$$

- BBSS simulations: PIC vs. Gaussian fitting model
  - PIC method predicts lower luminosity (~5%).
  - Using workstations(8 cores), one PIC simulation requires ~8 months, and a Gaussian-fitting simulation takes ~1.2 days.
  - Significant progress has been achieved recently in developing GPU-based BB codes. Preliminary tests showed a speed-up factor of ~50 for PIC simulations based on the CUDA compiler (K. Ohmi, in collaboration with Y. Zhang and Z. Li (IHEP), T. Yasui (J-PARC)).
    - This will speed up our investigations, especially of the interplay between beam-beam and machine imperfections.

	2021.12.21		Comments
	HER	LER	
I_{bunch} (mA)	0.8	1.0	
# bunch	-		
ε_x (nm)	4.6	4.0	w/ IBS
ε_y (pm)	35	20	Estimated from XRM data
β_x (mm)	60	80	Calculated from lattice
β_y (mm)	1	1	Calculated from lattice
σ_{z0} (mm)	5.05	4.60	Natural bunch length (w/o MWI)
ν_x	45.53	44.524	Measured tune of pilot bunch
ν_y	43.572	46.589	Measured tune of pilot bunch
ν_s	0.0272	0.0233	Calculated from lattice
Crab waist	40%	80%	Lattice design

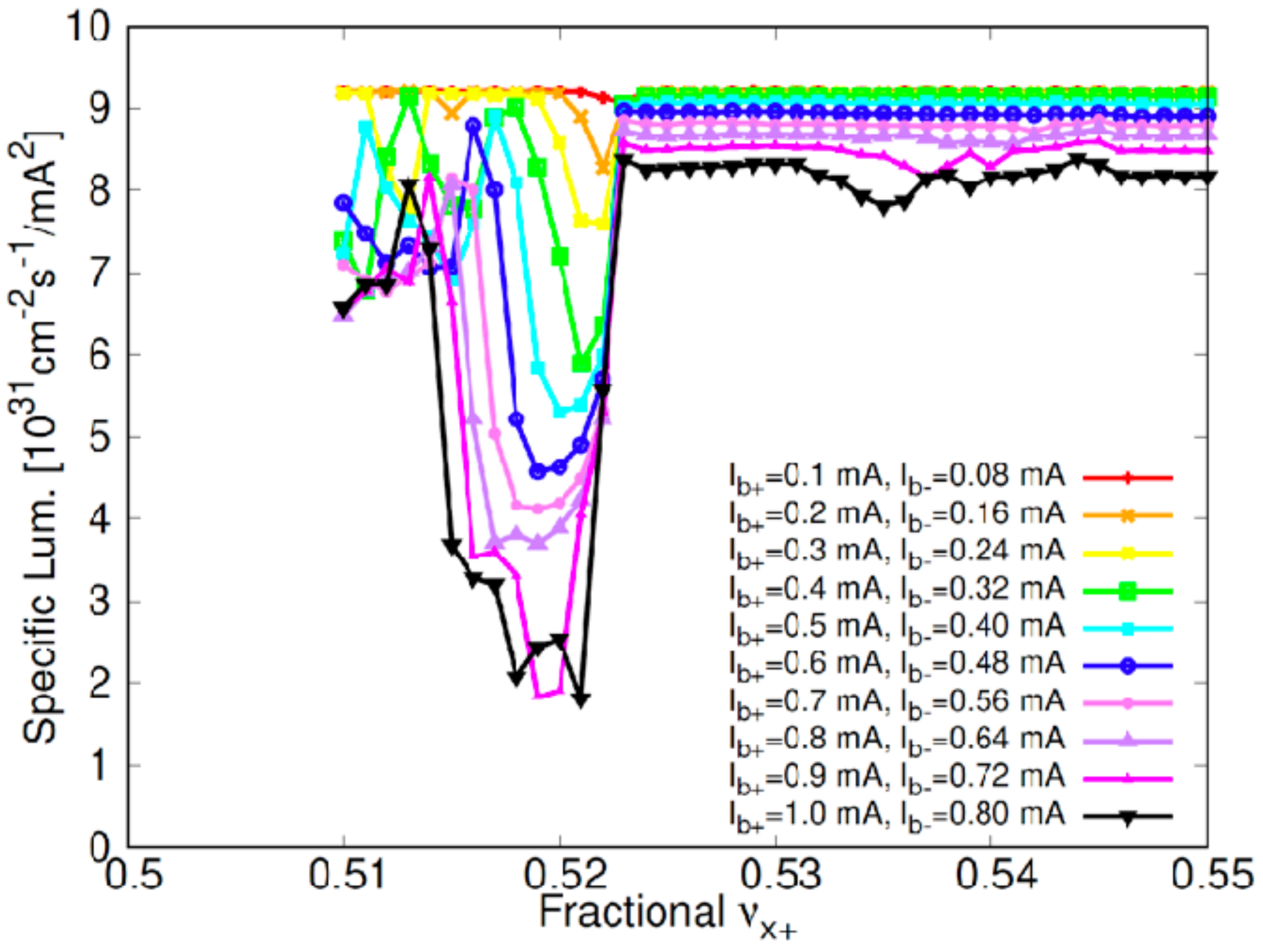


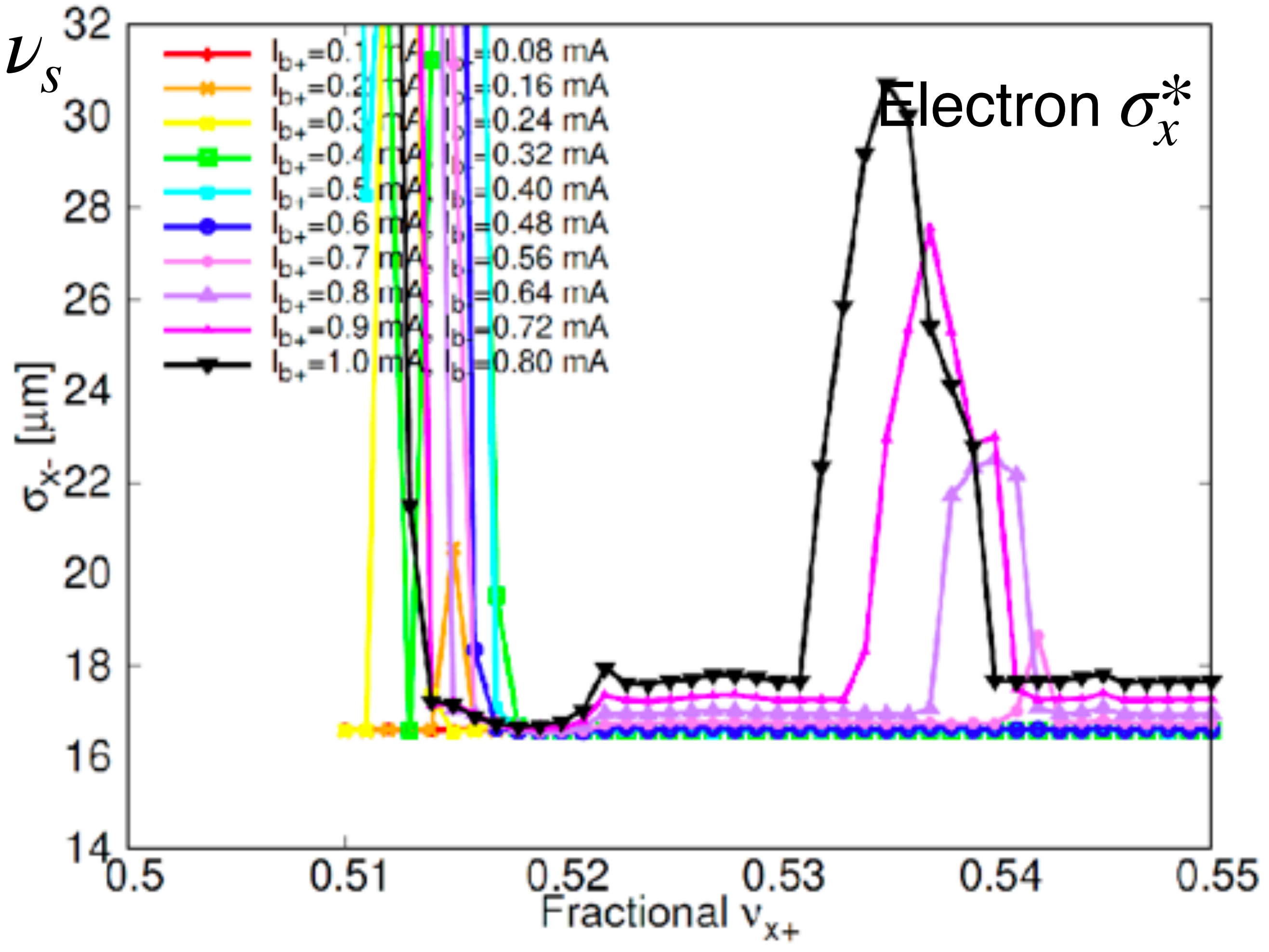
4 months for 6000 turns of tracking using 8 cores of 3-GHz workstation

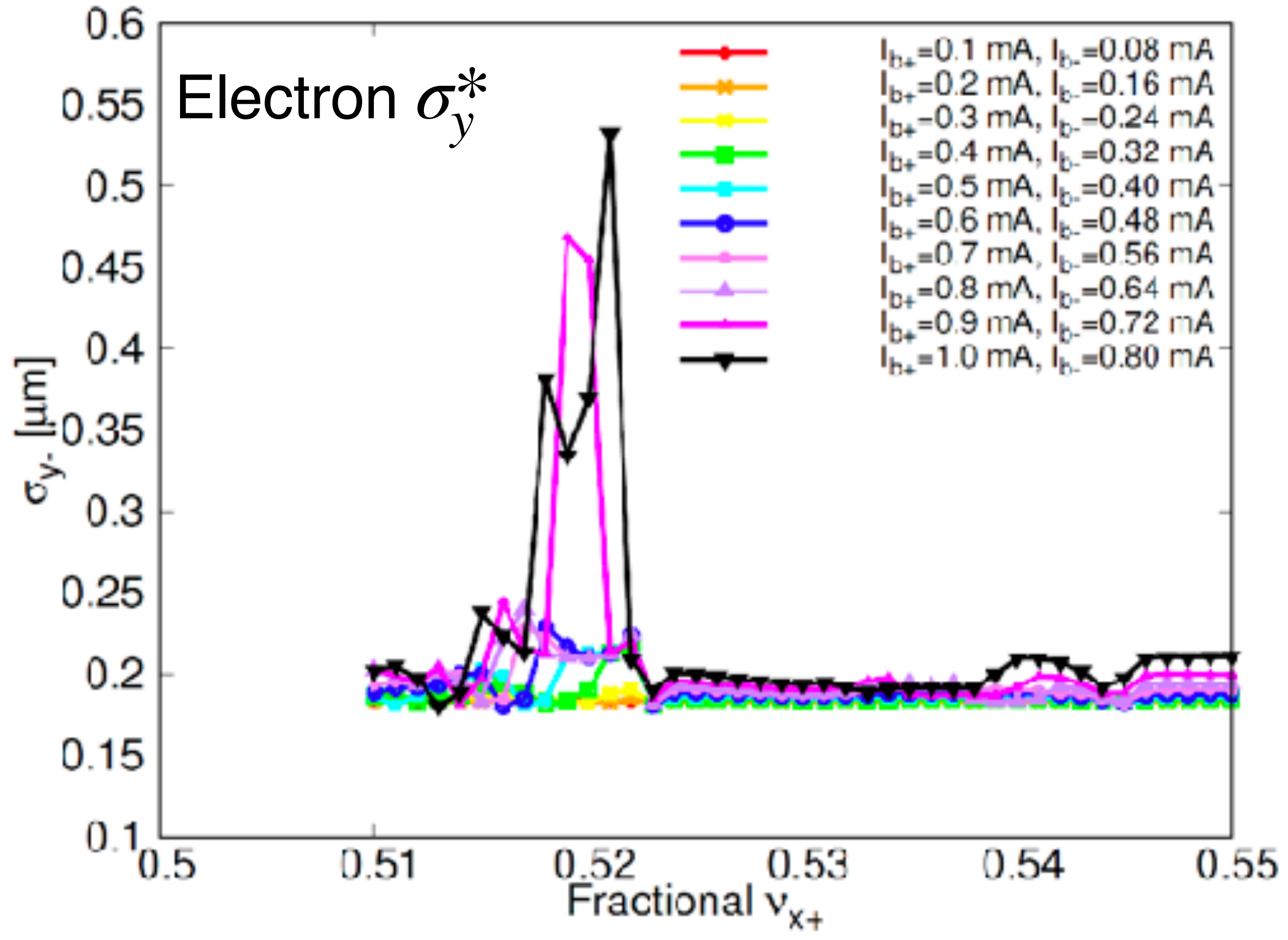
Status of beam-beam simulations

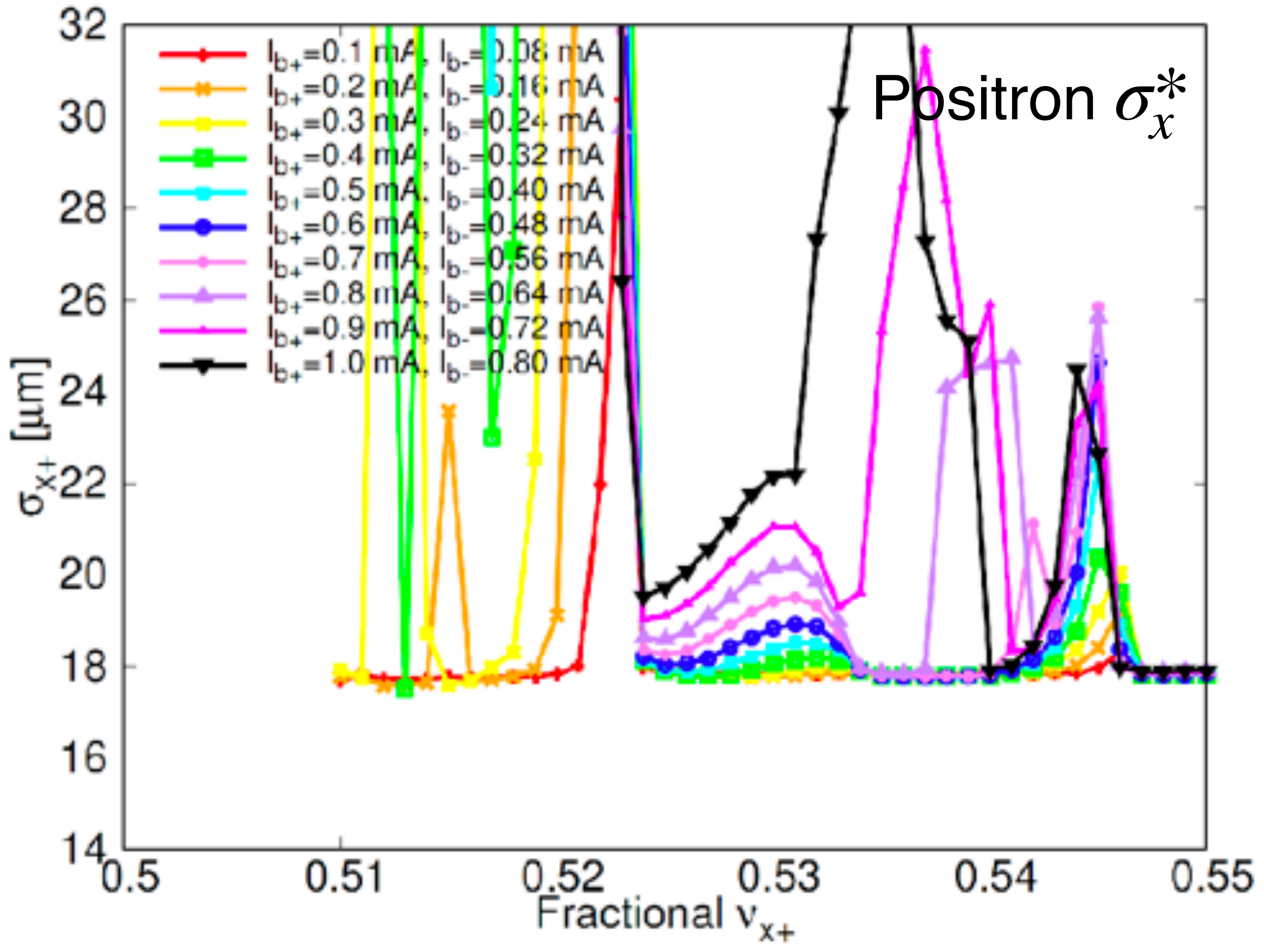
- Scan LER ν_x (with LER ν_y and HER $\nu_{x,y}$ fixed as the values of the parameter table of 2021.12.21)
  - Coupling impedances included
  - Weak horizontal blowup when $0.5 + \nu_s < [\nu_x] < 0.5 + 1.5\nu_s$

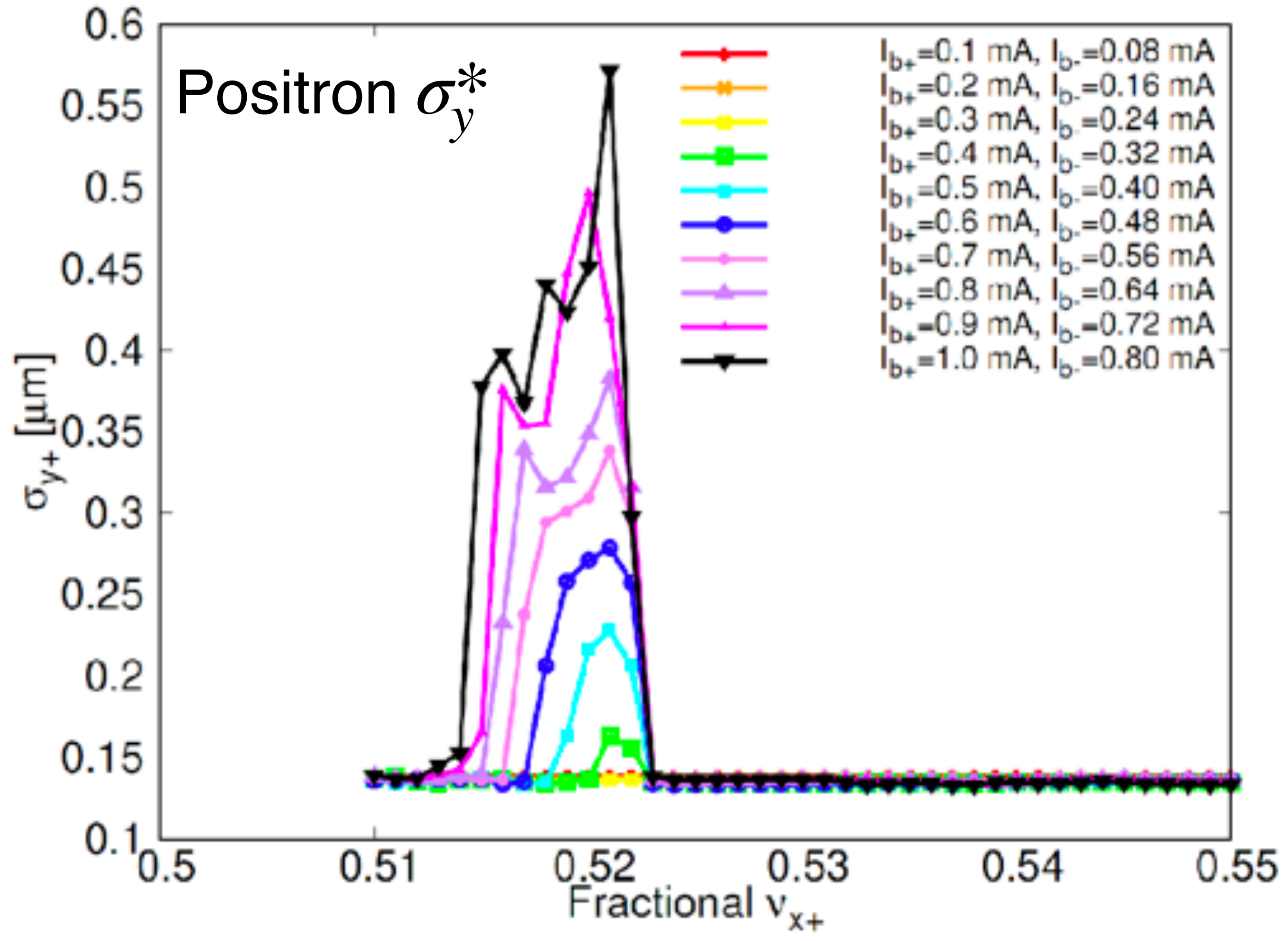
X-Z instability is sensitive to ν_x.









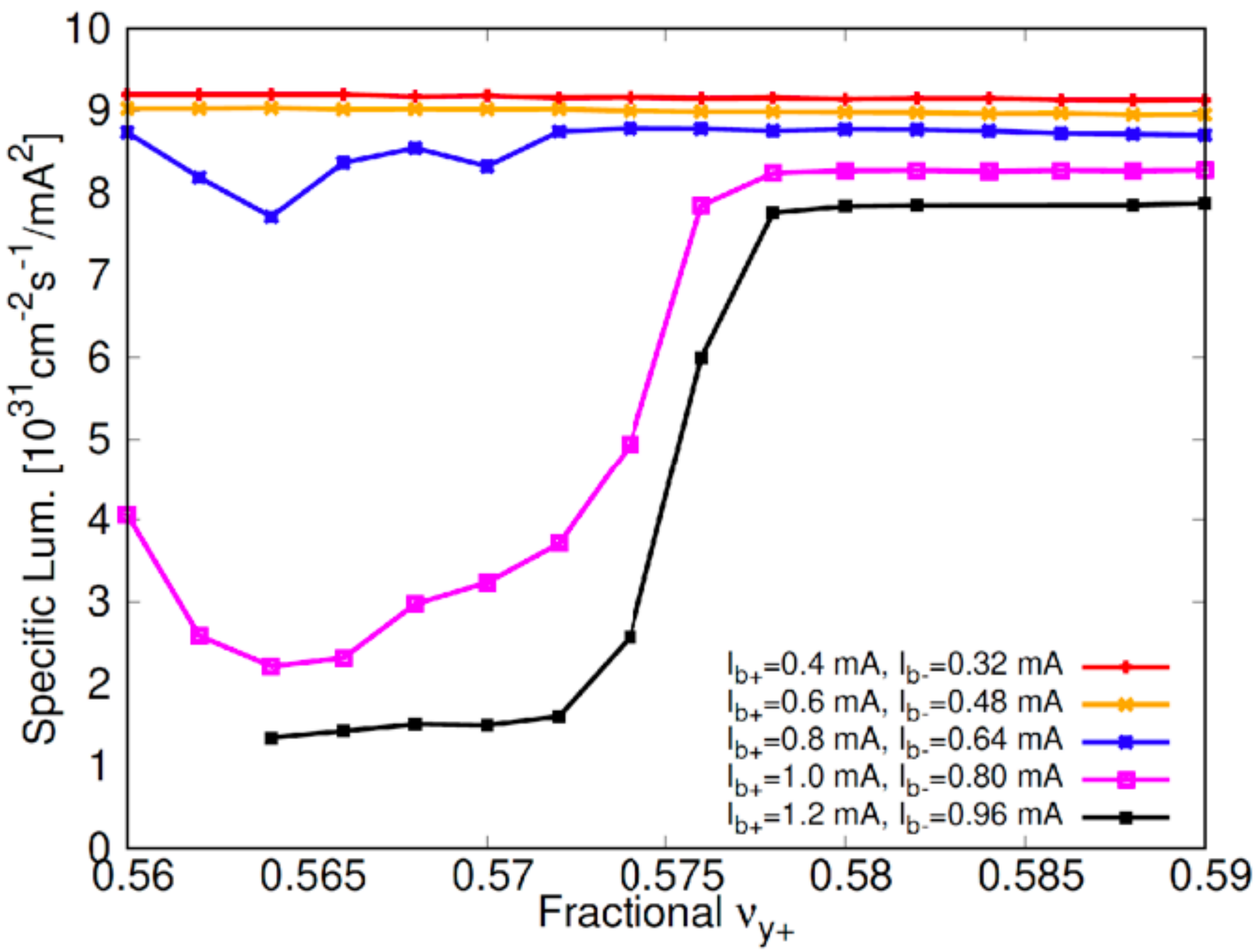


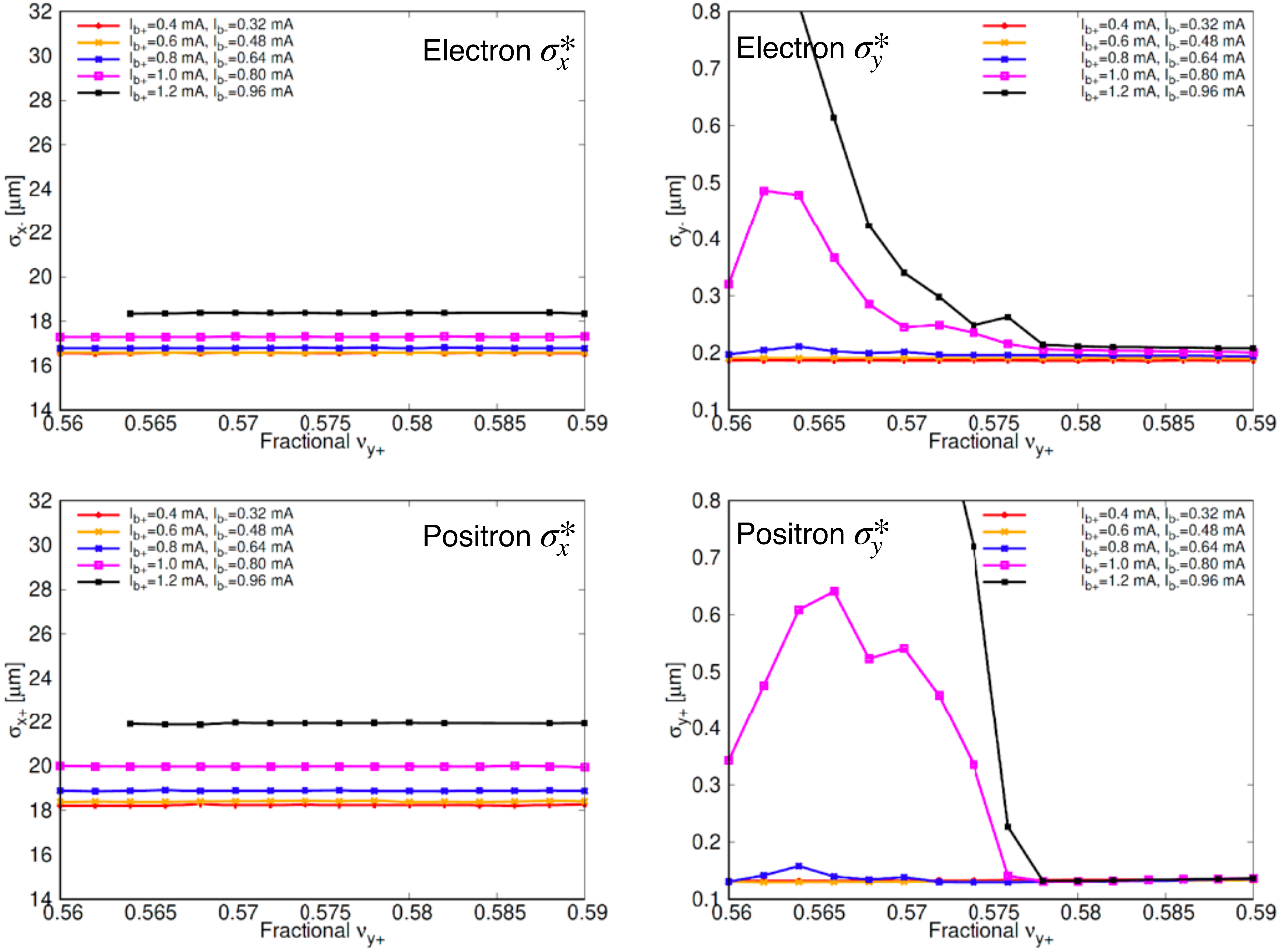
Status of beam-beam simulations

- BBSS simulations: Scan LER ν_y with bunch currents varied (with LER ν_x and HER $\nu_{x,y}$ fixed as the values of the parameter table of 2021.12.21, BB+Wxy+Wz)

* The interplay of BB+Wx,y+Wz causes instability, consistent with Y. Zhang and K. Ohmi's findings.

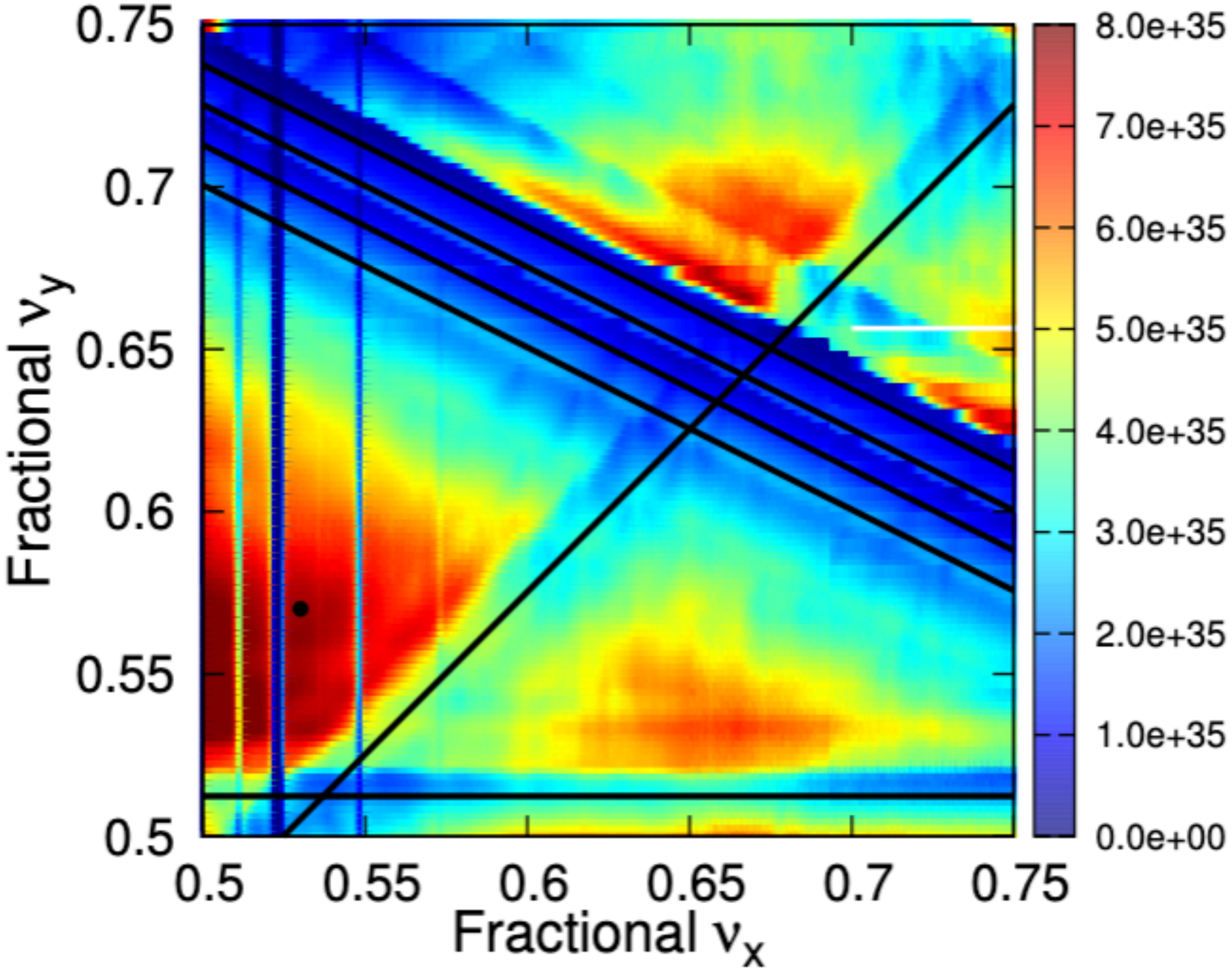
* This instability has a threshold that is ν_y-dependent.

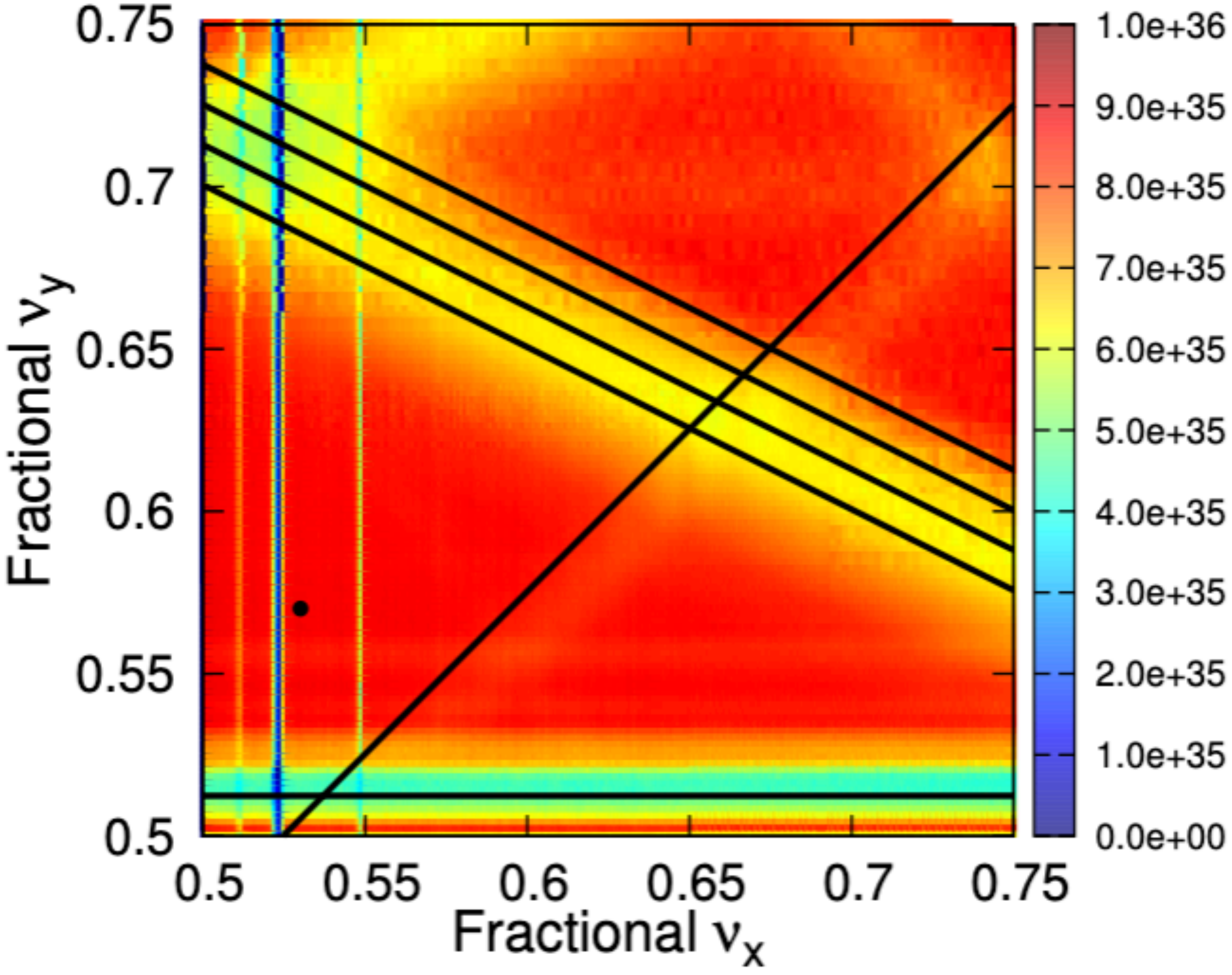




Crab waist applied to SuperKEKB

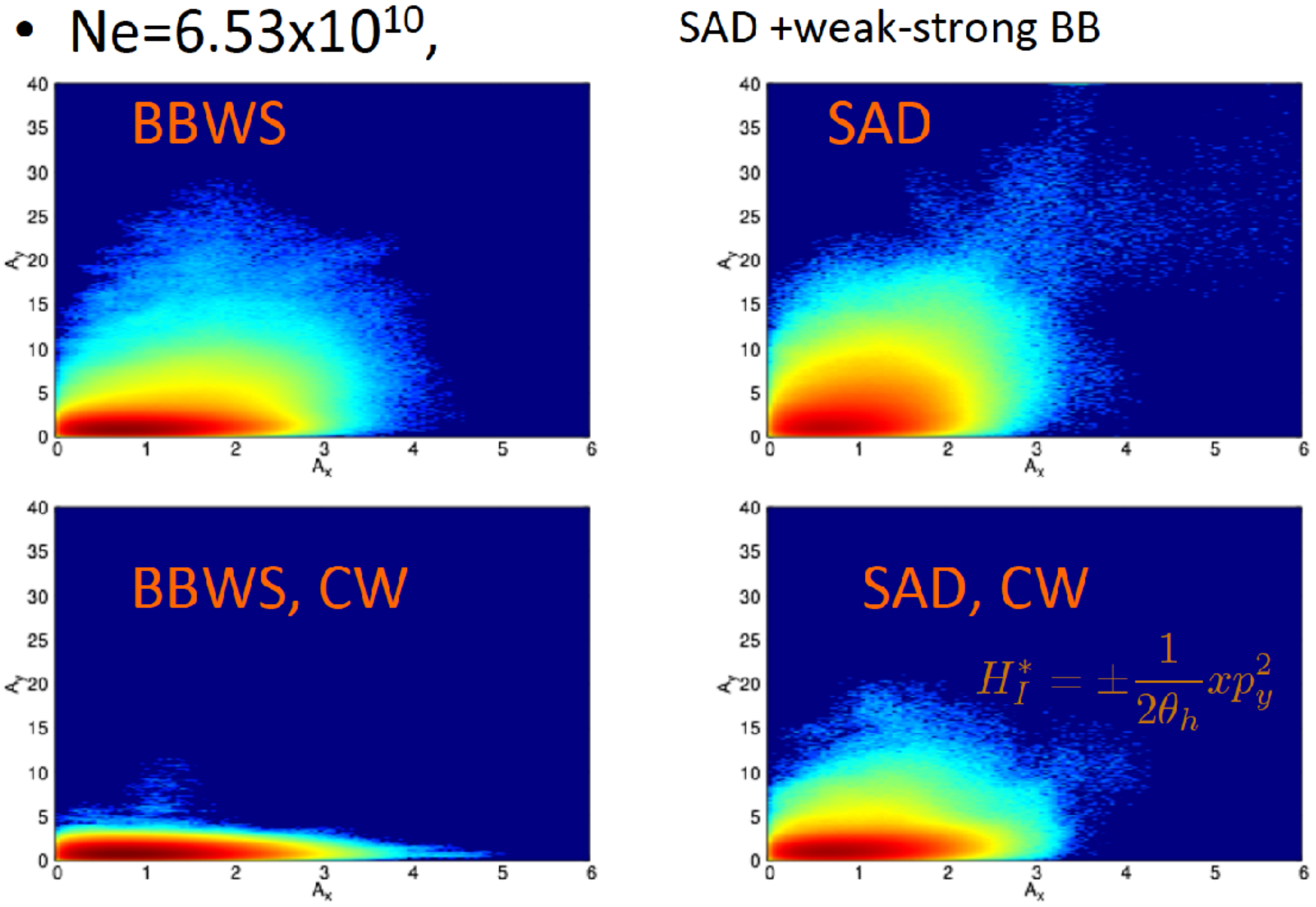
- SuperKEKB final design ($\beta_y^* = 0.3/0.27$ mm) with ideal crab waist
  - Tune scans using BBWS
  - Crab waist creates large area in tune space for choice of working point





Crab waist applied to SuperKEKB

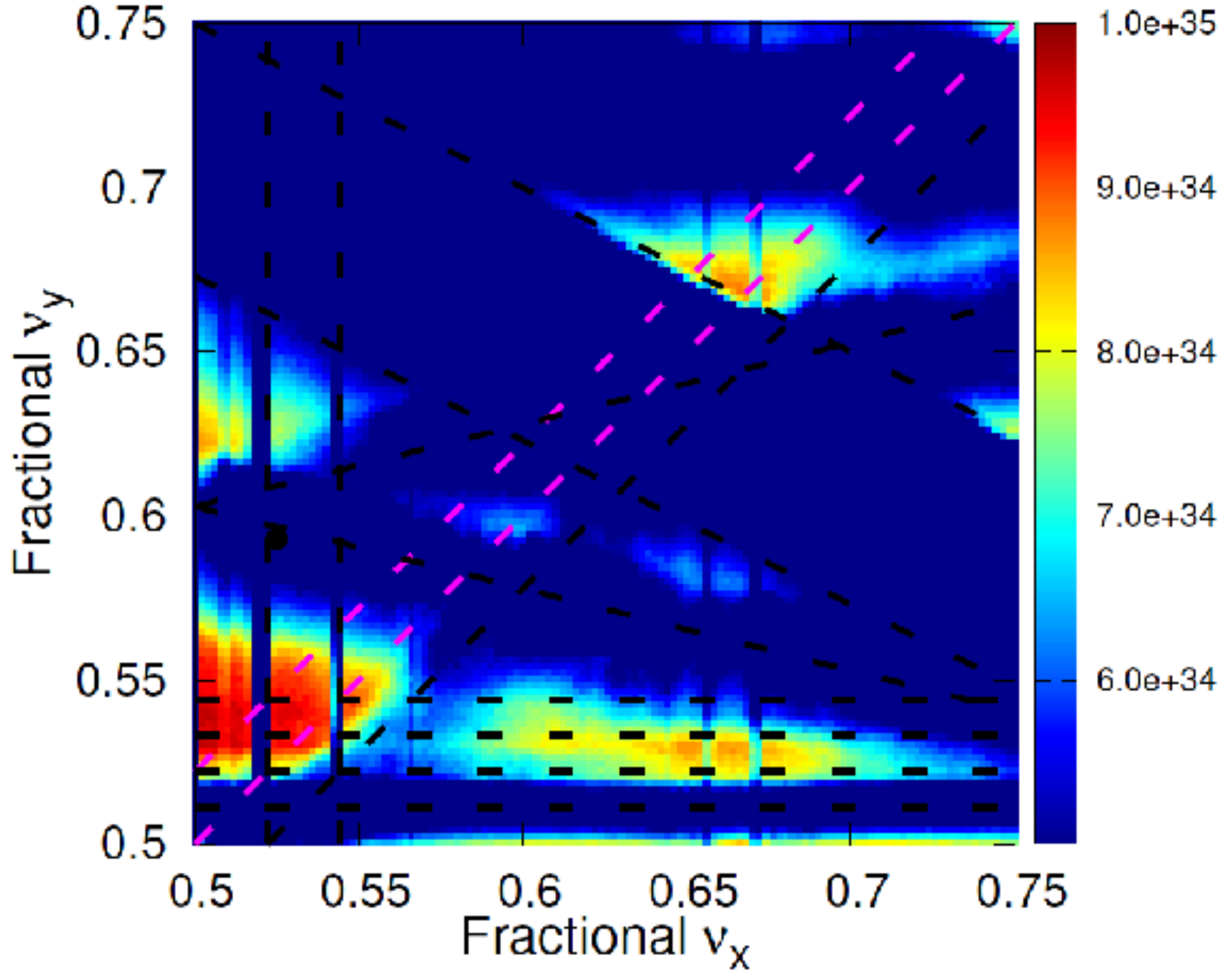
- SuperKEKB final design ($\beta_y^* = 0.3/0.27$ mm) with ideal crab waist
  - Beam-beam driven halo can be suppressed

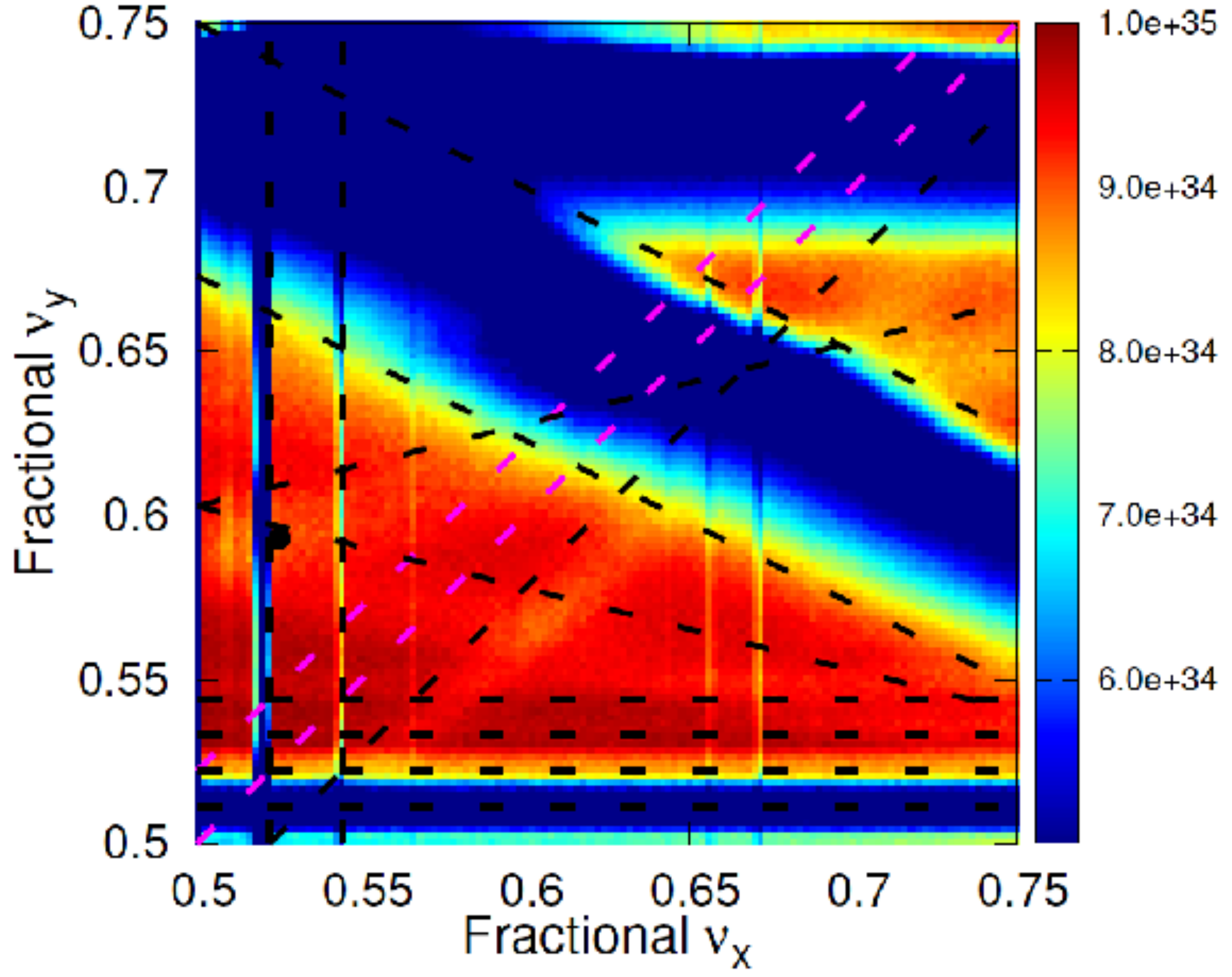


Crab waist applied to SuperKEKB

- SuperKEKB 2021b run ($\beta_y^* = 1$ mm) with ideal crab waist
  - Tune scan using BBWS showed that 80% crab waist ratio in LER is effective in suppressing vertical blowup caused by beam-beam resonances (mainly $\nu_x \pm 4\nu_y + \alpha = N$).

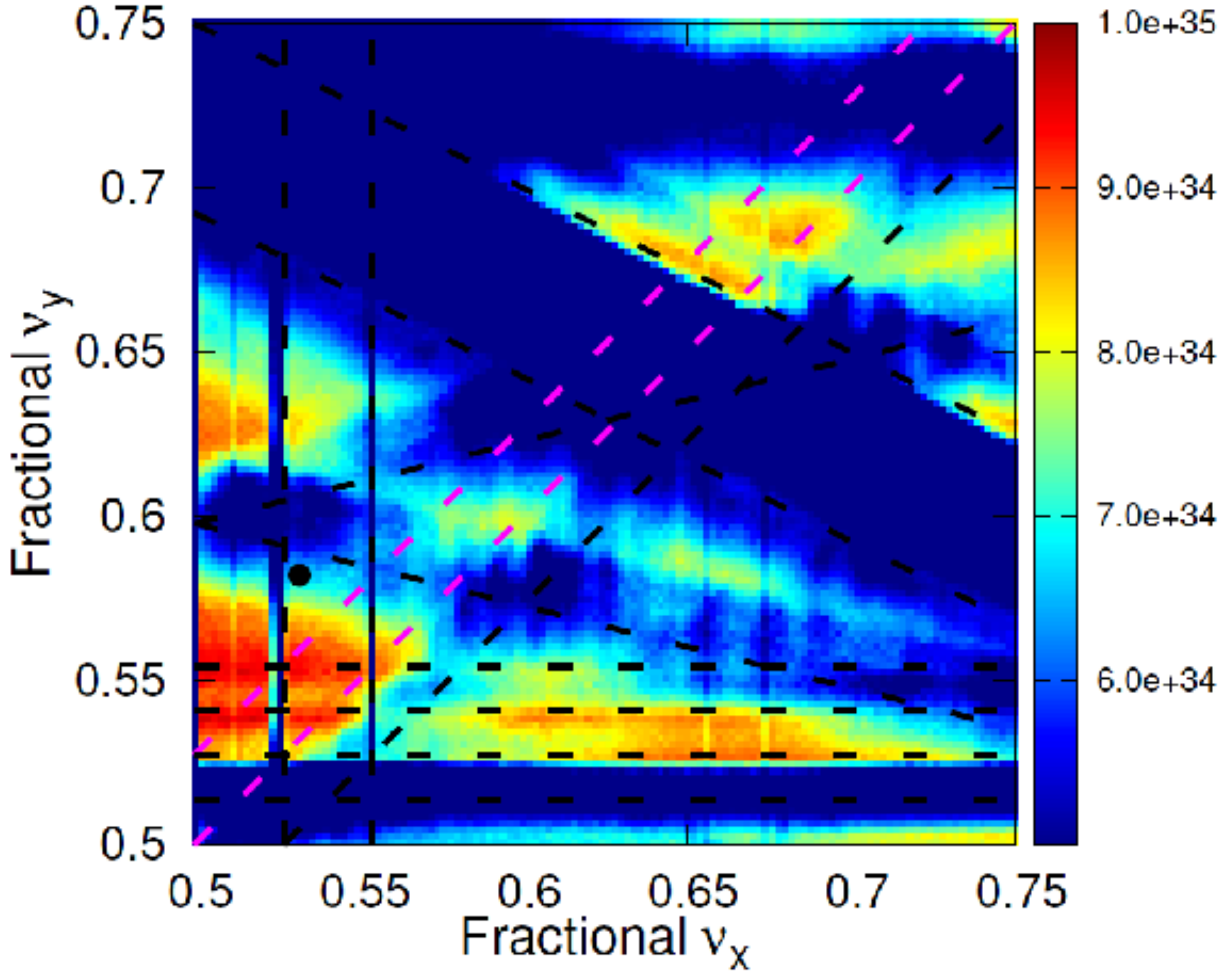
	2021.07.01		Comments
	HER	LER	
I_{bunch} (mA)	0.80	1.0	
# bunch	1174		Assumed value
ε_x (nm)	4.6	4.0	w/ IBS
ε_y (pm)	23	23	Estimated from XRM data
β_x (mm)	60	80	Calculated from lattice
β_y (mm)	1	1	Calculated from lattice
σ_{z0} (mm)	5.05	4.84	Natural bunch length (w/o MWI)
ν_x	45.532	44.525	Measured tune of pilot bunch
ν_y	43.582	46.593	Measured tune of pilot bunch
ν_s	0.0272	0.0221	Calculated from lattice
Crab waist	40%	80%	Lattice design

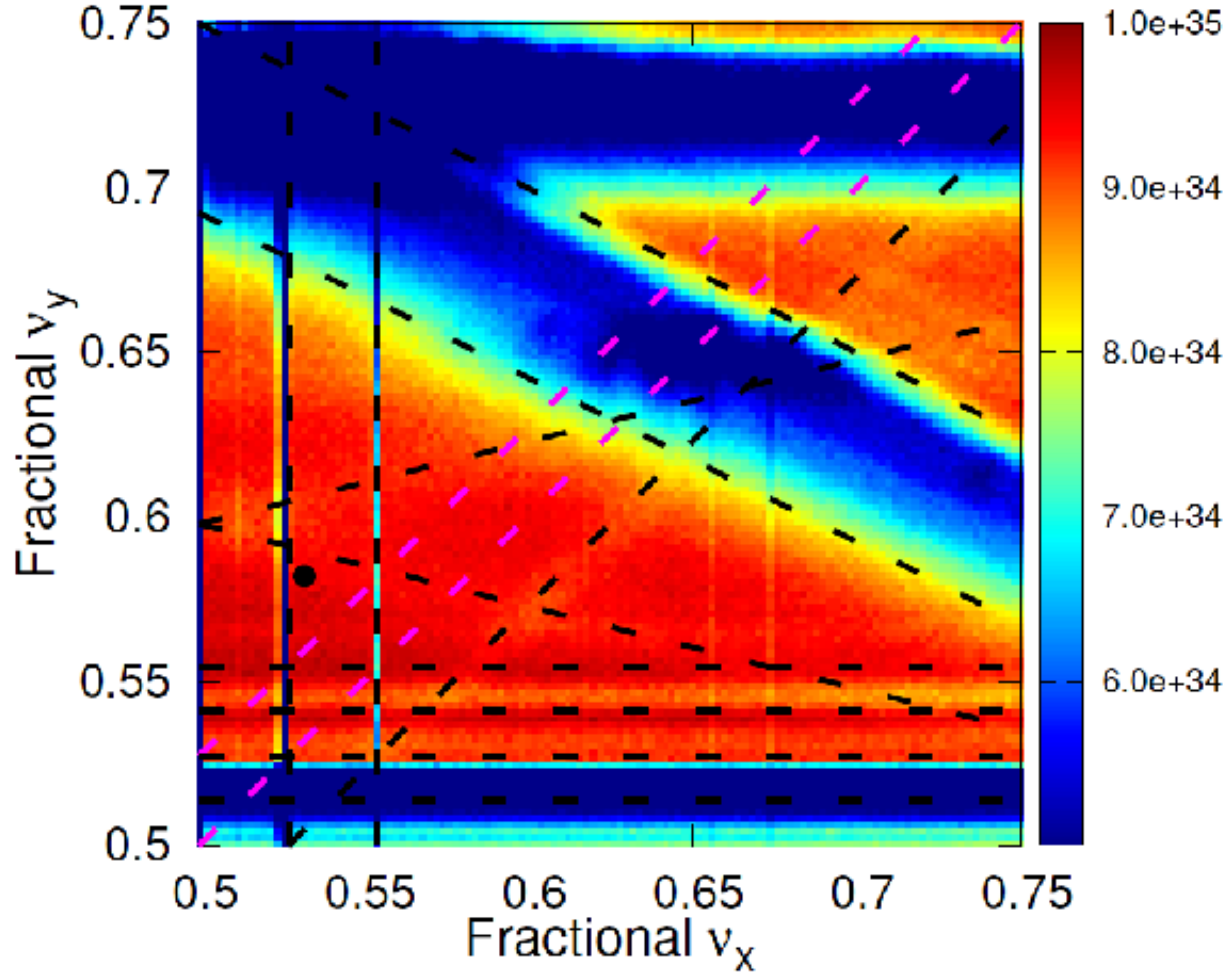

Lum. w/o crab waist in LER

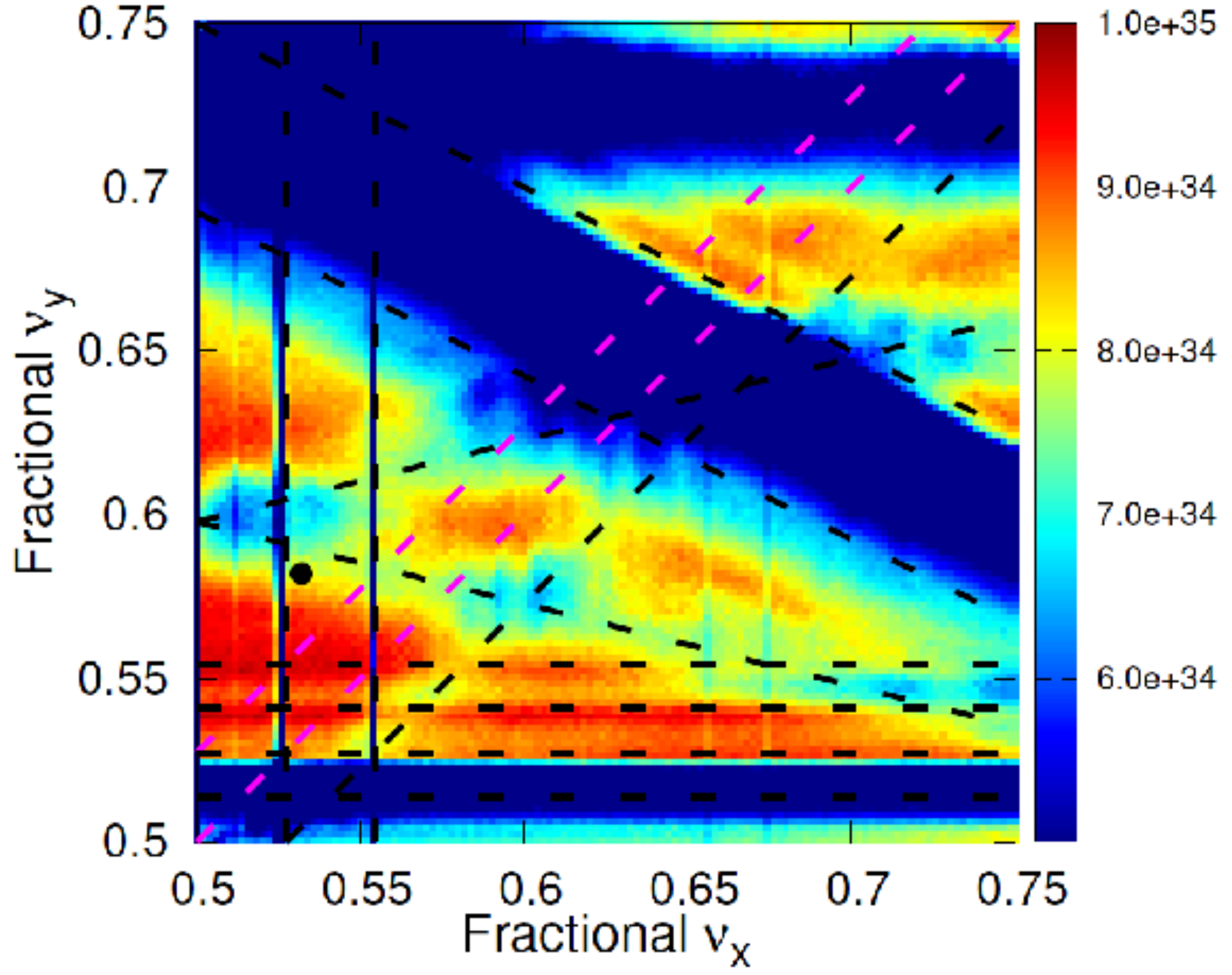

Lum. w/ 80% crab waist in LER

Crab waist applied to SuperKEKB

- SuperKEKB 2021b run ($\beta_y^* = 1$ mm) with ideal crab waist
    - Tune scan using BBWS showed that 40% crab waist ratio (current operation condition) in HER is not enough for suppressing vertical blowup caused by beam-beam resonances (mainly $\nu_x \pm 4\nu_y + \alpha = N$).

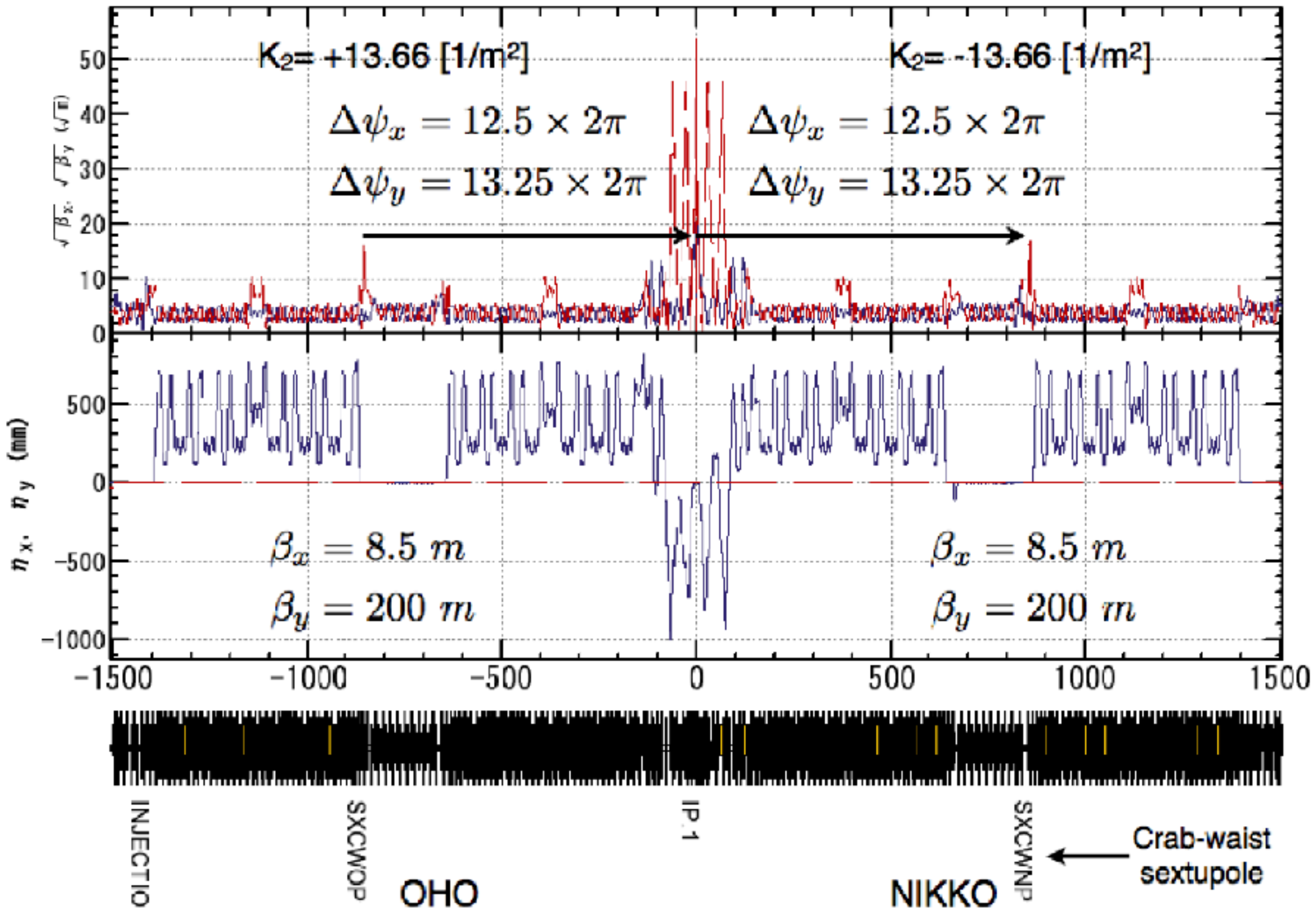

Lum. w/o crab waist in HER


Lum. w/ 80% crab waist in HER


Lum. w/ 40% crab waist in HER

Crab waist applied to SuperKEKB

- SuperKEKB final design ($\beta_y^* = 0.3/0.27$ mm) with practical crab waist [1]
  - CW scheme with CW sextupoles outside IR
  - CW reduces dynamic aperture and Touschek lifetime, and was not chosen as baseline for TDR



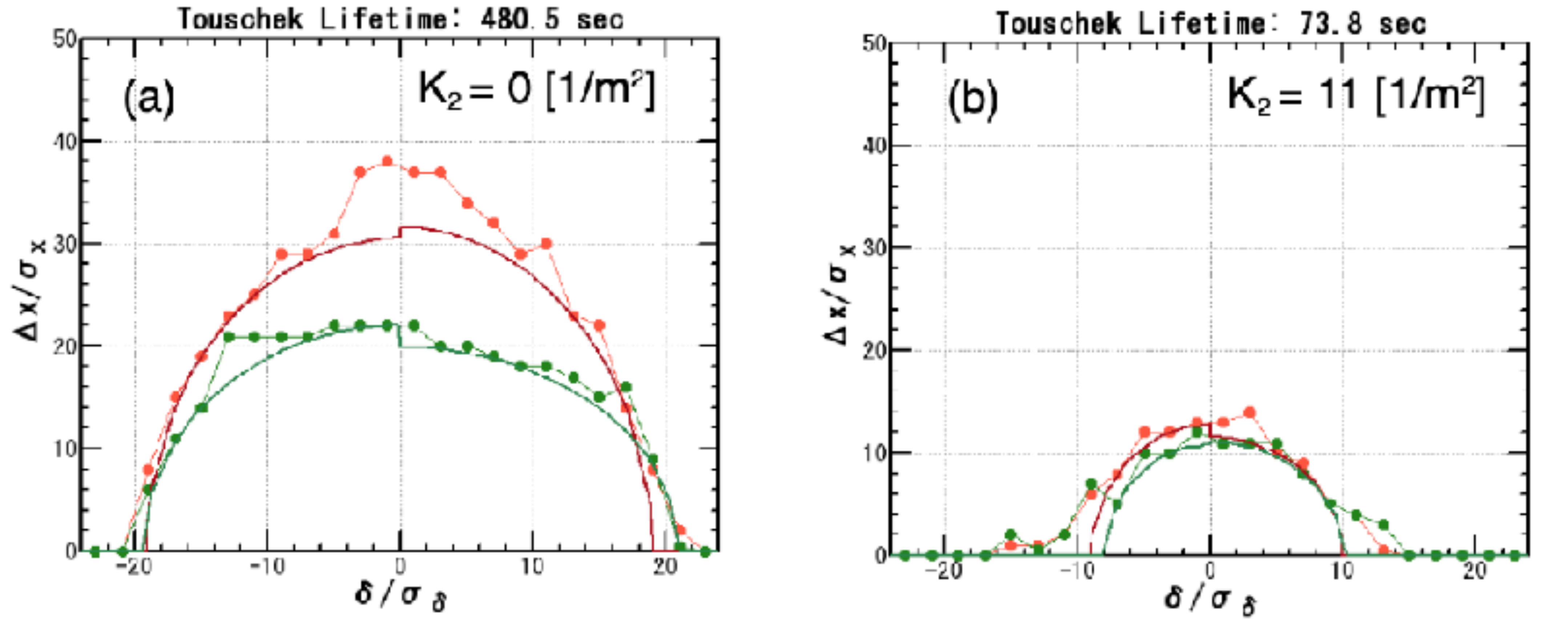
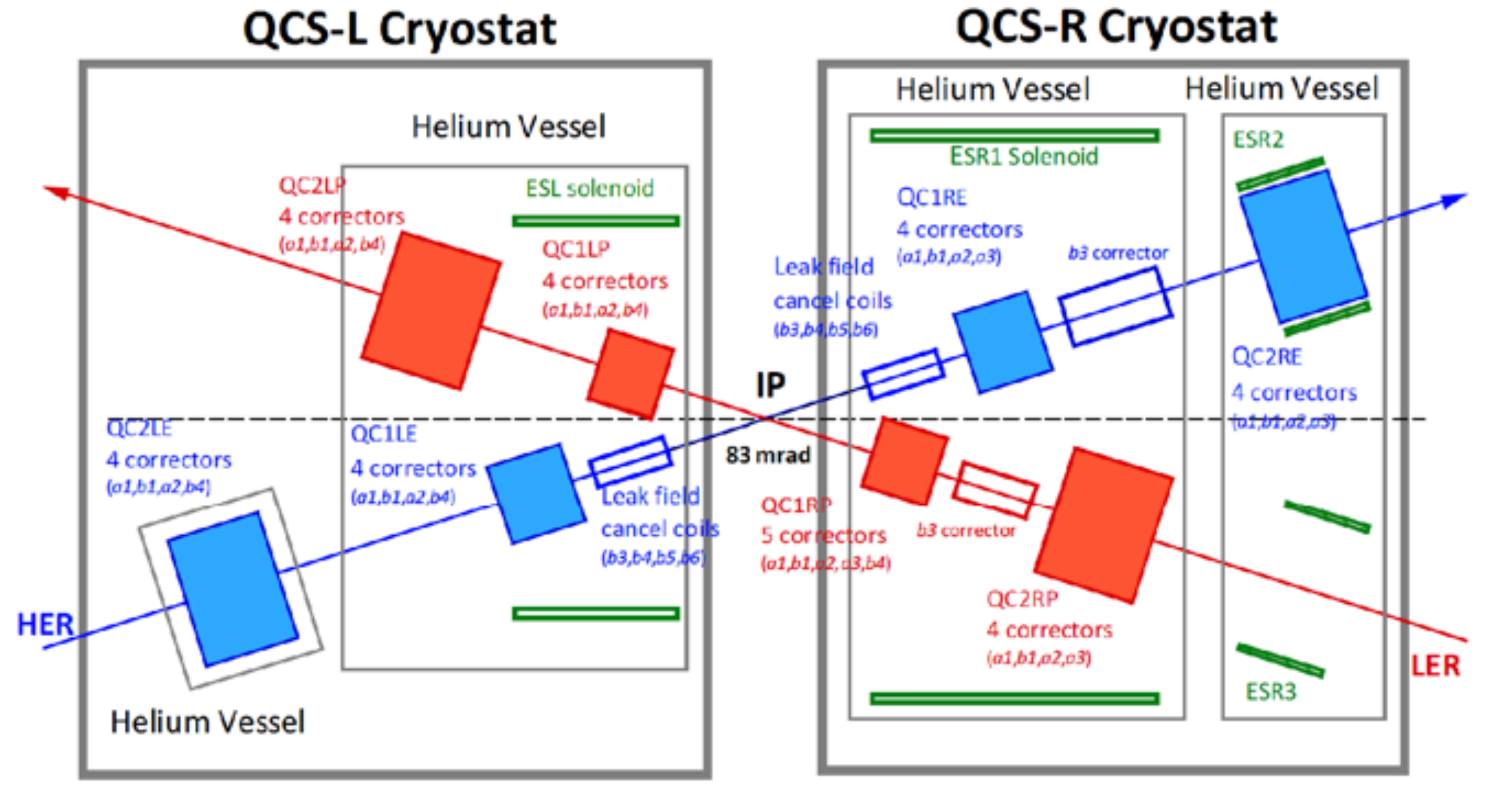


Figure 4.28: Dynamic aperture in the LER crab-waist lattice without beam-beam effect. Initial ratio of the vertical to the horizontal amplitude is 0.27 %. (a) $K_2 = 0$ [1/m^2], (b) $K_2 = 11$ [1/m^2].

[1] SuperKEKB TDR.

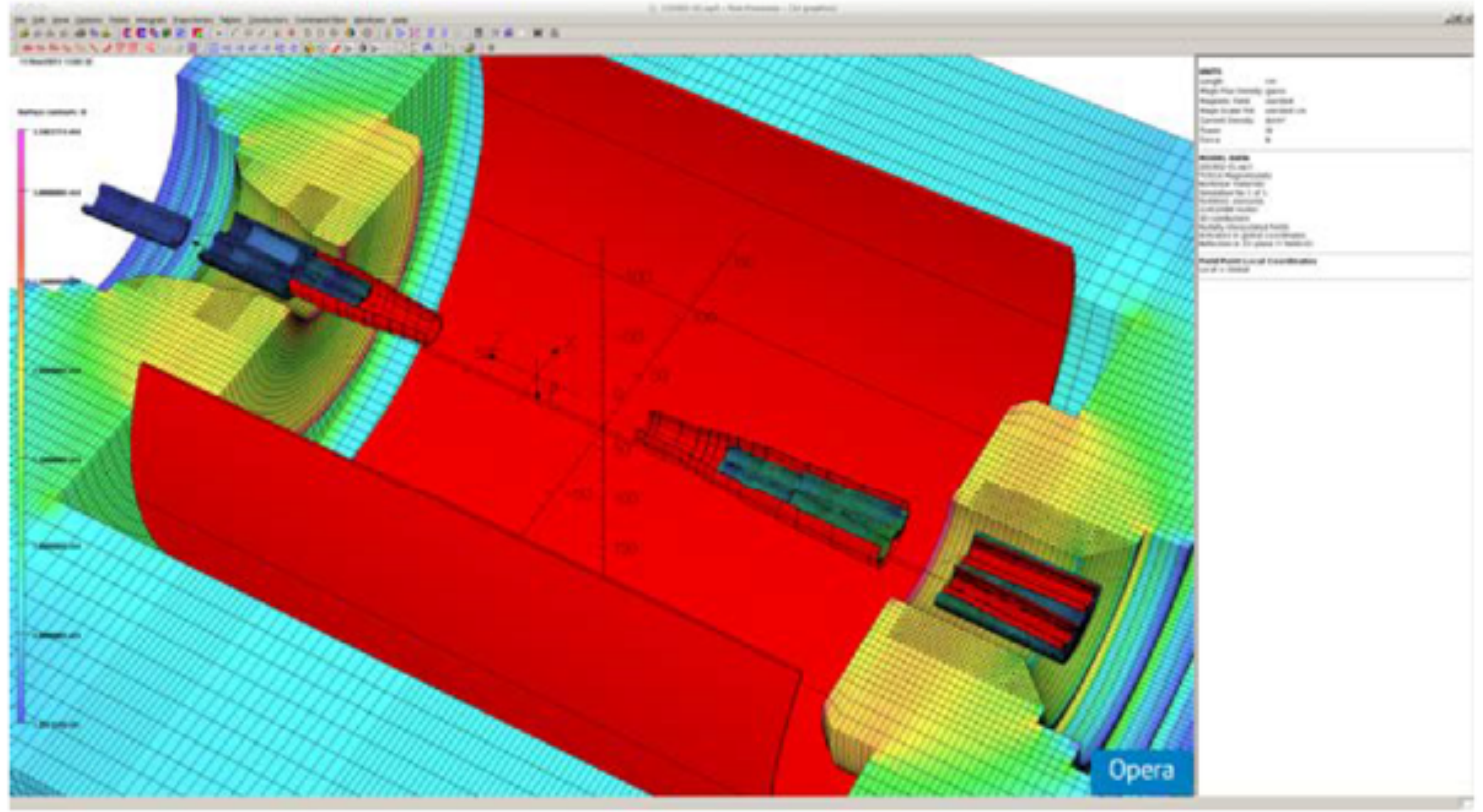
Crab waist applied to SuperKEKB

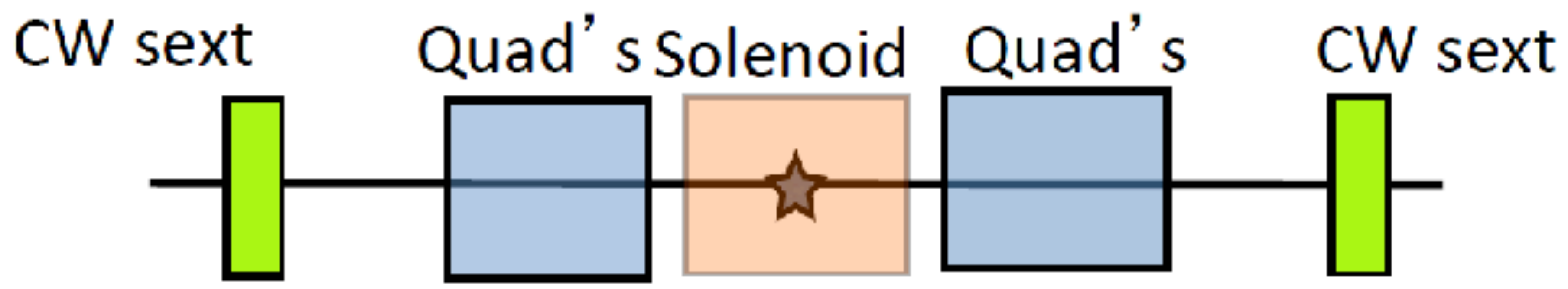
- SuperKEKB final design ($\beta_y^* = 0.3/0.27$ mm) with practical crab waist
  - CW does not work well because of the nonlinear IR. The nonlinearity scales as $1/\beta_y^*$ [1].
  - SuperKEKB design lattice includes nonlinear fields extracted from 3D model [2]



4 SC main quadrupole magnets: 1 collared magnet, 3 yoked magnets
16 SC correctors: a1, b1, a2, b4
4 SC leak field cancel magnets: b3, b4, b5, b6
1 compensation solenoid

4 SC main quadrupole magnets: 1 collared magnet, 3 yoked magnets
19 SC correctors: a1, b1, a2, a3, b3, b4
4 SC leak field cancel magnets: b3, b4, b5, b6
3 compensation solenoid





$$\mathcal{M}_{IR} = \underbrace{e^{-axy^2}}_{\text{crab waist sext}} e^{-H_{Q's}} e^{-H_{Sol}} e^{-H_{BB}} e^{-H_{Sol}} e^{-H_{Q's}} \underbrace{e^{-axy^2}}_{\text{crab waist sext}}$$

$$\neq$$

$$e^{-H_{Q's}} e^{-H_{Sol}} \overset{\text{ideal crab waist}}{e^{-xp_y^2/2\phi}} e^{-H_{BB}} \overset{\text{ideal crab waist}}{e^{-xp_y^2/2\phi}} e^{-H_{Sol}} e^{-H_{Q's}}$$

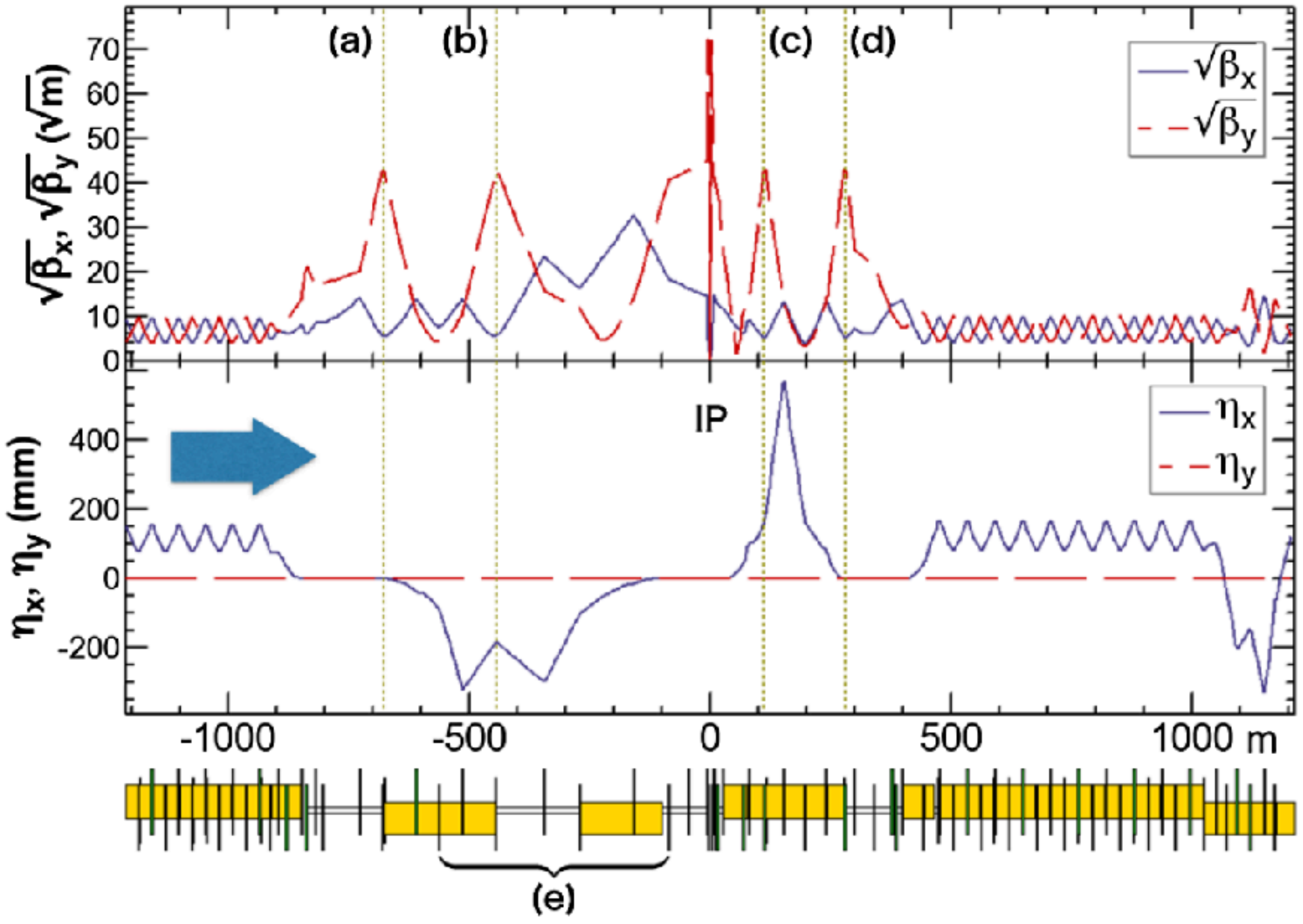
[1] K. Ohmi, EIC workshop, March, 2014.

[2] N. Ohuchi, SuperKEKB ARC, 2018.

Crab waist applied to SuperKEKB

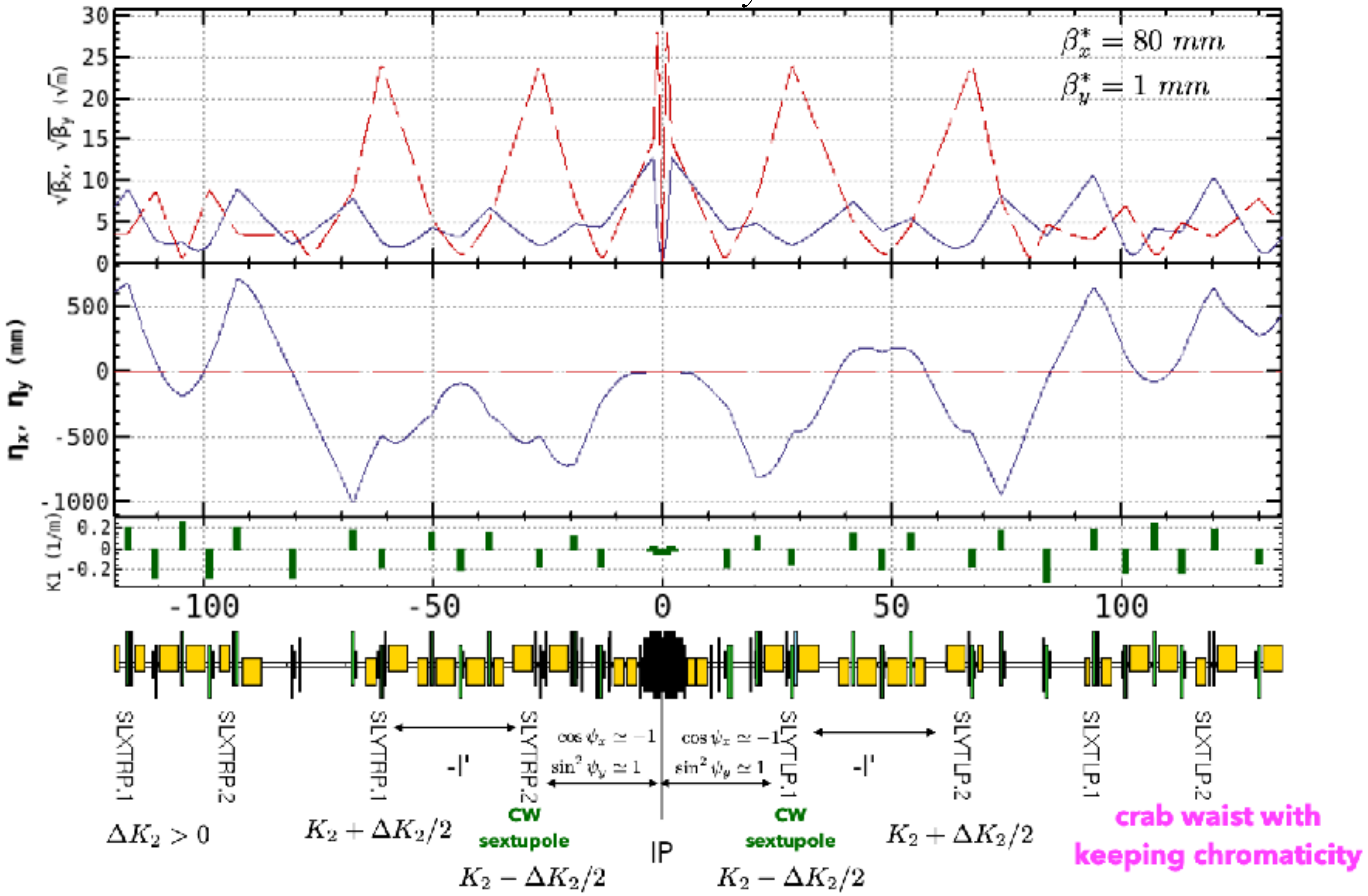
- Optics design with crab waist for $\beta_y^* = 1$ mm
  - In 2020, K. Oide introduced the FCC-ee CW scheme [1] to SuperKEKB [2].
  - FCC-ee CW scheme utilizes the sextupoles (a-d) for local chromaticity correction and crab waist.

FCC-ee



[1] K. Oide et al., PRAB 19, 111005 (2016).

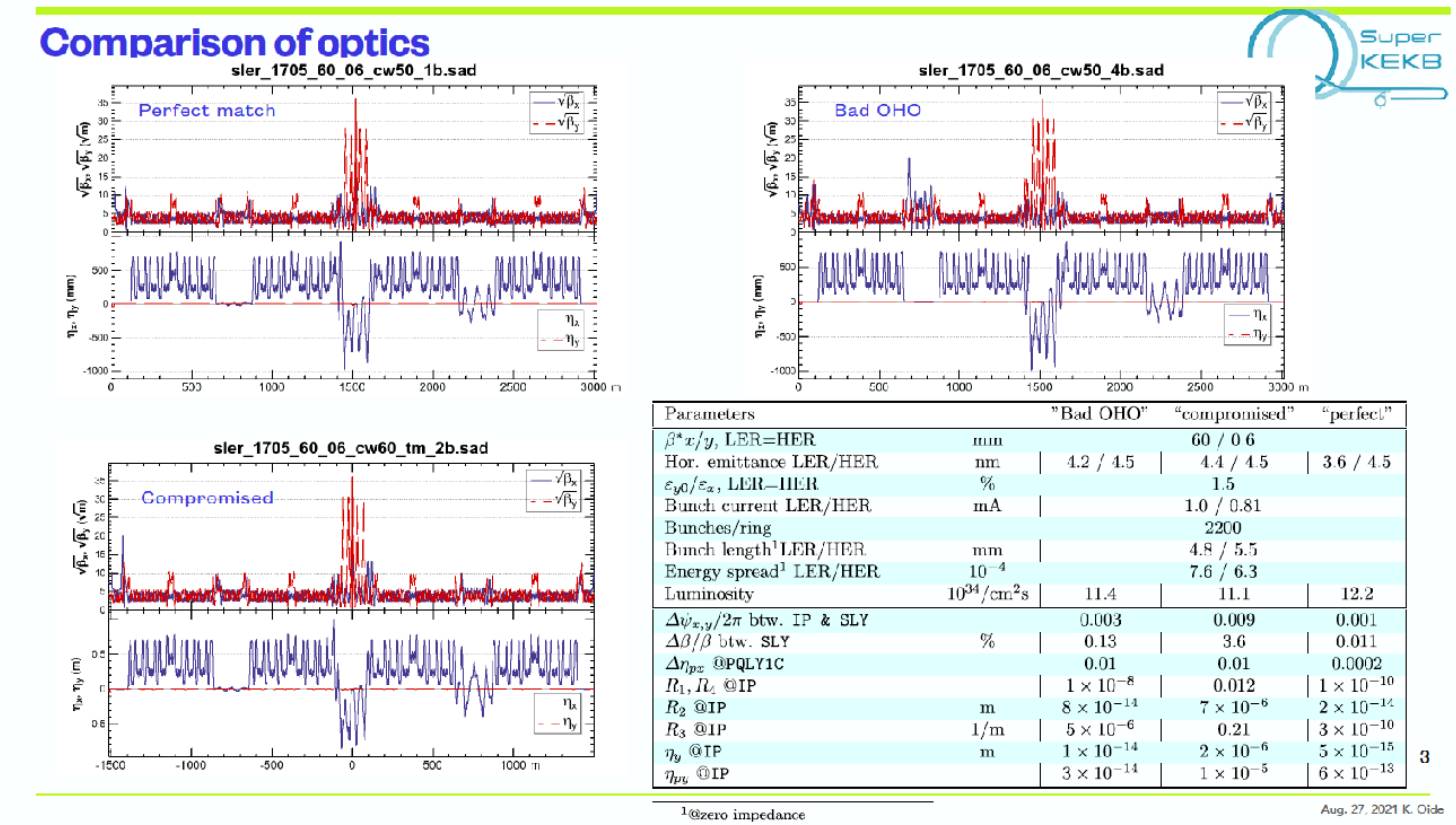
SuperKEKB w/ $\beta_y^* = 1$ mm



[2] Y. Ohnishi, SuperKEKB ARC 2020.

Crab waist applied to SuperKEKB

- Optics design with crab waist for $\beta_y^* = 0.6$ mm by K. Oide [1]



Comparison of optics

Parameters		"Bad OHO"	"compromised"	"perfect"
$\beta^* x/y$, LER=HER	mm	60 / 0 6		
Hor. emittance LER/HER	nm	4.2 / 4.5	4.4 / 4.5	3.6 / 4.5
$\varepsilon_{y0}/\varepsilon_x$, LER=HER	%	1.5		
Bunch current LER/HER	mA	1.0 / 0.81		
Bunches/ring		2200		
Bunch length[1] LER/HER	mm	4.8 / 5.5		
Energy spread[1] LER/HER	10^{-4}	7.6 / 6.3		
Luminosity	$10^{34}/\mathrm{cm}^2\mathrm{s}$	11.4	11.1	12.2
$\Delta\psi_{x,y}/2\pi$ btw. IP & SLY		0.003	0.009	0.001
$\Delta\beta/\beta$ btw. SLY	%	0.13	3.6	0.011
$\Delta\eta_{px}$ @PQLY1C		0.01	0.01	0.0002
R_1, R_4 @IP		1×10^{-8}	0.012	1×10^{-10}
R_2 @IP	m	8×10^{-14}	7×10^{-6}	2×10^{-14}
R_3 @IP	1/m	5×10^{-6}	0.21	3×10^{-10}
η_y @IP	m	1×10^{-14}	2×10^{-6}	5×10^{-15}
η_{py} @IP		3×10^{-14}	1×10^{-5}	6×10^{-13}

3

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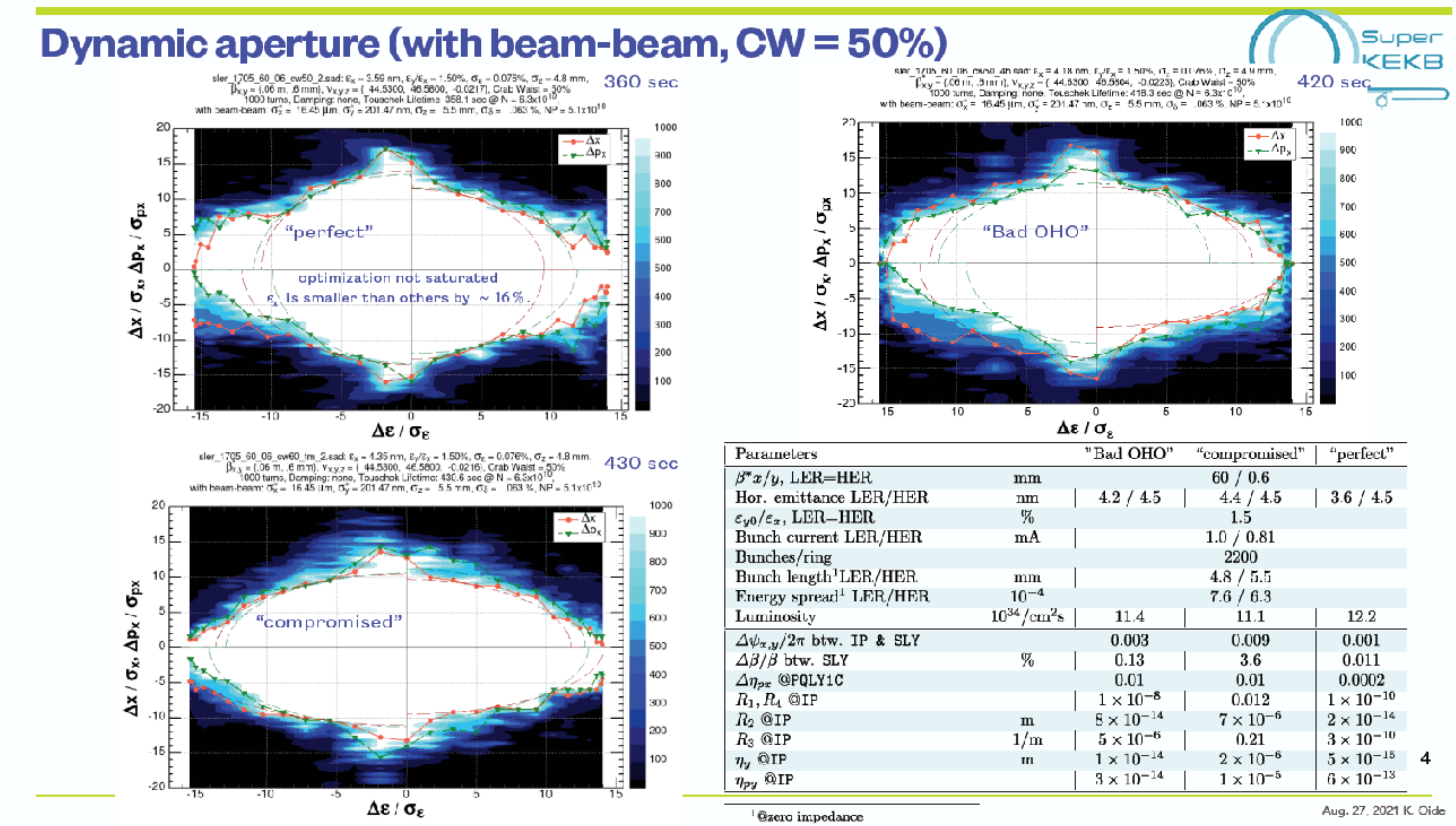
[1] @zero impedance

Aug. 27, 2021 K. Oide

[1] K. Oide, SuperKEKB ARC, 2021.

Crab waist applied to SuperKEKB

- Optics design with crab waist for $\beta_y^* = 0.6$ mm by K. Oide [1]
  - With 50% CW strength, lifetime is acceptable for beam operation



Dynamic aperture (with beam-beam, CW = 50%)

Parameters		"Bad OHO"	"compromised"	"perfect"
$\beta^* x/y$, LER=HER	mm	60 / 0.6		
Hor. emittance LER/HER	nm	4.2 / 4.5	4.4 / 4.5	3.6 / 4.5
$\varepsilon_{y0}/\varepsilon_x$, LER=HER	%	1.5		
Bunch current LER/HER	mA	1.0 / 0.81		
Bunches/ring		2200		
Bunch length[1] LER/HER	mm	4.8 / 5.5		
Energy spread[1] LER/HER	10^{-4}	7.6 / 6.3		
Luminosity	$10^{34}/\mathrm{cm}^2\mathrm{s}$	11.4	11.1	12.2
$\Delta\psi_{x,y}/2\pi$ btw. IP & SLY		0.003	0.009	0.001
$\Delta\beta/\beta$ btw. SLY	%	0.13	3.6	0.011
$\Delta\eta_{px}$ @PQLY1C		0.01	0.01	0.0002
R_1, R_4 @IP		1×10^{-8}	0.012	1×10^{-10}
R_2 @IP	m	8×10^{-14}	7×10^{-6}	2×10^{-14}
R_3 @IP	1/m	5×10^{-6}	0.21	3×10^{-10}
η_y @IP	m	1×10^{-14}	2×10^{-6}	5×10^{-15}
η_{py} @IP		3×10^{-14}	1×10^{-5}	6×10^{-13}

---

[1] @zero impedance

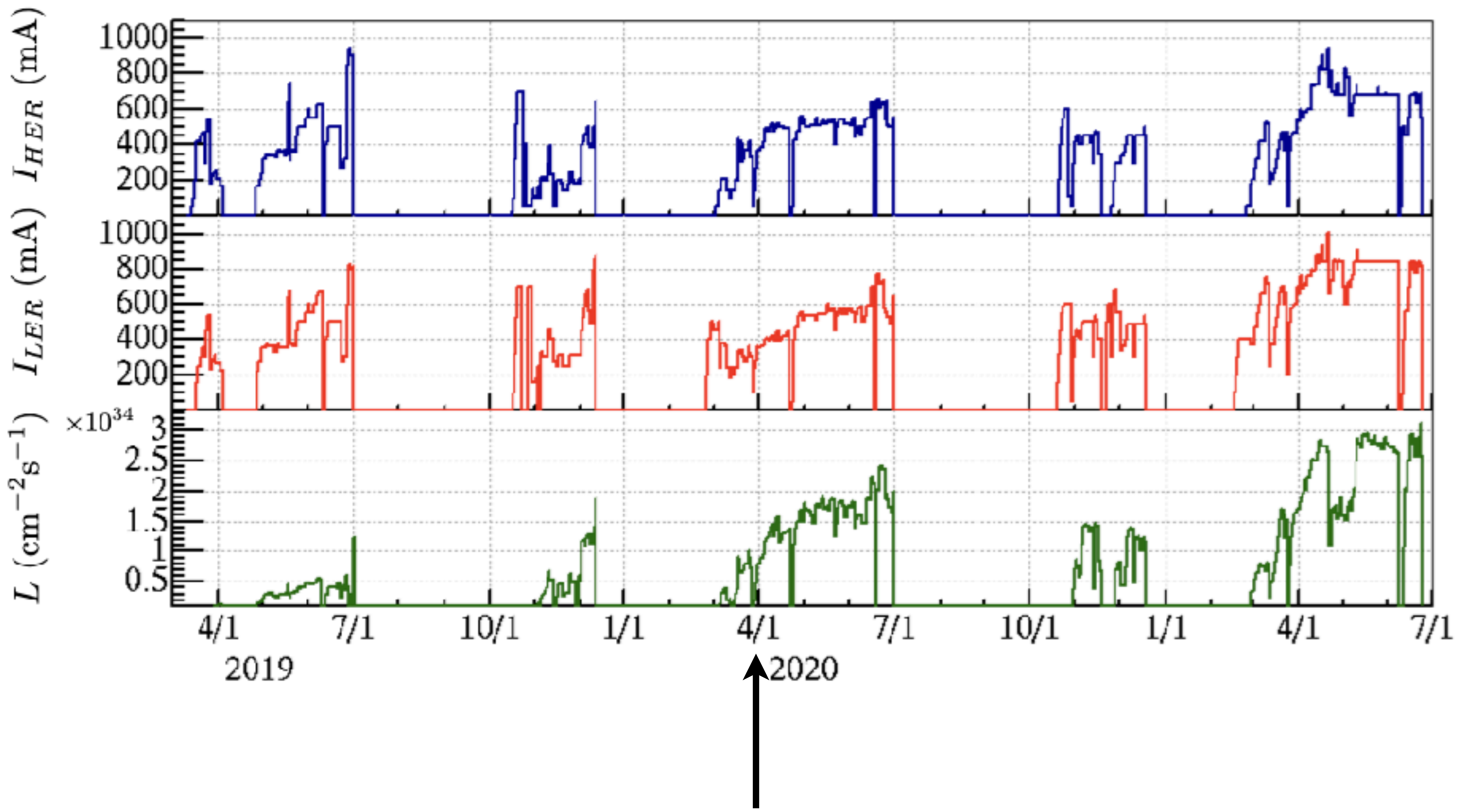
4

Aug. 27, 2021 K. Oide

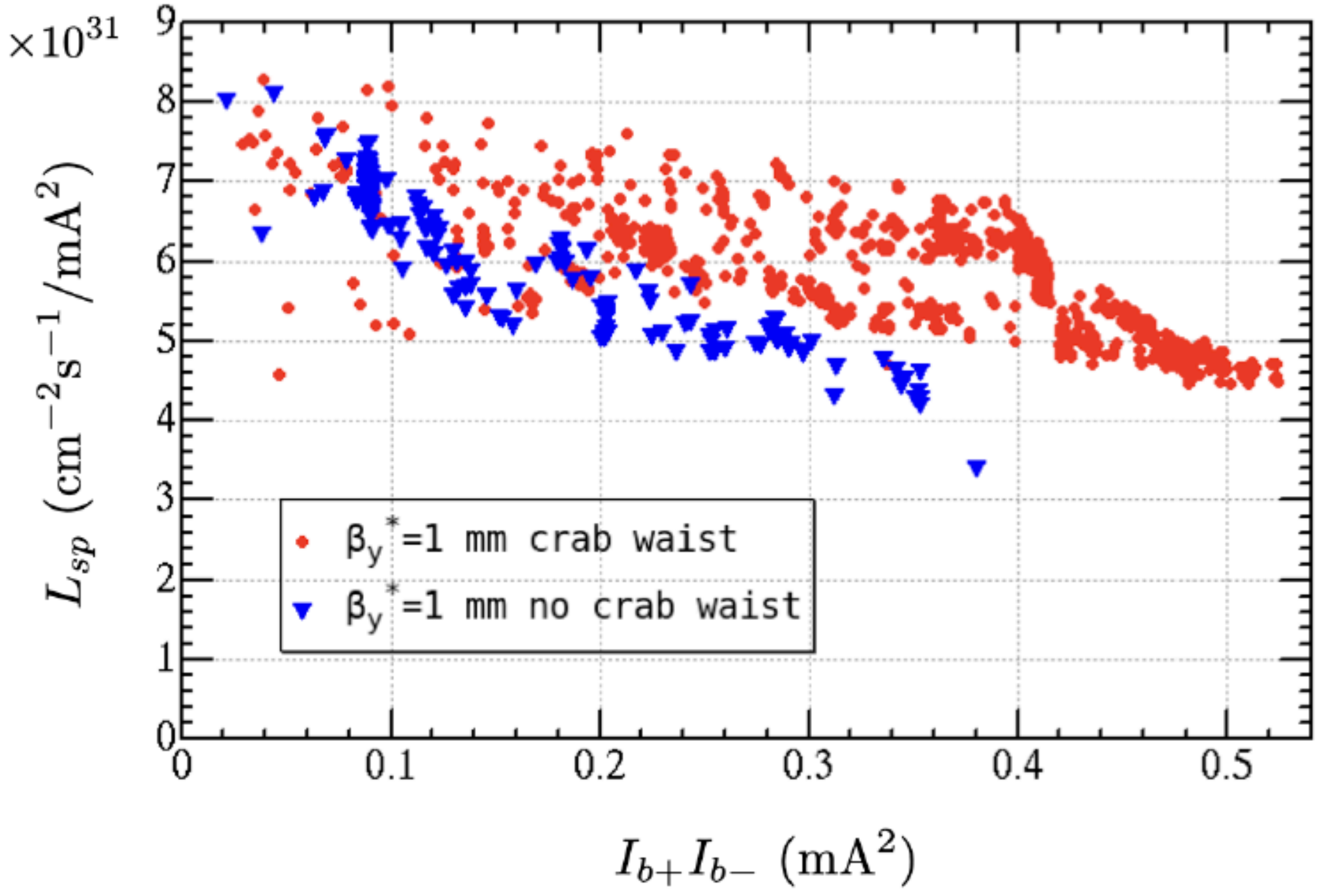
[1] K. Oide, SuperKEKB ARC, 2021.

Crab waist applied to SuperKEKB

- SuperKEKB beam operation with crab waist for $\beta_y^* = 1$ mm
  - Operation with CW has been successful [1].



Crab waist introduced since April 2020



[1] Y. Ohnishi, The European Physical Journal Plus volume 136, 1023 (2021).

Comparison of simulations and experiments

- Known sources of luminosity degradation
  - Bunch lengthening
  - Chromatic couplings
  - Single-beam blowup in LER (Impedance effects and its interplay with FB, see K. Ohmi's talk)
  - Optics distortion due to SR heating (see Y. Ohnishi's talk)
  - Luminosity "loss" correlated with injection.

} Identified in 2022

- Sources to be investigated via experiments
  - Imperfect crab waist
  - Beam-beam-driven synchro-betatron resonances
  - Interplay of BB, longitudinal and transverse impedances, and feedback system
  - Global couplings (side effects of IP knobs)
  - The interplay of BB and nonlinear lattices
  - Coupled bunch instabilities

BB simulations w/ final design configuration

- Findings [1]
    - K. Ohmi and K. Hirosawa developed a simple method to calculate the nonlinear terms. Good agreements were found with PTC results.
    - Then perturbation maps were made via MAP element in SAD to simulate luminosity loss. Finally, the term of $p_x^2 p_y$ was found to be important. Its sources were also well understood. Other chromatic terms can also be important in addition to chromatic couplings.
    - **Finally we arrived at a clear picture for the luminosity loss in beam-beam simulations (weak-strong model plus design lattice): The sources are beam-beam resonances and nonlinearity of the IR. But, the remedy is far from apparent.**

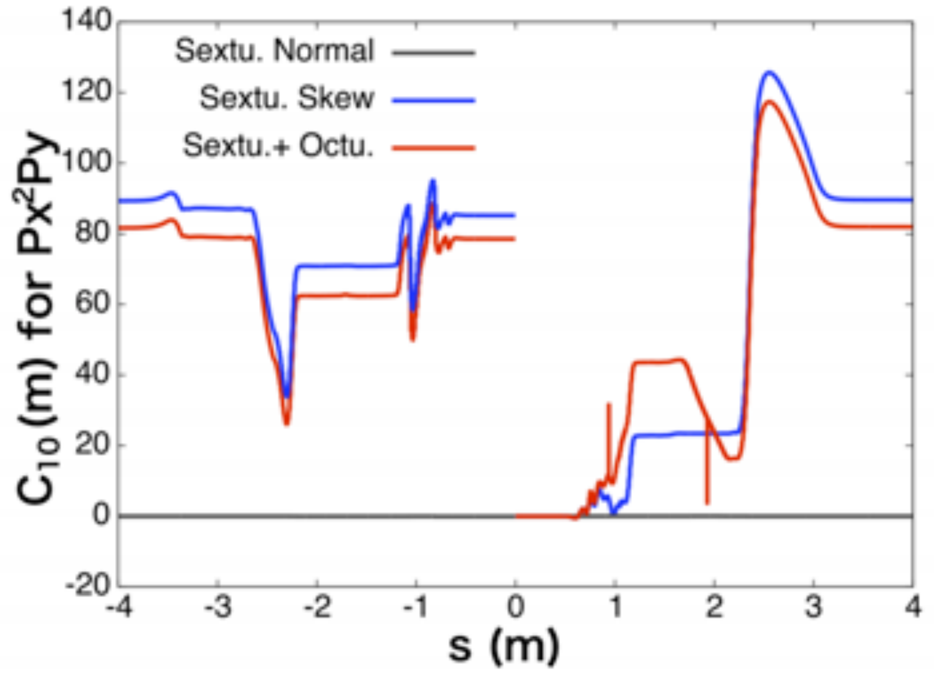


Figure 4: Coefficient of $P_X^2 P_Y$ caused by skew sextupole (SK_2) and octupole ($K_3 + SK_3$) fields.

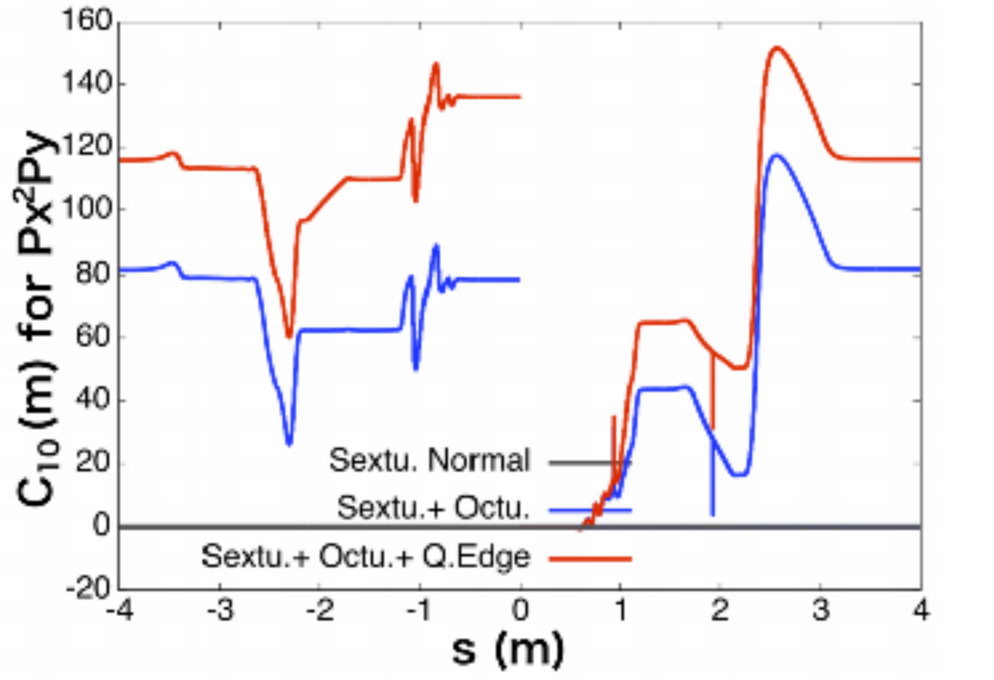


Figure 5: Coefficient of $P_X^2 P_Y$ for sextupole and octupole ($SK_2 + K_3 + SK_3$) and quadrupole hard-edge fringe ($SK_2 + K_3 + SK_3 + Q.edge$) fields.

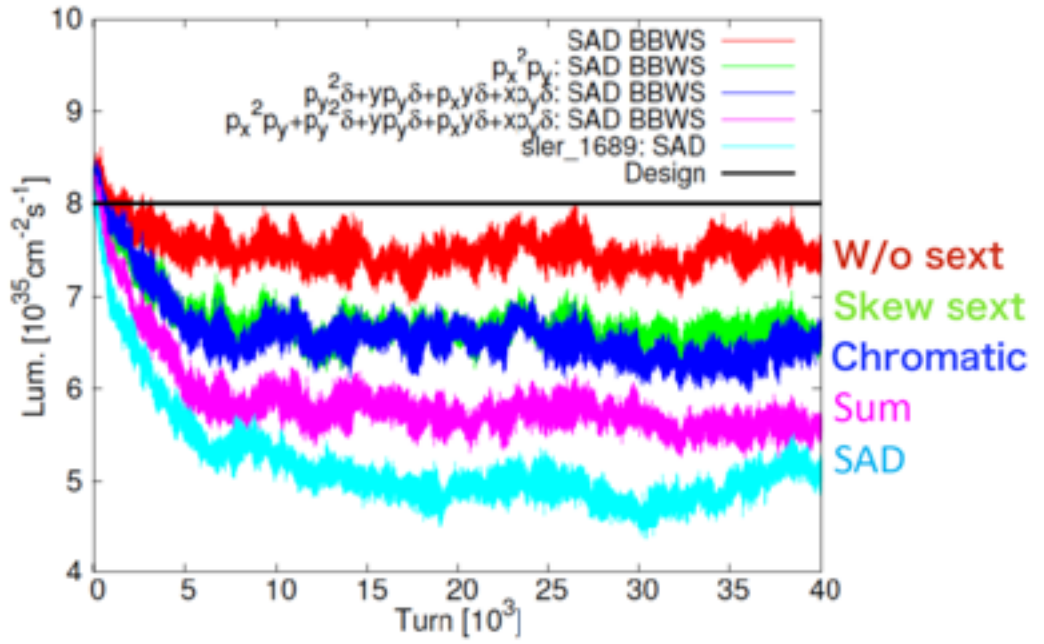
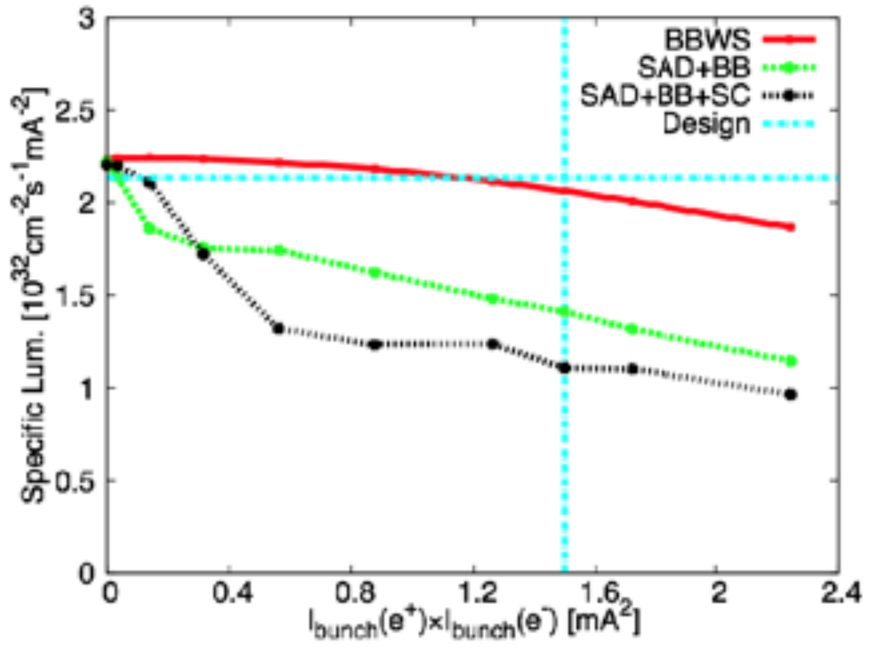
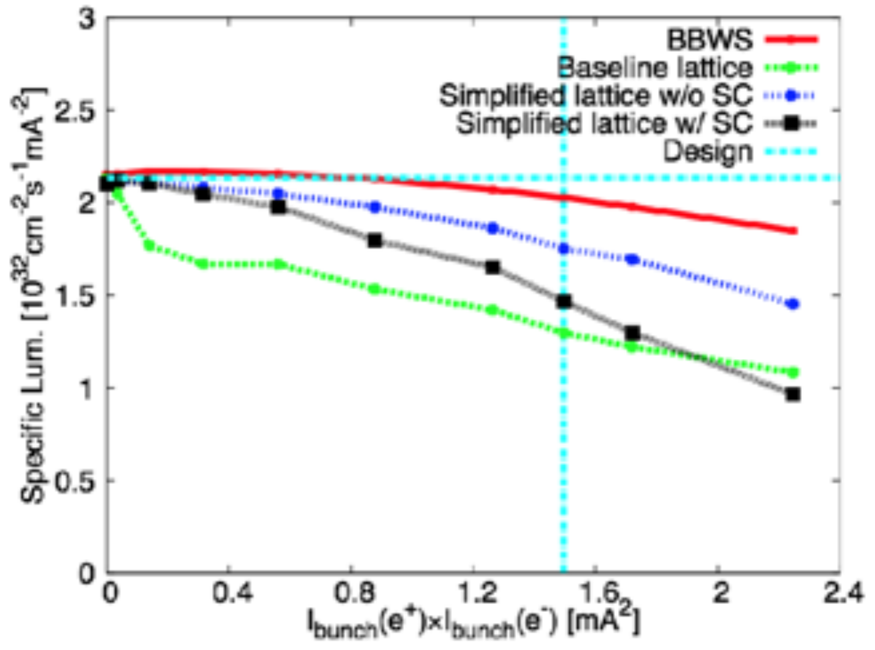


Figure 6: Luminosities for sextupole term (: $P_X^2 P_Y$), chromatic twiss, and SAD.





[1] K. Hirosawa et al., The influence of higher order multipoles of IR magnets on luminosity for SuperKEKB, in Proceedings of IPAC'18, Vancouver, BC, Canada, 2018.

Beam-beam viewpoints on achieving higher luminosity

- Assume balanced collision: $\beta^*_{y+} = \beta^*_{y-} = \beta^*_y$, $\epsilon_{y+} = \epsilon_{y-} = \epsilon_y$ and the hourglass effect is not strong, we can look into the formula of beam-beam parameter and discuss the challenges
- Note that we have to respect the constraints of real machines.

Beam-beam limit requires:
$\xi_y \leq 0.1$

To keep $\xi_y \leq 0.1$, higher currents requires smaller β^*_y

If we must accept $\xi_y \leq 0.1$, then smaller β^*_y is always preferred

$$\xi^i_{y\pm} \approx \frac{r_e}{2\pi e f_0 \gamma_\pm \tan\frac{\theta_c}{2}} \frac{I_{b\mp}}{\sigma_{z\mp}} \sqrt{\frac{\beta^*_y}{\epsilon_y}}$$

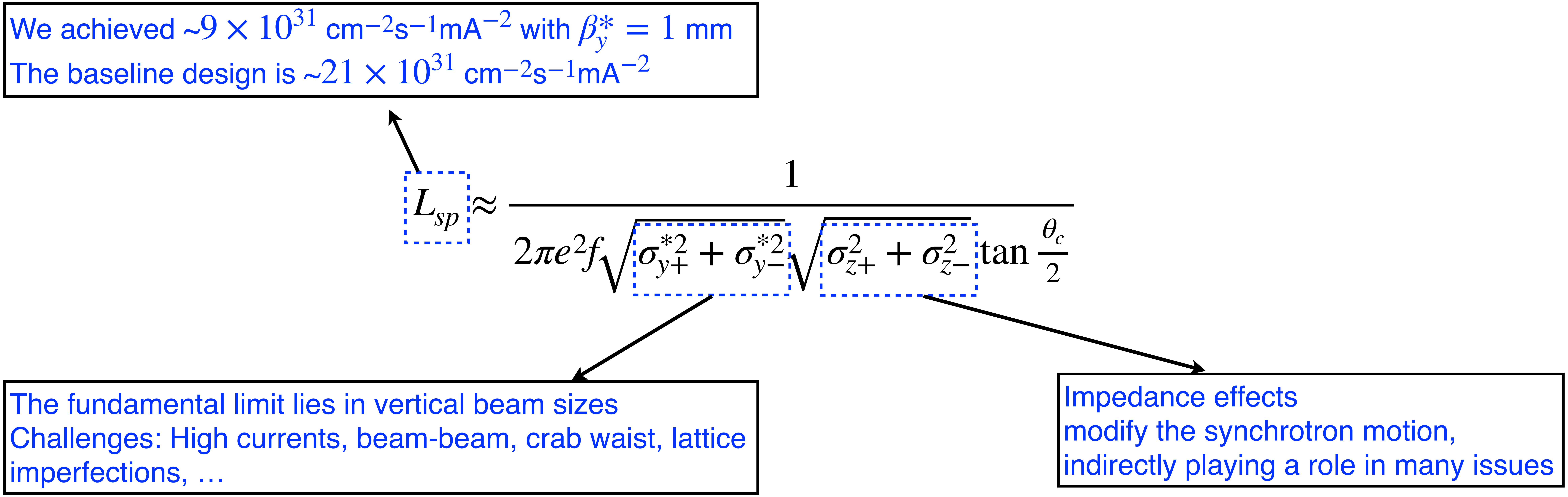
Crossing angle:
1) IR layout (constraints from optics design)
2) Reducing θ_c does not create a gain of luminosity if there the beam-beam limit exists.

Impedance effects
Longer σ_{z0} can be beneficial

We achieved $\epsilon_{y0} \approx 20$ pm
$\xi_y \leq 0.1$ sets a lower limit on the achievable ϵ_y (at a given β^*_y).
It is not feasible to achieve $\epsilon_y \approx 10$ pm at $\beta^*_y = 1$ mm

Beam-beam viewpoints on achieving higher luminosity

- Specific luminosity only depends on the geometric parameters (beam sizes and crossing angle).



We achieved ~9×10^{31} cm^{-2}s^{-1}mA^{-2} with $\beta_y^* = 1$ mm
The baseline design is ~21×10^{31} cm^{-2}s^{-1}mA^{-2}

$$\boxed{L_{sp}} \approx \frac{1}{2\pi e^2 f \sqrt{\boxed{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}} \sqrt{\boxed{\sigma_{z+}^2 + \sigma_{z-}^2}} \tan\frac{\theta_c}{2}}$$

The fundamental limit lies in vertical beam sizes
Challenges: High currents, beam-beam, crab waist, lattice imperfections, …

Impedance effects
modify the synchrotron motion,
indirectly playing a role in many issues